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An alternative method to specify the degree of resonator stability

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Abstract. We present an alternative method to specify the stability of real stable resonators. We introduce the degree of optical stability or the S parameter, which specify the stability of resonators in a numerical scale ranging from 0 to 100%. The value of zero corresponds to marginally stable resonator and $S < 0$ corresponds to unstable resonator. Also, three definitions of the S parameter are provided: in terms of $A\&D$, $B\&Z_{\mathrm{R0}}$ and g_1g_2. It may be noticed from the present formalism that the maximum degree of stability with $S = 1$ automatically corresponds to $g_1g_2 = 1/2$. We also describe the method to measure the S parameter from the output beam characteristics and B parameter. A possible correlation between the S parameter and the misalignment tolerance is also discussed.

Keywords. Degree of optical stability; S parameter; misalignment tolerance.

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1. Introduction

The stability of a two-mirror resonator is generally expressed in terms of the g_1g_2 parameter. The condition for the existence of a Gaussian beam can be specified as $0 < g_1g_2 < 1$ (exception confocal) [1,2]. When more than two mirrors are used, the convenient method is to express the stability in terms of the round-trip $ABCD$ matrices. The condition of stability transforms to $-1 < (A + D)/2 < 1$ [2,3]. However, they specify only a range over which the resonator is stable. Steffen *et al* [4] reported that the effect of pump-induced thermal lens on the cavity could be minimized, if the value of g_1g_2 is set as $1/2$. As a result, it has been a design practice to have resonator with g_1g_2 of $1/2$ or closer to it. However, a definition of stability, which expresses the stability in a numerical scale ranging from 0 to 100% with 100% corresponding to g_1g_2 of $1/2$, would be highly preferred. With this motivation, we present an alternative method to specify the stability of stable optical resonators. It would be an added benefit, if a resonator with 100% stability also has larger misalignment tolerance, so that it would be easy to align such a resonator in practice.

The degree of optical stability or S parameter specifies the stability of resonators in a numerical scale ranging from 0 to 100%. The value of zero corresponds to marginally stable resonator and $S < 0$ corresponds to unstable resonator. We also present three definitions of S parameter: in terms of $A\&D$, in terms of $B\&Z_{\mathrm{R0}}$ and in terms of g_1g_2, where A, B and D are the matrix elements of the round trip $ABCD$ matrix of the resonator. It may be noticed from the present formalism that the maximum value of the degree of stability corresponding to $S = 100\%$, automatically corresponds to $g_1g_2 = 1/2$. We also describe an experimental technique to measure the S parameter from the laser output beam characteristics and B parameter. The measured value of S parameter is in close agreement with the predicted value for the same configuration. A possible correlation between the S parameter and the misalignment tolerance is also discussed.

2. Three definitions of *S* parameter

The degree of optical stability (DOS) or the S parameter may be expressed in three different forms, i.e.

1. in terms of A and D,
2. in terms of B and Z_{R0}
3. in terms of g_1g_2 parameters,

where A, B and D are the elements of the round-trip matrix $ABCD$, Z_{R0} is the free space Rayleigh range of the fundamental TEM_{00} mode inside the cavity and g_1 and g_2 are the usual g parameters. Even though they all refer to the same physical quantity, S, yet they convey different physical meaning.

2.1 *In terms of A and D* (*Use: Resonator optimization*)

The degree of optical stability, S, is defined as

$$S = 1 - \left[\frac{A(z) + D(z)}{2}\right]^2, \quad \text{with } 0 < S \leq 1 \tag{1}$$

where $A(z)$ and $D(z)$ are the elements of the round-trip $ABCD$ matrix starting from a reference plane located at z. We note that S is independent of the location of the reference plane inside the resonator, whereas $A(z)$ and $D(z)$ are both functions of z. This can be used to identify cavity configuration, which may maximize the S parameter. When the S parameter is different in the sagital and tangential directions, this formalism is extremely helpful to identify the optimum location. It may also be noticed that eq. (1) is very close to the 'm^2 parameter' defined as $m^2 = [(A + D)/2]^2$ with $0 \leq m^2 < 1$ in ref. [2]. It may be noticed that a resonator corresponding to the maximum value of the DOS would correspond to $m^2 = 0$ and marginally stable DOS would correspond to $m^2 = 1$. In practice, we would be interested to have the stability characterizing parameter to reach 100% at its best location.

2.2 *In terms of B and* Z_{R0} (*Use: Experimental measurement of the S parameter*)

Let us consider a resonator with end mirrors M_1 and M_2 with radius of curvature R_1 and R_2 respectively. Consider a reference frame with $z = 0$ located at M_1. Let us assume that $q_{\mathrm{R}}(z)$ is the reduced complex radius of curvature at a reference plane located at a distance z from M_1. The existence of a Gaussian beam inside a stable resonator requires the following condition (self-consistency condition) to be satisfied by $q_R(z)$ [2]:

$$\frac{1}{q_a(z)}, \frac{1}{q_b(z)} = \frac{D(z) - A(z)}{2B(z)} \pm j\frac{\sqrt{1-m^2}}{B(z)}, \tag{2}$$

where the m parameter is given by [2]

$$m = \frac{A(z) + D(z)}{2}. \tag{3}$$

For a real stable resonator, $-1 < m < 1$, the second term in eq. (2) in all situations is imaginary. Depending on the sign of B, which can either be a positive or a negative quantity, one or the other of these solutions of eq. (2) then corresponds to a confined Gaussian beam with a positive real value of spot-size. Using eqs (3) and (1), we may re-write eq. (2) as

$$\frac{1}{q_{\mathrm{R}}(z)} = \frac{D(z) - A(z)}{2B(z)} - j\frac{\sqrt{S}}{|B(z)|}, \tag{4}$$

where $B(z)$ is the element of the round-trip $ABCD$ matrix of the resonator. The absolute value of B is taken, because $\sqrt{S} > 0$ for $B > 0$ and $\sqrt{S} < 0$ for $B < 0$. The reduced complex radius of curvature $q_{\mathrm{R}}(z)$ may also be defined as [2]

$$\frac{1}{q_{\mathrm{R}}(z)} = \frac{n}{R(z)} - j\frac{\lambda_0}{\pi\omega^2(z)}, \tag{5}$$

where n is the refractive index of the medium, $R(z)$ is the radius of curvature and $\omega(z)$ is the spot-size of the fundamental Gaussian beam inside the resonator at the location z, λ_0 being the free space wavelength and $j = \sqrt{-1}$. The spot-size, $\omega(z)$, at the reference plane is given by

$$\omega^2(z) = \frac{\lambda_0}{\pi}\frac{|B(z)|}{\sqrt{S}}. \tag{6}$$

The significance of eq. (6) is that we may use it along with eq. (1) to estimate the spot-size at any reference plane z in a cavity. Or, if we know the spot-size, and the B parameter at any location, we can estimate the S parameter. Since, the radius of curvature is infinity at the waist of a resonator, the real part of eq. (5) must be zero at the waist location, say $z = z_0$, which may be solved by equating $A(z_0)$ and $D(z_0)$. If ω_0 is the waist size, then we may re-arrange eq. (6) to get

$$S = \left[\frac{B(z_0)}{Z_{\mathrm{R0}}}\right]^2, \tag{7}$$

where the Z_{R0} is the free space Rayleigh range in free space and is defined as

$$Z_{\mathrm{R0}} = \frac{\pi\omega_0^2}{\lambda_0}. \tag{8}$$

We note that the DOS is to be calculated using the free space Rayleigh range, even though, the waist may be formed in a medium with refractive index, n. In fact, the refractive index dependence is actually absorbed in $B(z_0)$. As pointed out earlier, we can use eq. (7) to find the stability from experimental data.

2.3 *In terms of* g_1g_2 (*Use: To compare with other resonators*)

We can express the g_1g_2 parameter in terms of $A(z)$ and $D(z)$ as [3]

$$G = 2g_1g_2 - 1 = \frac{[A(z) + D(z)]}{2}, \tag{9}$$

where g_i is the g parameter of the resonator defined as $g_i = 1 - L/R_i$, $i = 1, 2$ with L being the equivalent free space distance between the two mirrors and R_i being the radius of curvature of the ith mirror used. Now, the S parameter can be expressed as

$$S = 1 - (2g_1g_2 - 1)^2. \tag{10}$$

Hence, it is obvious that when $S = 1$, it automatically ensures that $g_1g_2 = 1/2$. Another application of eq. (10) is that we may also estimate the S parameter if we know the g_1g_2 parameter.

3. Measurement of S parameter

We present an experimental technique to measure the S parameter of a simple plano-concave resonator. The setup is based on a diode end-pumped Nd : YVO_4 laser based on a plano-concave cavity with the plane-mirror acting as the output coupler. The gain medium is kept very close (<0.5 mm) to a concave mirror with 50 mm radius of curvature. The pump-induced thermal lens [5–7], can significantly affect the cavity stability and beam parameters. Care has been taken to minimize the effect of pump-induced thermal lens on the cavity by ensuring that the thermal lens focal length is approximately 20 times larger than the focal length of the mirror itself. In a separate experiment, the thermal lens focal length was measured to be 1662 mm. Hence, the effective radius of curvature of the mirror (R_{eq}) was 48.54 mm. We measure the waist-size at the plane output coupler and the corresponding M^2 parameter as a function of the cavity length to determine the Rayleigh range

Z_{R0}. The S parameter may be predicted by using eq. (1) and estimated from the experimental data using eq. (7) as follows:

$$A = D = 1 - \frac{L_{eq}}{f_{eq}}, \tag{11a}$$

$$B = 2L_{eq} - \frac{L_{eq}^2}{f_{eq}}, \tag{11b}$$

where

$$\frac{1}{f_{eq}} = \frac{1}{R_m} + \frac{1}{F_{th}}, \tag{12}$$

where it is assumed that the pump-induced thermal lens of focal length, F_{th}, is situated close to the rear mirror of radius of curvature R_m.

3.1 *Experimental setup*

The schematic of the experimental setup is shown in figure 1. It consists of a diode end-pumped $Nd:YVO_4$-based IR laser at 1064 nm operating at an output power of ~90 mW. The gain medium was a 1 at% doped a-cut $Nd:YVO_4$ crystal (AR coated on both sides at 1064 nm) with 1 mm thickness end-pumped by a fiber coupled laser diode at 809 nm. The focused pump spot-size was 117 μm in the gain medium and the absorbed pump power was 250 mW. An RG850 cut-off filter was used to block the leaked 809 nm.

A simple technique was applied to measure the laser beam parameters (maybe referred to as 2-spot method for simplicity). To measure the beam parameters – multimode waist-size W_0, far-field divergence Θ_0 and M^2 – we first collimated the 1064 nm output using a 100 mm lens (L1) and then focused it again with another 100 mm lens (L2). The separation between the lenses was 200 mm. A knife-edge KE1 was kept at 100 mm distance from L1 and another knife-edge was kept at 100 mm from L2. The length of the cavity was varied and the spot-size measured using the knife-edges KE1 and KE2 are noted down. Care was taken to ensure that the relative distance between the output coupler, L3, KE1, L4 and KE2 remained constant during the entire experiment. The laser beam parameters may be determined in this case as follows:

$$W_0 = W_{20}, \tag{13a}$$

$$\Theta_0 = \frac{W_{10}}{F}, \tag{13b}$$

$$Z_{R0} = \frac{W_0}{\Theta_0} \tag{13c}$$

and

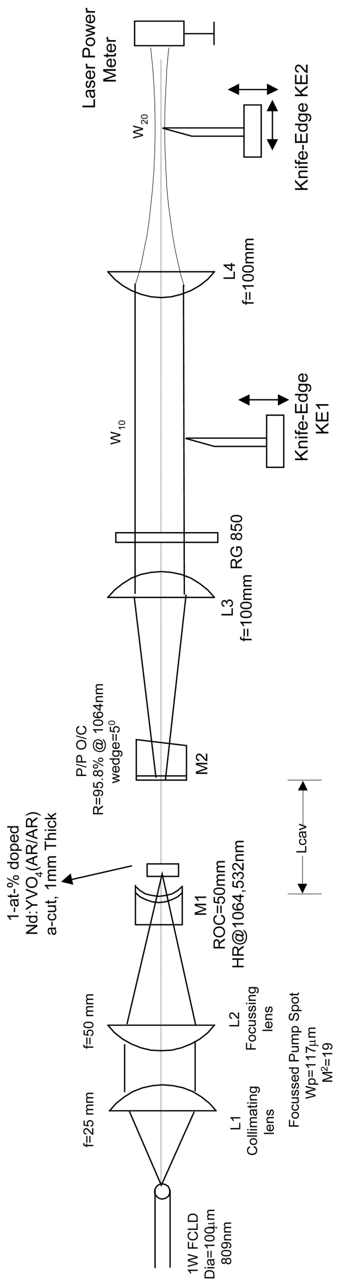

Figure 1. The experimental setup to measure the degree of stability of a plano-concave cavity.

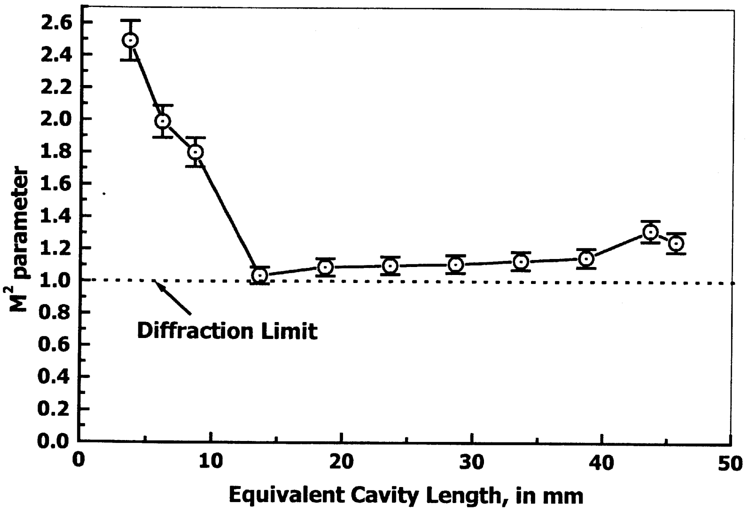

Figure 2. The variation of the beam quality factor, M^2, as a function of cavity length.

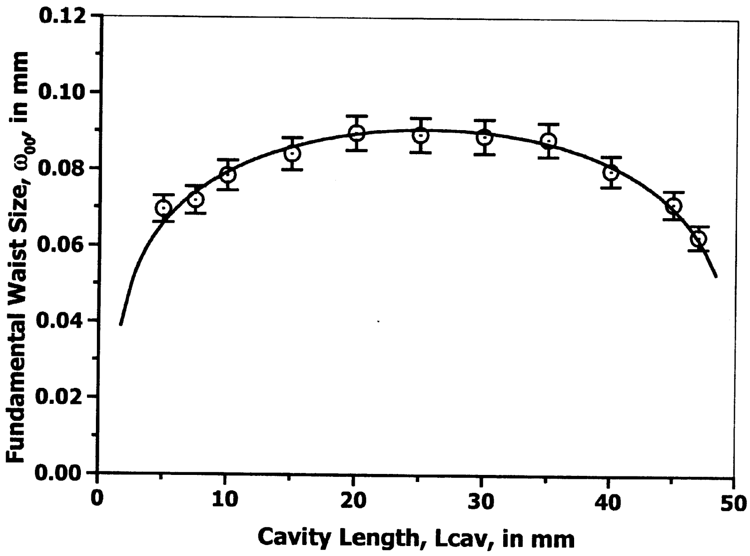

Figure 3. The variation of the fundamental cavity waist size, ω_0, with the cavity length, L_{cav} for a plano-concave cavity.

$$M^2 = \frac{\pi W_{10} W_{20}}{F \lambda_0}, \tag{13d}$$

where W_{10} and W_{20} are the spot-sizes measured by KE1 and KE2 respectively.

3.2 *Results and discussion*

Figure 2 shows the variation of the M^2 parameter as a function of the cavity length. Figure 3 shows the variation of the fundamental waist-size ω_0 as a function of the cavity length. To get the fundamental Gaussian waist size from the multimode value of W_0, it was divided by $\sqrt{M^2}$ (W_0 from eq. (13a) and M^2 from eq. (13d)). In order to support the accuracy of the measurement of the effective radius of curvature of 48.54 mm, we have also plotted the variation of the expected waist size using this value in figure 3. It may be observed that the agreement is very good. In fact, this correction to the radius of curvature ($\sim$3% only) was necessary to predict the S parameter accurately for cavity lengths approaching $L = R$ condition.

Figure 4 shows the predicted variation of the S parameter using eq. (1) and eq. (11a) for three different cases.

Case 1: Passive cavity. No thermal lens is present, so R_{eq} = ROC = 50 mm.
Case 2: Present cavity with negligible thermal lens: F_{th} = 1662 mm and ROC=50 mm.
Case 3: Strong thermal lens: ROC = F_{th} =50 mm.

The quadratic dependence of the S parameter as a function of the cavity length may be noted from figure 4. Also the S parameter differ significantly in Case 1 and Case 2, when the length of the resonator approaches the equivalent radius of curvature. The measured value of S parameter estimated using eqs (11b) and (13c) is also plotted in figure 4. It may be observed that the predicted and measured values are in good agreement.

4. A possible correlation between *S* parameter and misalignment tolerance

In order to check the possible correlation between the S parameter and the misalignment tolerance, we measured the misalignment tolerance for the above-mentioned cavity and correlated it with the S parameter variation.

We used a collimated He–Ne laser to measure the misalignment tolerance of the above-mentioned cavity as a function of the cavity length. In this method, the plane-output coupler was tilted away from the aligned maximum power location, and the angular tilt $\theta_{\text{mis},50\%}$ required to bring down the output power to half the maximum value was determined. Figure 5 shows the variation of $\theta_{\text{mis},50\%}$ as a function of the cavity length. It may be observed that the misalignment tolerance reduces for cavity lengths < 10 mm and > 40 mm, and it is nearly a constant within 10 to 40 mm range (within the error limit). It may be observed from figure 3 that the S parameter is more than 60% within the range of 10 to 40 mm. The S parameter also reduces to zero from 60%, when the length of the cavity is < 10 mm or > 40 mm. Hence, as a thumb rule, a cavity with S parameter greater than 60% is preferred to have better misalignment tolerance.

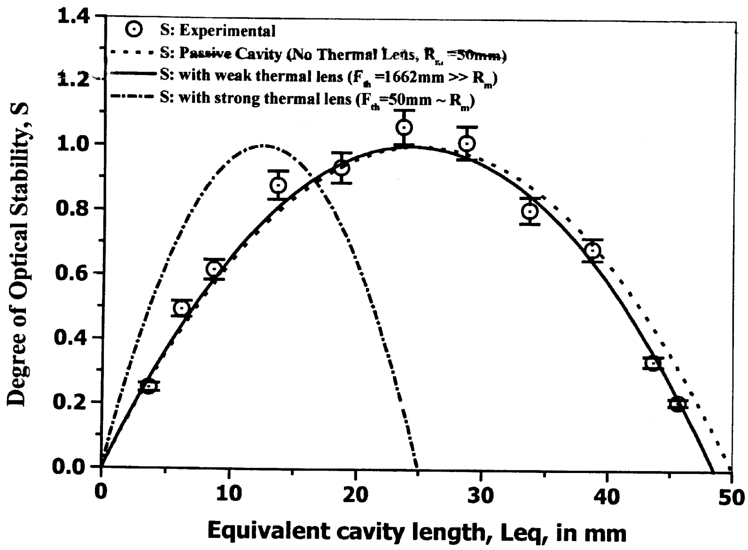

Figure 4. Variation of the degree of optical stability or the S parameter for a passive and active cavity. *Case* 1. The ROC of the mirror was 50 mm and the wavelength used was $\lambda_0 = 1064$ nm. *Case* 2. The active cavity S parameter is plotted in the weak thermal lens limit ($F_{\rm th} = 1662$ mm). *Case* 3. Strong thermal lens limit ($F_{\rm th} = R_{\rm m} = 50$ mm). The measured value of the S parameter is also plotted.

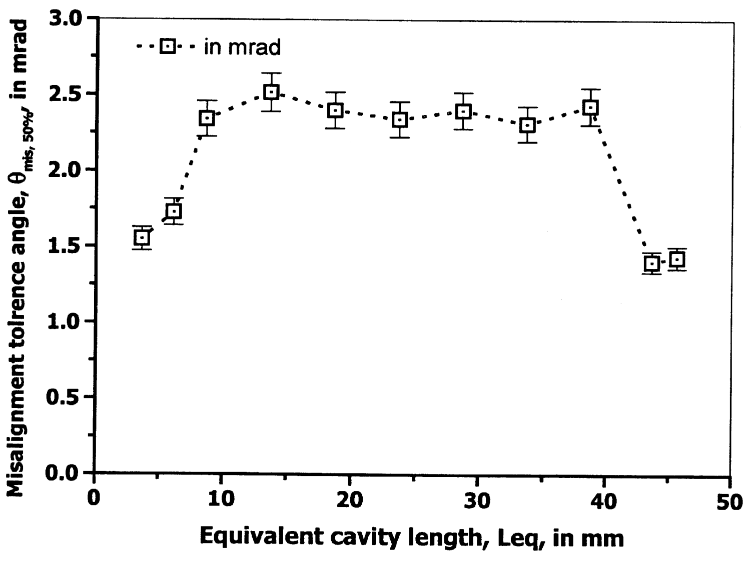

Figure 5. Variation of the misalignment tolerance measured as a function of the cavity length.

5. Conclusion

We report an alternative method to specify the stability of a resonator. The degree of optical stability or S parameter specify the stability in a numerical scale ranging from 0 to 100% with the maximum corresponding to $g_1 g_2$ of 1/2. The S parameter is expressed in three different forms and is given in eqs (1), (7) and (10). We also measured the value of the S parameter and compared it with the predicted value and the matching was found to be very good. We also infer from the measurement of misalignment tolerance that reduces significantly when the stability is closer to zero. Also, the misalignment tolerance is large when S lies in the range of 60% to 100%.

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