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A method for calculating active feedback system to provide vertical position control of plasma in a tokamak

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Abstract. In designing tokamaks, the maintenance of vertical stability of plasma is one of the most important problems. Systems of the passive and active feedbacks are applied for this purpose. Role of the passive system consisting of a vacuum vessel and passive coils is to suppress fast MHD (magnetohydrodynamic) instabilities. The active feedback system is applied to control slow motions of plasma.

The objective of the paper is to investigate two successive problems, solution of which allows to determine the possibility of controlling plasma motions. One of these is the problem of vertical stability under the assumption of ideal conductivity of plasma and passive stabilizing elements. The problem is solved analytically and on the basis of the obtained solution a criterion of MHD-stability is formulated. The other problem is connected with the control of plasma vertical position with active feedback system. Calculation of feedback control parameters is formulated as an optimization problem and an approximate method to solve the problem is suggested. Numerical simulations are performed with parameters of the T-15M tokamak in order to justify the suggested method.

Keywords. Tokamak; plasma; vertical stability; passive stabilizing structure; active feedback control; numerical simulation.

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1. Introduction

The control of plasma vertical motions is of special importance for tokamaks with divertor. Necessary condition for the control is the stability of plasma column under the assumption that plasma and passive stabilizing elements have ideal conductivity. If under this assumption the stability does not take place, the plasma deviates from equilibrium position on Alfvén time-scales ($\sim 10^{-6}$ s). This makes designing of an effective feedback control system impossible. The problem of plasma vertical stability within the frame of the model with ideal conductivity is investigated for a long time and is studied well [1–11]. Simple stability criteria, which are formulated in terms of coefficients of considered equations, are convenient in the analytical investigations of other problems of stabilization and control. Therefore, one of the purposes of the paper is to obtain a criterion of such kind. Section 2 describes the set of equations. In §3 the analytical solution of the problem is derived, on the basis of which a simple enough stability criterion is formulated.

The passive system, even if it is constructed successfully, can suppress only fast instabilities. Slow instabilities, development time of which is proportional to the characteristic decay time of the eddy currents in passive coils ($\sim 10^{-3}$ s), can be stabilized by an active feedback system. In §4 the stability problem for this case is considered and a necessary condition of vertical position control is obtained. In designing an active system, the problem of calculation of feedback control parameters arises, analysis of which is another purpose of this paper. The control parameters are determined by minimizing power requirement. In §5 the corresponding problem of optimization is formulated and the algorithm for its approximate solution is developed. Section 6 describes numerical calculations performed using T-15M parameters to justify the proposed algorithm. Finally, the conclusions are presented in §7.

2. General set of equations

The rigid-shift model will be applied to investigate plasma vertical stability. The model assumes that the entire plasma column moves in vertical direction as a solid body. The equilibrium distribution of the current in plasma is taken into account. But it is supposed that this distribution (therefore, the plasma total current I_{p}), and also the shape of the plasma remains unchanged as plasma moves. Thus, it is supposed that only the eddy currents, induced in the passive structure by the plasma motion, are changing. The vacuum vessel and passive stabilizer plates are modeled as sets of elementary toroidal coils. The finite conductivity of stabilizing elements is taken into account.

Mathematically, the model is described by plasma motion equation and circuit equations, and is represented by a system of linear differential equations. Unknowns are vertical displacement of plasma and eddy currents in passive conductors. The details of the model used are explained in [12,13].

We first consider the situation without active feedback. Let $\tilde{\xi}(t)$ be a small vertical displacement of the plasma column from a reference equilibrium position. Then, the equation of plasma motion is as follows:

$$M\frac{\mathrm{d}^2\tilde{\xi}}{\mathrm{d}t^2} = w_0\tilde{\xi}(t) + \sum_{j=1}^{N} w_j\tilde{I}_j(t). \tag{1}$$

Here M is the mass of the plasma, $w_0\tilde{\xi}$ represents the reverting Lorentz force on the moving plasma due to the external field gradient, N is the number of passive stabilizing coils, $\tilde{I}_j(t)$ is the eddy current in the jth passive coil and $w_j\tilde{I}_j$ represents the returning force on the plasma due to this eddy current.

The currents in passive coils are described by Kirchhoff's equations:

$$\sum_{j=1}^{N} L_{ij}\frac{\mathrm{d}\tilde{I}_j}{\mathrm{d}t} + R_i\tilde{I}_i(t) = -\Phi_i\frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}t} = -I_\mathrm{p}\frac{\mathrm{d}L_{ip}}{\mathrm{d}\tilde{\xi}}\frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}t}, \quad i = 1,\ldots,N, \tag{2}$$

where R_i is the ohmic resistance of the ith coil, L_{ii} and L_{ij} are the self and mutual inductances of the coils, $L_{ip}(\tilde{\xi})$ is the mutual inductance between ith coil and plasma. (The value of L_{ip} is calculated by taking the non-homogeneous distribution of plasma current into consideration.) It can be shown that, under the assumptions made, the relation $w_i = \Phi_i$ holds.

Let us represent solutions of the problems (1) and (2) in exponential form:

$$\tilde{\xi}(t) = \xi\,\mathrm{e}^{\gamma t}, \quad \tilde{I}_j(t) = I_j\,\mathrm{e}^{\gamma t}, \quad j = 1,\ldots,N. \tag{3}$$

Then the set of eqs (1) and (2), after dropping $\mathrm{e}^{\gamma t}$, is reduced to a generalized algebraic eigenvalue problem:

$$\begin{aligned} &\gamma\left\{\sum_{j=1}^{N} L_{ij}I_j + w_i\xi\right\} = -R_iI_i\,, \quad i = 1,\ldots,N,\\ &\gamma^2M\xi = \sum_{j=1}^{N} w_jI_j + w_0\xi. \end{aligned} \tag{4}$$

If we designate

$$\mathbf{I} = (I_1,\ldots,I_N)^T, \quad \mathbf{w} = (w_1,\ldots,w_N)^T,$$
$$L = \begin{bmatrix} L_{11} & \cdots & L_{1N}\\ \cdots & \cdots & \cdots\\ L_{N1} & \cdots & L_{NN}\end{bmatrix}, \; R = \begin{bmatrix} R_1 & \cdots & 0\\ \ldots & \ddots & \ldots\\ 0 & \cdots & R_N\end{bmatrix},$$

then problem (4) can be rewritten in the form:

$$R\mathbf{I} + \gamma\,L\mathbf{I} = -\gamma\,\mathbf{w}\xi, \quad \gamma^2M\xi = (\mathbf{w},\ \mathbf{I}) + w_0\xi, \tag{5}$$

where $(\mathbf{w}, \mathbf{I})$ denotes the scalar (inner or dot) product of vectors $\mathbf{w}$ and $\mathbf{I}$.

Solutions of the system (5), for some important cases, are given in the next sections.

3. Stability in the approach of ideal conductivity of stabilizing elements

Controlling plasma displacements is possible, only if the plasma is stable under the assumption of ideal conductivity of plasma and passive stabilizing elements. In this case $R = 0$. For $\gamma = 0$ the plasma is stable, and therefore this case is not interesting for us and in the following part we investigate the case $\gamma \neq 0$. Then the system (5) becomes

$$L\mathbf{I} = -\mathbf{w}\xi\,, \quad \gamma^2M\xi = (\mathbf{w},\ \mathbf{I}) + w_0\xi. \tag{6}$$

In particular, it can be found from these equations, that $w_0 = M\gamma_0^2$, where γ_0 is the Alfvén growth rate in the absence of stabilizing elements.

If we express $\mathbf{I}$ from the first equation of the system (6) and put it in the second equation, we obtain next formula for the growth rate γ in the ideal case:

$$\gamma_{\text{ideal}}^2 = \frac{w_0}{M} - \frac{1}{M}(L^{-1}\mathbf{w},\ \mathbf{w}). \tag{7}$$

The ideally conducting conductors stabilize plasma, if $\gamma_{\text{ideal}}^2 < 0$. Then eq. (7) leads to the following criterion of MHD-stability, or the necessary condition of stabilization, follows from (7):

$$(L^{-1}\mathbf{w},\ \mathbf{w}) > w_0, \tag{8}$$

which the system of passive conductors must satisfy. It is necessary to take into account this condition at all stages of design work.

4. Stabilization by applying active feedbacks

First of all let us note that the phrase 'stabilization of plasma vertical motion' means the same as 'control of plasma vertical position', if the rigid shift model is used.

We consider active feedback systems consisting of resistive toroidal coils. We suppose that at a given moment t, the displacement $\tilde{\xi}(t)$ of the plasma and its velocity $\tilde{\xi}'(t)$ can be measured. Using these measured values $\tilde{\xi}(t)$ and $\tilde{\xi}'(t)$, feedbacks produce a voltage $U_i(\tilde{\xi}(t), \tilde{\xi}'(t))$ in the ith active coil. Then the circuit equation (2) becomes

$$\sum_{j=1}^{N} L_{ij}\frac{\mathrm{d}\tilde{I}_j}{\mathrm{d}t} + R_i\tilde{I}_i(t) = -\Phi_i\frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}t} - U_i\left(\tilde{\xi}(t), \frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}t}\right).$$

The feedback can be linear or non-linear. In this paper the linear model is used. We assume that control coils are of two kinds: coils reacting to the displacement of the plasma and coils reacting to the velocity of displacement. Note that, in general, a single coil can be of both kinds at the same time. Let us assume that the feedbacks allow to generate voltage in the first case equal to $a_i\tilde{\xi}(t)$, and in the second case equal to $b_i\tilde{\xi}'(t)$:

$$\sum_{j=1}^{N} L_{ij}\frac{\mathrm{d}\tilde{I}_j}{\mathrm{d}t} + R_i\tilde{I}_i(t) = -\Phi_i\frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}t} - a_i\tilde{\xi}(t) - b_i\frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}t}. \tag{9}$$

Note that (9) is valid for all coils: both for active and passive, if it is accepted that $a_i = 0$ and $b_i = 0$ for passive coils.

We consider that positions of active coils are given. Then the problem of selection of active system leads to the determination of control parameters a_i and b_i, ensuring stabilization of plasma vertical motion.

If we represent solutions in the form (3), the problems (1) and (9) can be reduced to the following algebraic problem:

$$(R+\gamma\, L)\mathbf{I} = -\gamma\,(\mathbf{w}+\mathbf{b})\xi - \mathbf{a}\xi\,, \quad \gamma^2 M\xi = (\mathbf{w},\ \mathbf{I}) + w_0\xi\,, \tag{10}$$

where $\mathbf{a} = (a_1, \ldots, a_N)^T$, $\quad \mathbf{b} = (b_1, \ldots, b_N)^T$.

Hereinafter we assume that L is a positive definite symmetric matrix and $w_0 \geq 0$.

If we express $\mathbf{I}$ from the first equation of the system (10) and put in the second equation, we obtain

$$\gamma^2 M - w_0 + \gamma((R+\gamma\, L)^{-1}(\mathbf{w}+\mathbf{b}),\ \mathbf{w}) + ((R+\gamma\, L)^{-1}\mathbf{a},\ \mathbf{w}) = 0.$$

Using the formula $(A+B)^{-1} = A^{-1} - (A+B)^{-1}BA^{-1}$ we have:

$$\begin{aligned}\varphi(\gamma) = &\left[(R^{-1}\mathbf{w},\mathbf{a}) - w_0\right] \\ &+ \left[\gamma^2 M + \gamma((R+\gamma L)^{-1}\mathbf{w}, \mathbf{w}+\mathbf{b} - LR^{-1}\mathbf{a})\right] = 0.\end{aligned} \tag{11}$$

The root γ of the equation $\varphi\,(\gamma) = 0$ with the greatest real part determines the most unstable mode. Note that γ is a function of $\mathbf{a}$ and $\mathbf{b}$: $\gamma = \gamma(\mathbf{a},\ \mathbf{b})$.

Under the assumptions made the function $\varphi\,(\gamma)$ is continuous at $\gamma \geq 0$ and it is not difficult to see that $\varphi\,(+\infty) = +\infty$. Therefore, for the equation $\varphi\,(\gamma) = 0$ to have no positive root, the fulfillment of the following condition is necessary:

$$\varphi\,(0) = (R^{-1}\mathbf{w},\ \mathbf{a}) - w_0 \geq 0. \tag{12}$$

We will call (12) as the necessary condition of vertical position control by active feedbacks.

5. Problem of selection of active feedback system

Once the coordinates of active coils are defined, the problem of selection of the active feedback system consists of the following: Find values of control vectors $\mathbf{a}$ and $\mathbf{b}$ ensuring stabilization of plasma vertical motion and requiring minimal power consumption.

Let us calculate power requirement for realizing active feedbacks. The function $U_i = a_i\tilde{\xi}(t) + b_i\tilde{\xi}'(t)$, expressing feedback, has a meaning of voltage. According to the formula $P = U^2/R$, for total power necessary for realization of active feedbacks, we have

$$P(\mathbf{a},\ \mathbf{b}) = \sum \frac{a_i^2 + b_i^2\gamma^2(\mathbf{a},\ \mathbf{b})}{R_i}\,\xi_m^2,$$

where ξ_m is the parameter of the problem and expresses the maximum amplitude of displacements, for which the system is designed. Hereinafter summation is performed on active coils, though it can be accepted also as summation on all coils with allowance for $a_i = 0$ and $b_i = 0$ for passive coils.

The problem about selection of an active feedback system, in the general form, can be formulated as an optimization problem: Find values of parameters $\mathbf{a}$ and $\mathbf{b}$ satisfying the constraint $\gamma(\mathbf{a},\ \mathbf{b}) \leq 0$ and minimizing function $P(\mathbf{a},\ \mathbf{b})$.

In general case, it is difficult to find analytical expression for roots of eq. (11). Let us find an approximate estimation for the growth rate γ. If the necessary condition of stabilization is satisfied, i.e. if the plasma is stable in the ideal approach, second and higher powers of γ can be neglected. Then from the system (10) we have

$$\mathbf{I} = -(R + \gamma\, L)^{-1}(\gamma\, (\mathbf{w} + \mathbf{b}) + \mathbf{a})\xi\,, \quad -(\mathbf{I},\, \mathbf{w}) = w_0 \xi.$$

By eliminating $\mathbf{I}$ from these equations we obtain

$$((R + \gamma\, L)^{-1}\gamma(\mathbf{w} + \mathbf{b}),\, \mathbf{w}) + ((R + \gamma\, L)^{-1}\mathbf{a},\, \mathbf{w}) = w_0.$$

Using the approximate relation $(R + \gamma\, L)^{-1} \approx R^{-1} - \gamma R^{-1} L R^{-1}$ we have

$$(R^{-1}\gamma(\mathbf{w} + \mathbf{b}),\, \mathbf{w}) + ((R^{-1} - \gamma R^{-1} L R^{-1})\mathbf{a},\, \mathbf{w}) = w_0.$$

From here an approximate estimation for the growth rate is obtained:

$$\gamma = -\frac{(\mathbf{a},\, R^{-1}\mathbf{w}) - w_0}{(R^{-1}\mathbf{w},\, \mathbf{w}) + (\mathbf{b},\, R^{-1}\mathbf{w}) - (\mathbf{a},\, R^{-1} L R^{-1}\mathbf{w})}. \tag{13}$$

This relation can be obtained also from (11) using $(R+\gamma\, L)^{-1} \approx R^{-1} - \gamma R^{-1} L R^{-1}$ and neglecting second and higher powers of γ.

Let us note that w_0 and $s = (R^{-1}\mathbf{w},\, \mathbf{w}) > 0$ are determined by input data and consequently they can be considered as known parameters. For the active system to stabilize plasma, condition $\gamma(\mathbf{a},\, \mathbf{b}) \le 0$ must be satisfied. According to the necessary condition (12), the numerator of the fraction in expression (13) for γ must be non-negative. Therefore, for stability, it is necessary for the denominator to be positive. Thus, the parameters $\mathbf{a}$ and $\mathbf{b}$ must satisfy the restrictions

$$(\mathbf{a},\, R^{-1}\mathbf{w}) \ge w_0,$$
$$(\mathbf{b},\, R^{-1}\mathbf{w}) > (\mathbf{a},\, R^{-1} L R^{-1}\mathbf{w}) - s.$$

In order to prevent errors, concerned with the determination of input data, the fulfillment of these conditions with some reserves ε_1 and ε_2 ($\varepsilon_2 \ge \varepsilon_1$) will be required. That is, in expression (13) the numerator should not be less than ε_1, and the denominator should not be less than ε_2. Then the new restrictions are of form:

$$(\mathbf{a},\, R^{-1}\mathbf{w}) \ge w_0 + \varepsilon_1 = \hat{w}_0,$$
$$(\mathbf{b},\, R^{-1}\mathbf{w}) \ge (\mathbf{a},\, R^{-1} L R^{-1}\mathbf{w}) - (s - \varepsilon_2) = (\mathbf{a},\, R^{-1} L R^{-1}\mathbf{w}) - \check{s}.$$

In scalar products involving vector $\mathbf{a}$, actually only those components of vectors the numbers of which coincide with numbers of active coils of first kind are used. Therefore, it is convenient to use the corresponding projections of vectors. Let A be the subspace spanned by the basis vectors the numbers of which correspond to the number of the active coils of first kind. For example, if the active coils, reacting to displacement, are numbered with 1 to k, then $A = \text{Span}\,\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_k\}$, where $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_k$ are standard basis vectors. That is, A is subspace of vectors, for which only first k components can be non-zero. Let us denote $\mathbf{u} = \text{proj}_A R^{-1}\mathbf{w}$ and $\mathbf{v} = \text{proj}_A R^{-1} L R^{-1}\mathbf{w}$ (Note that $\text{proj}_A \mathbf{x}$ means the orthogonal projection of the vector $\mathbf{x}$ on the space A.) Then we have: $(\mathbf{a}, R^{-1}\mathbf{w}) = (\mathbf{a}, \mathbf{u})$ and $(\mathbf{a}, R^{-1} L R^{-1}\mathbf{w}) = (\mathbf{a}, \mathbf{v})$.

Similar things can be done for $\mathbf{b}$ also. We designate the corresponding subspace as B. If we denote $\mathbf{p} = \mathrm{proj}_B R^{-1}\mathbf{w}$, then $(\mathbf{b}, R^{-1}\mathbf{w}) = (\mathbf{b}, \mathbf{p})$.

Let us note that the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{p}$ are represented by input data of the problem and consequently we can consider them as known parameters.

Using new notations, we can reformulate the problem of selection of active feedbacks:

Find values of parameters $\mathbf{a}$ and $\mathbf{b}$, satisfying the restrictions

$$(\mathbf{a}, \mathbf{u}) \geq \hat{w}_0, \quad (\mathbf{b}, \mathbf{p}) \geq (\mathbf{a}, \mathbf{v}) - \check{s}, \tag{14}$$

and minimizing the function

$$P(\mathbf{a}, \mathbf{b}) = P_1(\mathbf{a}) + P_2(\mathbf{a}, \mathbf{b}) = \left\| \sqrt{R^{-1}}\, \mathbf{a} \right\|^2 \xi_m^2 + \gamma^2 \left\| \sqrt{R^{-1}}\, \mathbf{b} \right\|^2 \xi_m^2, \tag{15}$$

where $\gamma = -\frac{(\mathbf{a}, \mathbf{u}) - w_0}{s + (\mathbf{b}, \mathbf{p}) - (\mathbf{a}, \mathbf{v})}$. Above-mentioned w_0, $\hat{w}_0$, s, $\check{s}$, $\mathbf{u}$, $\mathbf{v}$, $\mathbf{p}$, ξ_m and R are known parameters.

In the following part we carry out the selection of a feedback system in two stages, sequentially solving two optimization problems. Actually it means a certain separation of roles between coils of the first and second kinds. At the first stage, by using coils of the first kind we minimize first addend P_1 in expression (15) under first restriction of (14). At the second stage we use coils of the second kind to satisfy second restriction and minimize addend P_2. (Total power required for the realization of the selected system is $P = P_1 + P_2$.)

The first problem: Find a value of $\mathbf{a}$, satisfying restriction $(\mathbf{a}, \mathbf{u}) = \hat{w}_0$ and minimizing the function $P_1(\mathbf{a}) = \xi_m^2 \sum (a_i^2 / R_i)$.

It is easy to see (for example, using Lagrange multipliers), that the solution of the given problem is the vector $\mathbf{a}^*$ with coordinates $a_i^* = (\hat{w}_0 / \sum R_i u_i^2) R_i u_i$. Thus $P_1(\mathbf{a}^*) = \xi_m^2 \hat{w}_0^2 / \sum R_i u_i^2$.

If the vector $\mathbf{a}^*$ also meets the condition $\check{s} - (\mathbf{a}^*, \mathbf{v}) \geq 0$, the pair of vectors $\mathbf{a}^*$ and $\mathbf{b} = \mathbf{0}$ satisfies both restrictions of the general problem (14) and (15) and, as can be easily checked, it is an optimal solution.

If the vector $\mathbf{a}^*$ does not satisfy the above-stated condition, and consequently, satisfies condition $\check{s} - (\mathbf{a}^*, \mathbf{v}) < 0$, the vector $\mathbf{b}$ can be selected by solving the problem, the formulation of which is given below. Let us designate $D = (\mathbf{a}^*, \mathbf{v}) - s > -\varepsilon_2$ and note that D is easily calculated and also can be considered as a known parameter. Let us note that $(\mathbf{a}^*, \mathbf{u}) - w_0 = \varepsilon_1$. Then $\gamma(\mathbf{a}^*, \mathbf{b}) = -\varepsilon_1 / [(\mathbf{b}, \mathbf{p}) - D]$.

The second problem: Find a value of $\mathbf{b}$, satisfying restriction $(\mathbf{b}, \mathbf{p}) = D + E$ (where $E \geq \varepsilon_2$ is a parameter of the problem) and minimizing the function $P_2(\mathbf{b}) = \xi_m^2 \sum (b_i^2 / R_i) \gamma^2$, where $\gamma = -\varepsilon_1 / E$.

This problem is similar to the first problem and its solution is the vector $\mathbf{b}^*$ with coordinates $b_i^* = ((D + E) / \sum R_i p_i^2) R_i p_i$. Let us note that $\mathbf{b}^*$ depends on the parameter E. It can be seen that $P_2(\mathbf{b}^*) = \xi_m^2 \left(1 + \frac{D}{E}\right)^2 \varepsilon_1^2 / \sum R_i p_i^2$. Therefore, an increase in E results in a decrease in the power required for feedbacks. However, an increase in E also results in an increase in the absolute values of feedback parameters b_i^*. The feedbacks are represented by products $b_i \tilde{\xi}'(t)$, second factor $(\tilde{\xi}'(t))$ of which is determined through measurements, so with some error. Therefore, the sharp increase in b_i^* can also lead to increase in the error of the right-hand side of eq. (9). Consequently, the operation of feedbacks can be disturbed. Therefore, in the selection of an optional value of E it is necessary to make a decision taking into consideration the power requirement and levels of measurement errors.

The pair of vectors $\mathbf{a}^*$ and $\mathbf{b}^*$ satisfies both restrictions of the general problem (14) and (15), thus, $\gamma(\mathbf{a}, \mathbf{b}) \leq 0$. Therefore, the active feedback system with parameters $\mathbf{a}^*$ and $\mathbf{b}^*$ provides stabilization of vertical motion. Let us note that this solution may not be optimum for the general problem. On the other hand, such feedback control means simple separation of roles of the first and second kind coils, which is preferable for technical implementation. Furthermore, it can be strictly shown that the indicated solution approximates the optimal solution with accuracy comparable with accuracy of used models. As it is known, most of the numerical methods for optimization problems require a good initial approximation, and the pair $(\mathbf{a}^*, \mathbf{b}^*)$ can be successfully used also for this purpose.

6. Numerical experiment

The goal of the numerical experiment is to overcome two problems:

– To determine the range of applicability of formula (13), which is the key point of the optimization method.

– To justify the suggested method for solving the optimization problem.

Two methods are used for the calculation of growth rate γ: the method based on solving the algebraic eigenvalue problem (10) and the method based on the approximate formula (13). The abbreviations AEP and F13 will hereinafter be used for them, respectively.

In simulations, the plasma with parameters of the tokamak T-15M project has been used [13]. The cross-section of the plasma, locations of passive structures and active feedback coils are represented in figure 1.

In this research, the active feedback system consisting of two coils with coordinates $(r_1, z_1) = (2.05\text{ m}, 0.80\text{ m})$, $(r_2, z_2) = (2.05\text{ m}, -0.60\text{ m})$ and cross-section 3 mm × 3 mm is considered. First of them is a coil of the first type (reacting on displacement of plasma), and second one is a coil of the second type (reacting on velocity of the plasma).

Results of the calculations based on the suggested method show the following values for feedback parameters: $a_1^* = 3.276 \cdot 10^4$ and $b_2^* = -1.374 \cdot 10^3$. Let us note that $\mathbf{a} = (a_1, 0, 0, \ldots, 0)$ and $\mathbf{b} = (0, b_2, 0, \ldots, 0)$. Using the mentioned values of $\mathbf{a}^*$and $\mathbf{b}^*$ we can examine behaviours of the growth rate γ and power P, required for active feedbacks, depending on $\mathbf{w}$ (figure 2 (left)), $\mathbf{a}$ (figure 2 (middle)) and $\mathbf{b}$ (figure 2 (right)).

For investigation of dependence on $\mathbf{w}$ the minor radius of the vacuum vessel is varied. We designate $d = a_V - a_{V0}$ as the deviation of minor radius from reference value. The locations (and width of cross-sections) of stabilizing plates and coils were changed (around or away from reference position) directly proportional to vessel minor radius. Let us note, that increase of d results in decrease of $\mathbf{w}$ and deterioration of stability. As seen from figure 2, values of γ, calculated by

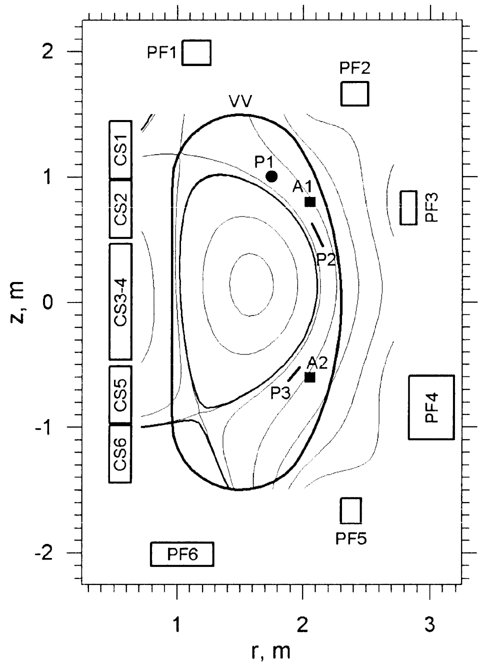

Figure 1. Cross-section of the plasma and locations of passive structure consisting of the vacuum vessel (VV) and passive stabilizers (P1, P2, P3). A1, A2 are active feedback coils, CS1–CS6 are central solenoid coils and PF1–PF6 are poloidal field coils.

different methods, are very close to each other because the values of γ are small and consequently the formula (13) can successfully be applied.

To investigate the dependence on $\mathbf{a}$ at fixed value $\mathbf{b}^*$, the value of $\mathbf{a}$ is varied according to expression $\mathbf{a} = (1 + \alpha/100)\mathbf{a}^*$, where $\mathbf{a}^*$ is the above-mentioned value and α expresses deviation of $\mathbf{a}$ from $\mathbf{a}^*$ in per cent. As seen from figure 2, by increasing $\mathbf{a}$ we could improve the stability. However, the required power increases.

For the appropriate application of formula (13), the numerator in this formula should not be less than ε_1, and the denominator should not be less than ε_2. So conditions (14) should be satisfied. Let us note that the dominant instability mode can be different from the mode expressed by (13) otherwise. In calculations, we took $\varepsilon_1 = 0.05w_0$ and $\varepsilon_2 = \varepsilon_1$. For the above-mentioned value $b_2^* = -1.374 \cdot 10^3$ the denominator in (13) is equal to ε_2, that is, $\mathbf{b}^*$ lays on the boundary of feasible region, determined by constraints (14).

To analyze dependence on $\mathbf{b}$, the value $\mathbf{a}^*$ is fixed and to vary values $\mathbf{b}$ the relation $\mathbf{b} = (1 + \beta/100)\mathbf{b}^*$ is used. As seen from figure 2, the decrease in $\mathbf{b}$ could improve stability and insignificantly reduce power requirement. However, as explained before, this circumstance can lead to the appearance of a new dominant unstable mode.

The second experiment has two main differences when compared to the first one:

– The optimal value of $\mathbf{b}$ is inside (not on boundary) the feasible region;

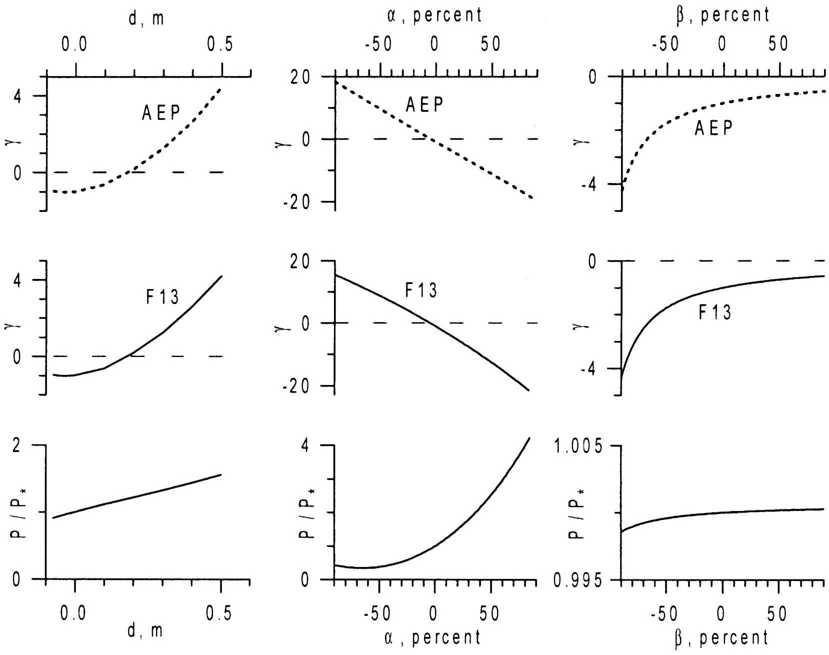

Figure 2. Behaviour of growth rate γ and power P depending on $\mathbf{w}$, $\mathbf{a}$ and $\mathbf{b}$. (1) Left: Dependences on d. Note that increasing d leads to decrease of $\mathbf{w}$. (2) Middle: Dependences on α. Here α is the deviation of $\mathbf{a}$ from $\mathbf{a}^*$. $\mathbf{a}$ is varied with α as $\mathbf{a} = (1 + \alpha/100)\mathbf{a}^*$. (3) Right: Dependences on β. Here β is the deviation of $\mathbf{b}$ from $\mathbf{b}^*$. $\mathbf{b}$ is varied with β as $\mathbf{b} = (1 + \beta/100)\mathbf{b}^*$. The continuous curves are obtained from formula (13), and dotted curves from solving the algebraic eigenvalue problem (10).

– The cases are considered, when the conditions to use formula (13) are not satisfied.

In this experiment the parameters of the first active coil are changed: width and thickness are increased 4 times; special resistance is reduced 4 times (it results in increase of resistive time $\tau_1 = L_{11}/R_1$ of the coil). The parameters of the second coil are not changed.

The calculation on the approximate optimization method results in $a_1^* = 5.136 \cdot 10^2$ and $b_2^* = -1.925 \cdot 10^3$ (corresponding voltages are 5.14 V and 19.25 V, if $\xi = 0.01$ m). Using these values the same calculations, as in the first example, are carried out (figure 3).

Let us discuss the conditions of applicability of formula (13). As seen from figure 3, in a broad range of values of parameters it gives close with AEP values of γ. However, at relatively large values of γ there are distinct behaviours. At $\alpha \geq 30$ a new mode becomes dominant. This mode also is stable, but has a growth rate expressed by a complex number (in figure 3 the real part of γ is presented at $\alpha \geq 30$). At $\beta \leq -70$ the dominant mode also has complex γ, but it becomes unstable as β decreases further.

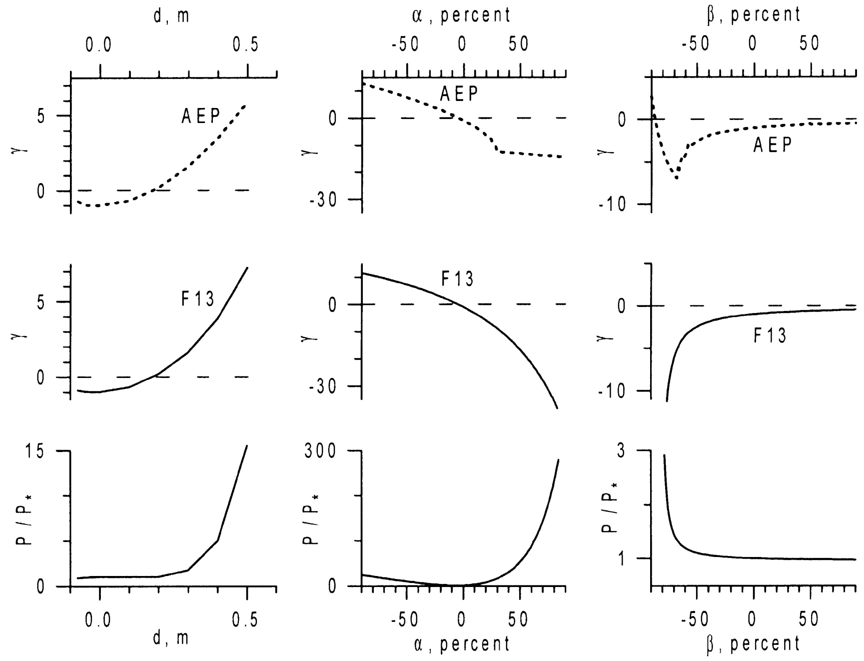

Figure 3. Same as in figure 2, but with changed parameters of the first active coil.

As seen from figure 3, the stability could be improved, if we increase **a** or decrease **b**. However, in both cases it is necessary to increase the power required. Thus it is justified that the obtained solution is really optimal.

So, the performed calculations show that formula (13), as well as the suggested method for calculation of active feedback parameters can be applied to investigate stabilization of plasma in tokamaks.

7. Conclusions

Two problems, emerging in the study of stabilization of plasma vertical motions in a tokamak, were considered. The results specified in the paper can be expressed in the following form.

On the basis of the rigid-shift model of the plasma, the vertical stability problem was analyzed. The problem was solved analytically under the assumption of ideal conductivity of plasma and passive stabilizing elements. The MHD stability criterion was obtained, which was formulated in terms of coefficients of the considered equations and expressed in a compact form. A system of passive conductors must satisfy this criterion, which is necessary for operating the active feedback system efficiently.

Control of vertical motions with application of an active feedback system was studied. Calculation of active feedback control parameters was formulated as an optimization problem and a method for its approximate solution was proposed. Numerical experiments have shown that the results of the proposed method comply with the theory.

The obtained results can be applied both in the design of the stabilization system of tokamaks and in theoretical studies.

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