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Beta decay rates of nuclei with $65 < A < 75$ for pre-supernova and supernova evolution

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Abstract. The half-lives are calculated for the β^- decay process for nuclei in the mass range ∼65–75 relevant for the core of a massive star at the late burning stage of stellar evolution and the collapse that leads to supernova explosion. These half-lives and rates are calculated by expressing the β^- Gamow–Teller decay strengths in terms of smoothed bivariate strength densities. These strength densities are constructed in the framework of spectral averaging theory for two-body nuclear Hamiltonian in a large nuclear shell model space. The method has a natural extension to electron captures as well as weak interaction rates for r and rp-processes.

Keywords. Beta decay rates; supernova evolution; spectral distribution method.

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1. Introduction

The late evolution of massive stars ($>8M_\odot$) is strongly influenced by the weak interaction processes, both in the pre-collapse stage as well as during the gravitational collapse leading to the supernova explosion. These weak processes influence the value of Y_e, the electron fraction. As Chandrasekhar mass is proportional to Y_e^2 and the energy of the shock wave depends very sensitively on Y_e, determination of its value at each stage is crucial. The electron captures reduce the number of electrons for pressure support, β^- decays increase the same. Thus accurate calculations of the rates are very important for supernova model. Fuller *et al* [1] did the early work for the weak interaction rates at finite temperatures for nuclei with A up to 60. Later, Aufderheide *et al* considered the problem and listed the important nuclei with $40 < A < 90$ [2] for both electron capture and beta decay at different densities and temperatures. Detailed shell model calculations for the weak interaction rates for pf-shell nuclei up to $A = 65$ by Martinez-Pinedo *et al* [3] and their incorporation in pre-supernova evolution [4] have stressed the need for the calculation of such rates. In this work we have investigated the rates and half-lives for some of the relevant nuclei heavier than $A = 65$ for beta decays at

different densities and temperatures. Kar *et al* [5] also calculated the beta decay rates of nuclei with $A > 60$. But in this work we focus our attention on nuclei with $A > 65$ for which the large shell model-based rates are not available but which are listed in [2] to be important at different density/temperature conditions at the pre-supernova and collapse stages.

We construct the bivariate strength densities for β^- operators under the framework of spectral averaging theory [6]. The spectral averaging theory which is based on statistical nuclear spectroscopy, was started with Bethe's statistical mechanical level density formula, Wigner's treatment of spectral fluctuations using matrix ensembles and French's embedded ensembles and Gaussian densities. The smoothed forms of spectroscopic observables follow from the action of central limit theorems (CLT) in nuclear shell model spaces. The statistical spectroscopy involves deriving and applying the smoothed forms in indefinitely large spaces with *interactions* by using unitary group decompositions (of Hamiltonians and the spectroscopic spaces), CLT's locally, and convolutions – the resulting theory is the spectral averaging theory in large shell model spaces (SAT-LSS) [6–11]. Here, it is seen that the essential role of interactions is to produce local spreadings of the noninteracting particle (NIP) densities and the spreadings are in general Gaussian in nature. The spectral averaging theory in large shell model spaces has important nuclear physics applications like calculations of nuclear state and level densities, occupation and spin cut-off densities and calculations of occupation numbers and spin cut-off factors [10].

The theory has been extended to calculate the smoothed form for interacting particle (IP) bivariate transition strength densities ($\mathbf{I}^H_{\mathcal{O}}(E_\mathrm{i}, E_\mathrm{f})$) for a one plus two body nuclear Hamiltonian H and a transition operator $\mathcal{O}$. This bivariate strength density ($\mathbf{I}^H_{\mathcal{O}}(E_\mathrm{i}, E_\mathrm{f})$) takes a convolution form [8,12,13] with the noninteracting particle (NIP) strength densities being convoluted with a spreading bivariate Gaussian due to irreducible two-body part of the interaction. Mathematically, $\mathbf{I}^H_{\mathcal{O}} \Rightarrow \mathbf{I}^{\mathbf{h}}_{\mathcal{O}} \otimes \rho^{\mathbf{V}}_{\mathcal{O}}$, where $\mathbf{I}^{\mathbf{h}}$ is the strength density due to effective one-body part $\mathbf{h}$ of the interacting Hamiltonian and $\rho^{\mathbf{V}}_{\mathcal{O}}$ is a zero centered bivariate Gaussian due to the irreducible two-body (off-diagonal) part $\mathbf{V}$ of the interaction. In spectral averaging theory, the NIP part $\mathbf{I}^{\mathbf{h}}_{\mathcal{O}}$ are constructed by calculating a few lower order moments of $\mathbf{I}^{\mathbf{h}}_{\mathcal{O}}$ and explicit analytical formulae for NIP strength densities are worked out in [13]. This method, termed as moment method, is a convenient way for rapid construction of NIP strength densities. The IP bivariate strength densities thus constructed have been used to calculate β^- decay rates for some fp-shell nuclei around the mass range $55 < A < 65$ [14], giant dipole resonance (GDR) cross-sections [15] etc.

The smoothed form for IP strength densities for β^- Gamow–Teller transition operator can therefore be constructed using the formalism of spectral avearaging theory in a large shell model space. With these, the half-lives and rates for beta decay processes at finite temperature relevant for pre-supernova environment are calculated for the nuclei mentioned earlier.

The paper is organised as follows. In §2 we give the formalism. Section 3 deals with the calculational methods that include choice of shell model space, the single particle energies (s.p.e.) etc. and presents results. In §4 we conclude with some discussions and future outlook.

2. Formalism

2.1 *Unitary decompositon of nuclear Hamiltonian*

Given m number of particles distributed in a shell model space, we have an m-particle shell model space. Each shell model orbit or single particle orbit α with degeneracy $N_\alpha = 2j_\alpha + 1$ is called a spherical orbit. A group of spherical orbits is called a unitary orbit $\boldsymbol{\alpha}$. In what follows we will use α, β etc. to denote spherical orbits and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ etc. for unitary orbits. Thus we have for m number of particles distributed in this shell model space, spherical configuration $\mathbf{m} \equiv m_\alpha, m_\beta, ...$ (m_α, m_β etc. are the number of particles in spherical orbits α, β etc. respectively) and unitary configuration $[\mathbf{m}] \equiv m_{\boldsymbol{\alpha}}, m_{\boldsymbol{\beta}}, ...$ Using the convention that the spherical orbits α belongs to unitary orbit $\boldsymbol{\alpha}$ the number of single particle orbits in unitary orbit $\boldsymbol{\alpha}$ is $N_{\boldsymbol{\alpha}} = \sum_{\alpha \in \boldsymbol{\alpha}} N_\alpha$.

A further decomposition of m-particle space is possible by attaching an s_α label to each spherical orbit α where s_α for lighter nuclei denotes $s\hbar\omega$ excitation value. With this the m-particle space can be decomposed into S-subspaces as follows:

$$m \rightarrow \sum S^\pi; \quad S^\pi \rightarrow \sum [\mathbf{m}]; \quad [\mathbf{m}] \rightarrow \sum \mathbf{m}; \quad S = \sum m_\alpha s_\alpha. \tag{1}$$

One can recognise here the appearence of $U(N)$ group which is generated by the N^2 operators $a^\dagger_{j_\alpha m_\alpha} a_{j_\alpha m_\alpha}$; α, $\beta = 1, 2, ..., N$. For m identical particles therefore there are $\binom{N}{m}$ antisymmetric states forming an irreducible representation (irrep) for the group $U(N)$ usually denoted by Young shape $\{1^m\}$. The only scalar operator is the number operator n as it remains invariant under the transformation of the $U(N)$ group. A given operator can be decomposed into tensor operators (belonging to a definite irrep of $U(N)$) with respect to $U(N)$ group.

Thus the unitary group $U(N_\alpha)$ acting on each spherical orbit α generates m_α of spherical configurations $\mathbf{m}$, i.e. $\mathbf{m}$ behaves as $\{1^{m_\alpha}\} \otimes \{1^{m_\beta}\} \otimes ....$ with respect to the direct sum group $U(N_\alpha) \oplus U(N_\beta) \oplus ...$ and the scalar operators are m_α's. Therefore, for a given nuclear two-body Hamiltonian $H = h(1) + V(2)$, it should be obvious that the noninteracting particle part $h(1)$ of H is a scalar with respect to spherical configuration group and

$$h = \sum \epsilon_\alpha n_\alpha = h^{[0]}, \tag{2}$$

where ϵ_α is the single particle energy (s.p.e.) of the spherical orbit α. For the spherical configuration scalar part $V^{[0]}$ (must be a second-order polynomial in n_α) of residual interaction $V(2)$, one has

$$V^{[0]} = \sum_{\alpha \geq \beta} V_{\alpha\beta} \frac{n_\alpha (n_\beta - \delta_{\alpha\beta})}{(1 + \delta_{\alpha\beta})}, \tag{3}$$

where $V_{\alpha\beta}$ is the average two-body interaction given by

$$V_{\alpha\beta} = \{N_{\alpha\beta}\}^{-1} \{\sum_J (2J+1) V^J_{\alpha\beta\alpha\beta} (1 + \delta_{\alpha\beta})\}. \tag{4}$$

Here $N_{\alpha\beta} = N_\alpha(N_\beta - \delta_{\alpha\beta})$, $V^J_{\alpha\beta}$ is the two-body matrix elements with J the angular momentum. The remaining non-diagonal part $\mathbf{V} = V(2) - V^{[0]}$ of $V(2)$ is an irreducible two-body part as there cannot be an effective one-body part of $V(2)$ with respect to the spherical configuration group.

The same idea can be applied for the decomposition of Hamiltonian under unitary configuration group. In this case, the unitary group $U(N_{\boldsymbol{\alpha}})$ acting in each unitary orbit $\boldsymbol{\alpha}$ generates $m_{\boldsymbol{\alpha}}$ of unitary configuration $[\mathbf{m}]$; i.e. $[\mathbf{m}]$ behaves as $\{1^{m_{\boldsymbol{\alpha}}}\} \otimes \{1^{m_{\boldsymbol{\beta}}}\} \otimes ...$ with respect to the direct sum group $U(N_{\boldsymbol{\alpha}}) \oplus U(N_{\boldsymbol{\beta}}) \oplus ...$ and the scalar operators are $m_{\boldsymbol{\alpha}}$'s. The decomposition relevant for the present is the spherical scalar part $h^{[0]} + V^{[0]}$ of the Hamiltonian. They will have $[\mathbf{0}] \oplus [\mathbf{1}] \oplus [\mathbf{2}]$ tensorial parts with respect to unitary configuration direct sum group.

Thus, after the unitary decomposition of the nuclear Hamiltonian H we have

$$\begin{aligned} H &= h(1) + V(2) \\ &\Rightarrow h^{[0]} + V^{[0]} + \mathbf{V} \\ &\Rightarrow h^{[0][\mathbf{0}]} + h^{[0][\mathbf{1}]} + V^{[0][\mathbf{0}]} + V^{[0][\mathbf{1}]} + V^{[0][\mathbf{2}]} + \mathbf{V}. \end{aligned} \tag{5}$$

It has been demonstrated that the contribution of $V^{[0][\mathbf{2}]}$ part is small ($\leq 5\%$) all across the periodic table by calculating its norm for the case of ds, fp, 10-orbit and 15-orbit interactions and phenomenological interactions like surface delta interaction and pairing + QQ interactions [6,10]. Therefore, $V^{[0][\mathbf{2}]}$ can be negleced for all practical purposes.

Thus eq. (5) reduces to

$$\begin{aligned} H &= h^{[0][\mathbf{0}]} + h^{[0][\mathbf{1}]} + V^{[0][\mathbf{0}]} + V^{[0][\mathbf{1}]} + \mathbf{V} \\ &= \mathbf{h} + \mathbf{V}, \end{aligned} \tag{6}$$

where $\mathbf{h}$ is the effective one-body part of H and $\mathbf{V}$ is the irreducible two-body part.

For a unitary configuration $[\mathbf{m}]$, the unitary decompositions of nuclear Hamiltonian H are given by

$$\begin{aligned} h^{[0][\mathbf{0}]} &= \sum_{\boldsymbol{\alpha}} \epsilon_{\boldsymbol{\alpha}} n_{\boldsymbol{\alpha}}; \quad \epsilon_{\boldsymbol{\alpha}} = \left(\sum_{\alpha \in \boldsymbol{\alpha}} \epsilon_\alpha N_\alpha \right) N^{-1}_{\boldsymbol{\alpha}}, \\ h^{[0][\mathbf{1}]} &= \sum_\alpha \epsilon^{[\mathbf{1}]}_\alpha n_\alpha \ ; \ \epsilon^{[\mathbf{1}]}_\alpha = \epsilon_\alpha - \epsilon_{\boldsymbol{\alpha}}, \\ V^{[0][\mathbf{0}]} &= \sum_{\boldsymbol{\alpha} \geq \boldsymbol{\beta}} [V_{\boldsymbol{\alpha\beta}}] \frac{n_{\boldsymbol{\alpha}}(n_{\boldsymbol{\beta}} - \delta_{\boldsymbol{\alpha\beta}})}{(1 + \delta_{\boldsymbol{\alpha\beta}})}; \ V_{\boldsymbol{\alpha\beta}} = \left[\sum_{\alpha \in \boldsymbol{\alpha}, \beta \in \boldsymbol{\beta}} N_{\alpha\beta} V_{\alpha\beta} \right] [N_{\boldsymbol{\alpha\beta}}]^{-1}, \\ V^{[0][\mathbf{1}]} &= \sum_\alpha \left\{ \sum_{\boldsymbol{\beta}} (m_{\boldsymbol{\beta}} - \delta_{\boldsymbol{\alpha\beta}}) \left[\epsilon^{[\mathbf{1}];\boldsymbol{\beta}}_\alpha \right] \right\} n_\alpha, \\ \epsilon^{[\mathbf{1}];\boldsymbol{\beta}}_\alpha &= \left\{ \left[\sum_{\beta \in \boldsymbol{\beta}} (N_\beta - \delta_{\alpha\beta}) V_{\alpha\beta} \right] - (N_{\boldsymbol{\beta}} - \delta_{\boldsymbol{\alpha\beta}}) V_{\boldsymbol{\alpha\beta}} \right\} \\ &\quad \times \{ N_{\boldsymbol{\beta}} - 2\delta_{\boldsymbol{\alpha\beta}} \}^{-1}. \end{aligned} \tag{7}$$

In the above and for the rest of the calculations we consider S-conserving part (see discussions above eq. (1) for S quantum number) of the interaction. The S-mixing part $V_{S\text{-mix}}$ of $V(2)$ represents admixing between distant configurations (at least $2\hbar\omega$ away from each other) and this leads to multimodal form of densities [16] unlike the unimodal forms. Moreover, GT $\beta^{\pm}$ operator does not connect different S-subspaces. Hence, $V_{S\text{-mix}}$ is not considered for the rest of the calculations.

Given $H = \mathbf{h} + \mathbf{V}$ the IP strength density $\mathbf{I}_{\mathcal{O}}^{H=\mathbf{h}+\mathbf{V}}(E_{\rm i}, E_{\rm f})$ for a transition operator $\mathcal{O}$ will take a bivariate convolution form [12] with the two convoluting functions being NIP strength densities $I_{\mathcal{O}}^{\mathbf{h}}$ and a normalised spreading bivariate Gaussian $\rho_{\mathcal{O};\text{BIV-}\mathcal{G}}^{\mathbf{V}}$ due to $\mathbf{V}$ (interactions),

$$\mathbf{I}_{\mathcal{O}}^{H=\mathbf{h}+\mathbf{V}}(E_{\rm i}, E_{\rm f}) = I_{\mathcal{O}}^{\mathbf{h}} \otimes \rho_{\mathcal{O};\text{BIV-}\mathcal{G}}^{\mathbf{V}}[E_{\rm i}, E_{\rm f}]. \tag{8}$$

For our case, $\mathcal{O} \equiv \mathcal{O}(\text{GT})$. In large spectroscopic spaces with protons and neutrons (pn), this GT bivariate strength density can be partitioned in different unitary configuration subspaces and S-subspaces (eq. (1)) and can be written as (identifying that $\mathcal{O}(\text{GT})$ does not connect two different subspaces)

$$\mathbf{I}_{\mathcal{O}(\text{GT})}^{H=\mathbf{h}+\mathbf{V}}(E_{\rm i}, E_{\rm f}) = \sum_{S} \sum_{[\mathbf{m}_{\rm p}^{\rm i}, \mathbf{m}_{\rm n}^{\rm i}], [\mathbf{m}_{\rm p}^{\rm f}, \mathbf{m}_{\rm n}^{\rm f}] \in S}$$

$$\mathbf{I}_{\mathcal{O}(\text{GT})}^{\mathbf{h};[\mathbf{m}_{\rm p}^{\rm i}, \mathbf{m}_{\rm n}^{\rm i}], [\mathbf{m}_{\rm p}^{\rm f}, \mathbf{m}_{\rm n}^{\rm f}]} \otimes \rho^{\mathbf{V};[\mathbf{m}_{\rm p}^{\rm i}, \mathbf{m}_{\rm n}^{\rm i}], [\mathbf{m}_{\rm p}^{\rm f}, \mathbf{m}_{\rm n}^{\rm f}]}[E_{\rm i}, E_{\rm f}]. \tag{9}$$

In general, for a transition operator $\mathcal{O}$ and for a nuclear Hamiltonian H, the strength density $\mathbf{I}^{H;(m,m')}$ is given as

$$\begin{aligned} \mathbf{I}^{H;(m,m')} &= I^{m'}(E') |\langle E' m' | \mathcal{O} | E M \rangle|^2 I^m(E) \\ &= \langle\langle \mathcal{O}^\dagger \delta(H - E') \mathcal{O} \delta(H - E) \rangle\rangle^m, \end{aligned} \tag{10}$$

where $I^{m'}(E')$ and $I^m(E)$ are final and initial state densities and $\langle\langle\ \rangle\rangle^m$ represents a trace over the m-particle space. By the action of CLT in the spectroscopic space, the density will be a bivariate Gaussian and it is demonstrated in [10,14] by constructing the exact NIP strength densities (and its S-decomposition) for $\mathcal{O}(\text{GT})$ operator and then comparing this with the smoothed Gaussian form.

The smoothed form for strength density is constructed with marginal centroids ϵ_1, ϵ_2 and variances σ_1^2, σ_2^2 and calculating few lower-order central bivariate moments given by

$$\begin{aligned} \epsilon_1 &= \langle \mathcal{O}^\dagger \mathcal{O} H \rangle^m / \langle \mathcal{O}^\dagger \mathcal{O} \rangle^m \\ \epsilon_2 &= \langle \mathcal{O}^\dagger H \mathcal{O} \rangle^m / \langle \mathcal{O}^\dagger \mathcal{O} \rangle^m \\ \sigma_1^2 &= \langle \mathcal{O}^\dagger \mathcal{O} H^2 \rangle^m / \langle \mathcal{O}^\dagger \mathcal{O} \rangle^m \\ \sigma_2^2 &= \langle \mathcal{O}^\dagger H^2 \mathcal{O} \rangle^m / \langle \mathcal{O}^\dagger \mathcal{O} \rangle^m \\ \mu_{pq} &= \left\langle \mathcal{O}^\dagger \left(\frac{H - \epsilon_2}{\sigma_2} \right)^q \mathcal{O} \left(\frac{H - \epsilon_1}{\sigma_1} \right)^p \right\rangle^m \Big/ \langle \mathcal{O}^\dagger \mathcal{O} \rangle^m. \end{aligned} \tag{11}$$

Therefore, the calculation of moments are in fact calculation of m-particle averages or traces of the operator in question. This is done by first calculating a few basic traces in spaces with low particle number and then propagating them to the m-particle space. As here we are dealing with unitary configurations $[\mathbf{m}]$ and unitary configuration densities (eq. (9)), we would require the unitary configuration traces of the type $\langle\ \rangle^{[\mathbf{m}]}$. These moments $M_{pq}([\mathbf{m}])$ for the construction of NIP strength densities $\mathbf{I}^{\mathbf{h}}_{\mathcal{O}}$ for a one-body transition operator $\mathcal{O}$ with $p+q \leq 2$ are calculated in details in refs [10,14]. As the GT operator is of the type $\mathcal{O}(\mathrm{GT}) = \epsilon_{\alpha\beta} a^{\dagger}_{\alpha} a_{\beta}$ ($\epsilon_{\alpha\beta}$ is the single-particle matrix elements), for a given initial configuration $[\mathbf{m}_{\mathrm{i}}]$, the final configuration $[\mathbf{m}_{\mathrm{f}}]$ then is obtained uniquely as $[\mathbf{m}_{\mathrm{f}}] = [\mathbf{m}_{\mathrm{i}}] \times \left(1^{\dagger}_{\alpha} 1_{\beta}\right)$. Thus one also obtains the partial moments $M_{pq}([\mathbf{m}_{\mathrm{i}}], [\mathbf{m}_{\mathrm{f}}])$ for $\mathbf{I}^{\mathbf{h}}$. For a proton neutron (pn) configuration one writes $[\mathbf{m}]$ as $[\mathbf{m}_{\mathrm{p}},\mathbf{m}_{\mathrm{n}}]$.

For the construction Gaussian spreadings $\rho_{\mathcal{O}}^{\mathbf{V};[\mathbf{m}^{\mathrm{i}}_{\mathrm{p}},\mathbf{m}^{\mathrm{i}}_{\mathrm{n}}];[\mathbf{m}^{\mathrm{f}}_{\mathrm{p}},\mathbf{m}^{\mathrm{f}}_{\mathrm{n}}]}(\mathrm{GT})(x,y)$, in eq. (9), the following approximations are adopted. The marginal centroids, $M_{10} = \langle \mathcal{O}^{\dagger}\mathcal{O}\mathbf{V}\rangle^{m^{\mathrm{i}}_{\mathrm{p}},m^{\mathrm{i}}_{\mathrm{n}}} / \langle \mathcal{O}^{\dagger}\mathcal{O}\rangle^{m^{\mathrm{i}}_{\mathrm{p}},m^{\mathrm{i}}_{\mathrm{n}}} \simeq \langle \mathbf{V}\rangle^{m^{\mathrm{i}}_{\mathrm{p}},m^{\mathrm{i}}_{\mathrm{n}}} = 0$, as $\mathbf{V}$ is traceless; $M_{01} = \langle \mathcal{O}^{\dagger}\mathbf{V}\mathcal{O}\rangle^{m^{\mathrm{f}}_{\mathrm{p}},m^{\mathrm{f}}_{\mathrm{n}}} / \langle \mathcal{O}^{\dagger}\mathcal{O}\rangle^{m^{\mathrm{f}}_{\mathrm{p}},m^{\mathrm{f}}_{\mathrm{n}}} \simeq \langle \mathbf{V}\rangle^{m^{\mathrm{f}}_{\mathrm{p}},m^{\mathrm{f}}_{\mathrm{n}}} = 0$ and the marginal variances given by traces of the type $\langle \mathcal{O}^{\dagger}\mathcal{O}\mathbf{V}^2\rangle / \langle \mathcal{O}^{\dagger}\mathcal{O}\rangle$ and $\langle \mathcal{O}^{\dagger}\mathbf{V}^2\mathcal{O}\rangle / \langle \mathcal{O}^{\dagger}\mathcal{O}\rangle$ are equal to $\langle \mathbf{V}^2\rangle$.

The calculations are done with fixed neutron and proton number (m_{p}, m_{n}) spaces, i.e. with T_{iso_Z} specified rather than T_{iso}. However, it is found that the polarised GT strength averaged over all initial space do not change very much from the lowest T_{iso} ground state strength. This GT strength goes to three possible final state isospins, given by $T'_{\mathrm{iso}} = T_{\mathrm{iso}} - 1$, T_{iso} and $T_{\mathrm{iso}} + 1$. Again, it is seen that for neutron-rich nuclei with ground state isospin greater than 2, most of the polarised strength goes to the lowest final isospin, i.e. $T'_{\mathrm{iso}} = T_{\mathrm{iso}} - 1$ [17]. The relevant Clebsch–Gordon coefficient is responsible for this. Thus, calculations in fixed (m_{p}, m_{n}) spaces without explicit isospin projection is still a reasonable procedure.

2.2 *Formalism for β^- decay half-lives and electron capture rates at finite temperature and high density*

The β^- decay rate $T(E_{\mathrm{i}} \to E_{\mathrm{f}})$ is the number of weak processes per second from a given initial state $|E_{\mathrm{i}}\rangle$ of the parent nucleus to the final nuclear state $|E_{\mathrm{f}}\rangle$ and $T(E_{\mathrm{i}} \to E_{\mathrm{f}}) \propto [g^2_{\mathrm{V}} B_{\mathrm{F}}(E_{\mathrm{i}} \to E_{\mathrm{f}}) + g^2_{\mathrm{A}} B_{\mathrm{GT}}(E_{\mathrm{i}} \to E_{\mathrm{f}})]$, where g_{V} and g_{A} are respectively the vector and axial vector coupling constants and B_{F} and B_{GT} are the Fermi and Gamow–Teller transition strengths respectively. Including the phase space factor f that incorporates the dependence of the rate on nuclear charge Ze and the available energy for the weak process under consideration, T takes the form $T(E_{\mathrm{i}} \to E_{\mathrm{f}}) = Cf[g^2_{\mathrm{V}} B_{\mathrm{F}}(E_{\mathrm{i}} \to E_{\mathrm{f}}) + g^2_{\mathrm{A}} B_{\mathrm{GT}}(E_{\mathrm{i}} \to E_{\mathrm{f}})]$ where C is a constant ($= 6250$ s). With Q, the Q-value of the weak process β^- decay, from ground state (GS), one can write down the expressions for ground state half-lives and β^- decay rates at finite temperature. For the present beta decay calculations we have neglected Fermi term B_{F} as the Fermi strength is concentrated in a narrow domain and high up in energy (the centroid $\sim 1.44ZA^{-1/3}$ MeV, width $\sim 0.157ZA^{-1/3}$ MeV). Writing $B_{\mathrm{GT}}(J_{\mathrm{i}}E_{\mathrm{i}} \to J_{\mathrm{f}}E_{\mathrm{f}}) = (2J_{\mathrm{i}}+1)^{-1} \sum |\langle E_{\mathrm{f}} J_{\mathrm{f}} M_{\mathrm{f}}|(\mathcal{O}_{\mathrm{GT}})^k_{\mu}|E_{\mathrm{i}} J_{\mathrm{i}} M_{\mathrm{i}}\rangle^2$ which in continuous version becomes $\{I(E_{\mathrm{i}})\}^{-1} \sum_{\alpha\in E_{\mathrm{i}}, \beta\in E_{\mathrm{f}}, \mu} |\langle E_{\mathrm{f}}\alpha|(\mathcal{O}_{\mathrm{GT}})^k_{\mu}|E_{\mathrm{i}}\beta\rangle|^2$, the

expression for GS half-life is

$$t_{1/2}(\mathrm{GS}) = \{6250(s)\} \times \left\{ \int_0^Q \left[\left(\frac{g_\mathrm{A}}{g_\mathrm{V}} \right)^2 3\pounds \right] \left[\frac{\mathbf{I}^H_{\mathcal{O}(\mathrm{GT})}(E_\mathrm{GS}, E_\mathrm{f})}{I^H(E_\mathrm{GS})} \right] f(Z)\mathrm{d}E_\mathrm{f} \right\}^{-1}. \tag{12}$$

In the above equation $\pounds$ is the so-called quenching factor and the factor 3 comes because of the definition of $\mathbf{I}^h$. The usual values of $\pounds$ is 0.6 [18] for β^- decay and $(g_\mathrm{A}/g_\mathrm{V})^2 = 1.4$ as given in [19]. For the present calculations $\pounds = 0.5$. The β^- decay rate $\lambda(T)$ at a finite temperature T is given by

$$\begin{aligned} \lambda(T) &= \frac{\ln 2(s^{-1})}{6250} \left[\int \mathrm{e}^{-E_\mathrm{i}/k_\mathrm{B}T} I^H(E_\mathrm{i})\mathrm{d}E_\mathrm{i} \right]^{-1} \\ &\quad \times \left[\int \mathrm{d}E_\mathrm{i} \mathrm{e}^{-E_\mathrm{i}/k_\mathrm{B}T} I^H(E_\mathrm{i}) \left[\int_0^{Q_\mathrm{i}} \mathrm{d}E_\mathrm{f} \left\{ \left(\frac{g_\mathrm{A}}{g_\mathrm{V}} \right)^2 3\pounds \right\} \right. \right. \\ &\quad \left. \left. \times \left[\frac{I^H_{\mathcal{O}(\mathrm{GT})}(E_\mathrm{i}, E_\mathrm{f})}{I^H(E_\mathrm{i})} \right] f(Z,T) \right] \right] \\ &= \frac{\ln 2(s^{-1})}{6250} \left[\int \mathrm{e}^{-E_\mathrm{i}/k_\mathrm{B}T} I^H(E_\mathrm{i})\mathrm{d}E_\mathrm{i} \right]^{-1} \\ &\quad \times \left[\int \mathrm{d}E_\mathrm{i} \mathrm{e}^{-E_\mathrm{i}/k_\mathrm{B}T} \left[\int_0^{Q_\mathrm{i}} \mathrm{d}E_\mathrm{f} \left\{ \left(\frac{g_\mathrm{A}}{g_\mathrm{V}} \right)^2 3\pounds \right\} I^H_{\mathcal{O}(\mathrm{GT})}(E_\mathrm{i}, E_\mathrm{f}) f(Z,T) \right] \right]. \end{aligned} \tag{13}$$

For the case of β^- decay the phase-space factor reads as

$$f = \int_1^{\epsilon_0} \frac{F_\mathrm{c}(Z,\epsilon_\mathrm{e})\epsilon_\mathrm{e}(\epsilon_\mathrm{e}^2 - 1)^{1/2}(\epsilon_0 - \epsilon_\mathrm{e})^2}{\{1 + \exp[(\mu_\mathrm{e} - \epsilon_\mathrm{e})/(k_\mathrm{B}T)_\mathrm{e}]\}} \mathrm{d}\epsilon_\mathrm{e}. \tag{14}$$

In the above, k_B stands for Boltzmann constant, μ_e is the chemical potential for electron; $\epsilon_0 = E_0/m_\mathrm{e}$; $(E_0 = Q_\mathrm{i} - E_\mathrm{f})$, $\mu_\mathrm{e} = \mu/m_\mathrm{e}$ and $(k_\mathrm{B}T)_\mathrm{e} = k_\mathrm{B}T/m_\mathrm{e}$. $F_\mathrm{c}(Z,E)$ in the above equations is the Coulomb correction factor.

For β^- process the Coulomb factor is taken as given in Schenter and Vogel [20] and the expression reads as

$$\begin{aligned} F_\mathrm{c}(Z,\epsilon) &= \frac{\epsilon}{\sqrt{\epsilon^2 - 1}} \exp[\alpha(Z) + \beta(Z)\sqrt{\epsilon - 1}]; \\ \alpha(Z) &= -0.811 + 4.46(-2)Z + 1.08(-4)Z^2, \quad (\epsilon - 1) < 1.2 \\ &= -8.46(-2) + 2.48(-2)Z + 2.37(-4)Z^2, \quad (\epsilon - 1) \geq 1.2, \\ \beta(Z) &= 0.673 - 1.82(-2)Z + 6.38(-5)Z^2, \quad (\epsilon - 1) < 1.2 \\ &= 1.15(-2) + 3.58(-4)Z - 6.17(-5)Z^2, \quad (\epsilon - 1) \geq 1.2. \end{aligned} \tag{15}$$

For the electron chemical potential we have used the expression given in [21],

$$\mu_{\rm e} = 1.11(\rho_7 Y_{\rm e})^{1/3} \left[1 + \left(\frac{\pi}{1.11}\right)^2 \frac{\bar{T}^2}{(\rho_7 Y_{\rm e})^{2/3}}\right]^{-1/3}. \quad (16)$$

In the above, ρ_7 is the matter density ρ in units of 10^7 g/cc, $\bar{T}$ is the temperature T expressed in MeV and $Y_{\rm e}$ is the electron fraction. We take μ_ν, the neutrino chemical potential to be zero as for the densities we consider the neutrinos are freestreaming. In the calculation of half-lives and rates, low lying log ft's wherever known are incorporated with the total strength suitably adjusted. So the method uses the known experimental information to make the rates more realistic.

3. Calculational procedure

For the construction of strength densities we have selected a 9-orbit shell model space both for proton and neutron with ^{40}Ca as core, consisting of the spherical orbits $3f_{7/2}$, $3p_{3/2}$, $3f_{5/2}$, $3p_{1/2}$, $4g_{9/2}$, $4d_{5/2}$, $4g_{7/2}$, $4s_{1/2}$ and $4d_{3/2}$. The s.p.e. in MeV are 24.5 26.58, 26.19, 29.09, 33.91, 38.52, 42.47, 42.30, 43.15 respectively. The initial values of s.p.e.'s are taken from ref. [22]. The renormalisation effects due to the closed core (in this case s shell, p shell, ds shell) are then incorporated. This effect not only renormalises fp–sdg separation but also renormalises the single particle energies. They can be evaluated from $V_{\alpha\beta}$ and $V_{\boldsymbol{\alpha\beta}}$ discussed in §2.1. The unitary orbits are $\{3f_{7/2}, 3p_{3/2}, 3f_{5/2}, 3p_{1/2}\}$, $\{4g_{9/2}\}$, $\{4d_{5/2}, 4g_{7/2}, 4s_{1/2}, 4d_{3/2}\}$. Thus each of the proton and neutron shell model space has been divided into three unitary orbits. For the two-body residual interaction, we have used a phenomenological interaction, namely pairing + QQ interaction [23] with the strength $\chi = 242/A^{-5/3}$. Calculations for Hamiltonian with explicit isospin-dependent terms are being looked into.

We apply this formalism for half-lives and decay rates for 10 representative nuclei in the range $65 < A < 75$. Most of these nuclei appear among the top 70 nuclei for β^- decay as given in ref. [2].

Using the formalism given in §2, one can construct the weak interaction strength densities (bivariate Gaussian form). For the state densities $I^H(E)$, required for the calculation of half-lives or rates (eqs (12), (13)), we adopt the formula given in Dilg *et al* [24]. It has been demonstrated in [10,11] that state densities $I^H(E)$ obtained from spectral averaging theory represent very well the results obtained from Dilg *et al* state density formula. Besides the assumption of marginal centroids and variances (§2.1), we also assume the bivariate correlation coefficient ζ to be independent of configuration. Moreover, for two-body EGOE (embedded Gaussian orthogonal ensemble) [8], one has the result $\zeta \simeq 1 - 2/m$, where m is the number of active nucleons. Therefore, in the present calculation, ζ is first evaluated by treating it as a parameter with the EGOE form for $\zeta = a + b/m$ and a and b are evaluated by calculating β^- decay half-lives for 10 nuclei in the mass range considered and then comparing them with the experimental values of the same.

To this end, we fix a and b by calculating ζ for various values of a and b with the constraint that the value of ζ lies within 0.6 and 0.9. The value of bivariate variance (approximated as state density variance, $\langle \mathbf{V}^2 \rangle$ (§2.1)) is taken to be 15.5 MeV^2. This has been demonstrated in [10,11]. The values of a and b are found out

by minimising the quantity $\sum_{i=\text{nuclei}}(\log(\tau^i_{1/2})_{\text{cal}} - \log(\tau^i_{1/2})_{\text{expt}})^2$. The values of ζ and β^- half-lives for the nuclei considered are listed in table 1. One sees that the half-lives are reproduced quite well for almost all the nuclei.

Table 2 gives the beta decay rates for two typical nuclei of the ten considered, one even–even and the other an odd–A nucleus. The rates are shown for four typical temperatures 2–5 $\times 10^9$ K and for three densities 10^7, 10^8 and 10^9 g/cc. Rates for other intermediate values can easily be calculated. Similarly, the rates for other nuclei can also be calculated and supplied if needed. Two realistic values of Y_{e} are used for the calculations. These rates decrease with increasing density as the phase-space factor falls off fast with electron chemical potential increasing along with density. The rates also rise with higher temperature as more excited states start contributing when the temperature increases.

4. Discussions and conclusions

We have calculated the half-lives and rates for weak interactions for a number of nuclei in the range of $65 < A < 75$. These calculations are performed by explicitly constructing smoothed form for bivariate strength densities for the weak interaction operator (in this case Gamow–Teller operator) using convolution form within the framework of spectral averaging theory in nuclear physics. These calculations can easily be extended to other nuclei in this range as well as to heavier nuclei. The rates for heavier nuclei at much lower densities are useful for the r-process nucleosynthesis.

We have used here, the principles of spectral averaging theory for the calculation of beta decay rates for different temperatures, densities and electron fraction values in pre-supernova environment. Earlier, such calculations with spectral averaging theory have been performed for β^- decay rates for pre-supernova stars and for the middle fp-shell nuclei [25]. But the present method is well suited for handling nuclei with many particles spread over a large number of orbits and also can be used as a method for calculating half-lives more globally. The same method should be

Table 1. β^- decay half lives and comparison with experimental values.

Nucleus	Z	Q_{val} (MeV)	$T^{\text{expt.}}_{1/2}$	$T^{\text{calc}}_{1/2}$	ζ
^{66}Co	27	10.0	0.23	0.32	0.6627
^{67}Ni	28	3.385	21.0	26.91	0.6603
^{68}Co	27	9.30	0.18	0.20	0.6581
^{68}Ni	28	2.06	19.0	18.50	0.6581
^{68}Cu	29	4.46	31.1	31.23	0.6581
^{69}Co	27	9.30	0.27	0.36	0.6561
^{69}Ni	28	5.36	11.4	12.04	0.6561
^{71}Ni	28	6.90	1.86	1.62	0.6524
^{72}Cu	29	8.22	6.60	6.13	0.6507
^{74}Cu	29	9.99	1.59	0.40	0.6507

Table 2. β^- decay rates for densities and temperatures relevant for supernova core.

			Temperature in K			
			2×10^9	3×10^9	4×10^9	5×10^9
Nucleus	ρ (g/cc)	Y_e	Rates (s^{-1})			
^{69}Co	10^9	0.47	7.25×10^{-3}	1.18×10^{-1}	2.96×10^{-1}	3.63×10^{-1}
	10^8	0.47	1.64×10^{-2}	2.62×10^{-1}	6.37×10^{-1}	7.53×10^{-1}
	10^7	0.47	1.94×10^{-2}	3.05×10^{-1}	7.32×10^{-1}	8.55×10^{-1}
	10^9	0.45	7.47×10^{-3}	1.22×10^{-1}	3.05×10^{-1}	3.73×10^{-1}
	10^8	0.45	1.66×10^{-2}	2.64×10^{-1}	6.41×10^{-1}	7.57×10^{-1}
	10^7	0.45	1.94×10^{-2}	3.06×10^{-1}	7.33×10^{-1}	8.56×10^{-1}
^{68}Ni	10^9	0.47	8.38×10^{-6}	7.81×10^{-4}	6.00×10^{-3}	1.59×10^{-2}
	10^8	0.47	2.18×10^{-3}	4.38×10^{-2}	1.34×10^{-1}	1.93×10^{-1}
	10^7	0.47	4.55×10^{-3}	8.08×10^{-2}	2.20×10^{-1}	2.90×10^{-1}
	10^9	0.45	1.08×10^{-5}	9.18×10^{-4}	6.76×10^{-3}	1.75×10^{-2}
	10^8	0.45	2.25×10^{-3}	4.50×10^{-2}	1.36×10^{-1}	1.97×10^{-1}
	10^7	0.45	4.57×10^{-3}	8.11×10^{-2}	2.20×10^{-1}	2.91×10^{-1}

extended to the calculation of electron capture rates for nuclei in this region. That will be complimentary to evaluation of beta decay rates and will be equally useful for pre-supernova and collapse stages of the supernova evolution. Such calculations are in progress. We note that at higher densities and consequently at higher A, as the electron chemical potential increases, beta decays in the supernova case get blocked and eventually do not take place. So the range of nuclei for which electron capture rates need to be calculated are wider than that for the beta decay case studied. So we shall report them separately in future.

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