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On relativistic models of strange stars

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Abstract. The superdense stars with mass-to-size ratio exceeding 0.3 are expected to be made of strange matter. Assuming that the 3-space of the interior space–time of a strange star is that of a three-paraboloid immersed in a four-dimensional Euclidean space, we obtain a two-parameter family of their physically viable relativistic models. This *ansatz* determines density distribution of the interior self-gravitating matter up to one unknown parameter. The Einstein's field equations determine the fluid pressure and the remaining geometrical variables. The information about mass-to-size ratio together with the conventional boundary conditions lead to the determination of total mass, radius and other parameters of the stellar configuration.

Keywords. Relativistic stellar models; superdense stars; strange stars; equation of state; general relativity.

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1. Introduction

The ground state of hadronic matter is the state in which quarks are confined in individual hadrons. This state, though very long lived, is not a naturally stable one. It is believed [1,2] that in the natural ground state, matter may be in a de-confined state of quark, which is called strange matter. A universal process in which the confined state could slowly evolve into strange matter is through the formation of neutron stars with dense cores in which phase transition to quark matter is likely to occur. If the strange matter is true ground state, the entire star may get converted into a strange matter star. The strange matter is the most likely candidate for the interior of any fast rotating pulsar compared to a hardonic matter of a neutron star. Accordingly, formulation of reliable criteria to distinguish a strange star from other superdense stars such as a neutron star is highly desirable. Apart from the information about density profile and equation of state (EOS) of interior matter, the mass–radius relationship of the object provides a vital clue to distinguish different superdense stars – white dwarf, neutron stars and ultracompact stars – from one another. Recently stellar objects such as SAX, Her. X-1 and other low-mass X-ray binaries (LXMB) have been differently interpreted as neutron stars and strange stars [3,4]. We observe that the information about density profile and mass-size relationship for such objects helps conclusively in determining their types as well. Like their neutron star counterparts, the matter content of the strange stars is believed to be extremely hot at the time of their formation in the core of supernova explosion which cools down rapidly in an exotic manner to extremely cold state in thermal equilibrium with the surrounding exterior (close to vacuum, void).

In the conventional approach, the relativistic models of superdense stars in equilibrium are obtained by assuming that their interior is filled with cold catalytic matter at a temperature of zero Kelvin with *a priori* furnished equation of state. The exterior of these stellar structures is vacuum, devoid of radiation of thermal or any other origin. The Einstein's field equations relate the dynamical variables such as the energy density and fluid pressure of the matter with the interior geometry. The non-linearity of the equations of the highly complex hydrodynamical system is analysed by adopting numerical procedures for obtaining their plausible solutions. Vaidya–Tikekar [5] and Tikekar [6] showed that in the absence of reliable information about equation of state of matter content, the alternative approach of prescribing suitable geometry for their 3-spaces leads to physically viable, easily tractable models of superdense stars in equilibrium. Several aspects of physical relevance of compact star models, based on Vaidya–Tikekar *ansatz*, have been investigated [7–10] by a number of workers. Mukherjee *et al* [11–13] indicated the possibility of using this set-up to describe models of the compact star like Her. X-1 and observed that the equation of state of the ReSS (realistic equation of state of strange star) approximated to linear form is derivable from models based on Vaidya–Tikekar *ansatz*. We have shown elsewhere [14] that the alternative *ansatz* suggested by Tikekar and Thomas [15] also has these features and the general three-parameter solution based on it also leads to physically plausible relativistic models of strange stars.

We have considered here the *ansatz* used by Finch and Skea [16] leading to a two-parameter family of physically viable relativistic models of neutron stars and shown that it also admits possibilities of describing strange stars like Her. X-1 and other highly compact configurations of matter in equilibrium. In §2 we have discussed the geometric aspects of the *ansatz* and the general physical features of the fluid distributions in equilibrium in view of the Einstein's field equations on the associated background space–time. The two-parameter family of the solution of the relativistic field equations is described in convenient notation in §3. In §4, we will discuss the family of relativistic models of compact stars based on the two-parameter solution of §3. The physical viability of strange star models admitted in the set-up with m/a greater than 0.3 is established using numerical procedures. The usefulness of the $m \sim a$ diagram for compact stars in view of the available estimate for observed configurations is also analysed.

2. Fluid distributions on paraboloidal space–times

The metric of the interior space–time of the star models based on the *ansatz* discussed by Finch and Skea will be expressed in the form

$$ds^2 = e^{\nu(r)}\, c^2\, dt^2 - \left(1 + \frac{r^2}{R^2}\right) dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{1}$$

This facilitates the comparison of the models with those based on the Vaidya–Tikekar *ansatz*. The 3-space of this space–time obtained at t = constant section has the geometry of the 3-paraboloid with Cartesian equation $x^2 + y^2 + z^2 = 2wR$, immersed in four-dimensional Euclidean space which is distinct from the spheroidal or pseudo-spheroidal geometry in [14,15].

The matter content of the star is assumed to be a perfect fluid in equilibrium, with the energy–momentum tensor and the Einstein's field equations (EFEs) written as

$$R_{ij} - \frac{1}{2}R g_{ij} = -\frac{8\pi G}{c^2} T_{ij} = \left(\rho + \frac{p}{c^2}\right) u_i u_j - \frac{p}{c^2} g_{ij}, \quad u^i = \left(0, 0, 0, e^{-\frac{\nu}{2}}\right), \tag{2}$$

with ρ and p respectively denoting matter density and fluid pressure.

This set-up implies the following system of three equations:

$$\frac{8\pi G}{c^2}\rho = \frac{1}{R^2}\left(3 + \frac{r^2}{R^2}\right)\left(1 + \frac{r^2}{R^2}\right)^{-2}, \tag{3}$$

$$\frac{8\pi G}{c^4} p = \left(1 + \frac{r^2}{R^2}\right)^{-1}\left[\frac{\nu'}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2}, \tag{4}$$

$$\left(1 + \frac{r^2}{R^2}\right)\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'}{2r}\right) - \frac{r\nu'}{2R^2} + \frac{r^2}{R^4} = 0. \tag{5}$$

Equation (3) indicates that the paraboloidal geometry of the interior space–time of the star ensures the positivity of matter density and provides the law of the variation of density of its matter content as well. The matter density attains its maximum value ρ_0 at the centre $r = 0$ with expression

$$\frac{8\pi G}{c^2}\rho_0 = \frac{3}{R^2}. \tag{6}$$

It decreases radially outward from this positive maximum value and on the boundary $r = a$ of the configuration attains the minimum value ρ_a

$$\frac{8\pi G}{c^2}\rho_a = \frac{1}{R^2}\left(3 + \frac{a^2}{R^2}\right)\left(1 + \frac{a^2}{R^2}\right)^{-2}. \tag{7}$$

The ratio $\alpha = \rho_a/\rho_0$ which represents density variation parameter has the explicit expression

$$\alpha = \frac{\rho_a}{\rho_0} = \left(1 + \frac{a^2}{3R^2}\right)\left(1 + \frac{a^2}{R^2}\right)^{-2}. \tag{8}$$

The space–time in the exterior region $r > a$ is described by Schwarzschild metric

$$\mathrm{d}s^2 = \left(1 - \frac{2m}{r}\right)c^2\,\mathrm{d}t^2 - \left(1 - \frac{2m}{r}\right)^{-1}\mathrm{d}r^2 - r^2\left[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right], \tag{9}$$

where $m = GM/c^2$ denotes mass in geometric units of a star of mass M. The interior space–time of (1) is joined smoothly with the exterior space–time of (9) at the boundary $r = a$ of the stellar configuration by stipulating the continuity of the metric coefficients and also the continuity of the fluid pressure p across the boundary. The continuity of the metric coefficient of $\mathrm{d}r^2$ leads to the mass–radius relation

$$\frac{m}{a} = \frac{a^2}{2R^2}\left(1 + \frac{a^2}{R^2}\right)^{-1} \tag{10}$$

of the configuration. Equations (8) and (10) determine a^2/R^2 and m/a in terms of the density variation parameter α as

$$\frac{a^2}{R^2} = \frac{\left[1 + \sqrt{1 + 24\alpha} - 6\alpha\right]}{6\alpha} \tag{11}$$

and

$$\frac{m}{a} = \frac{1 - 6\alpha + \sqrt{1 + 24\alpha}}{2\left(1 + \sqrt{1 + 24\alpha}\right)}. \tag{12}$$

Equivalently, α has the following dependence on m/a:

$$\alpha = \left(1 - \frac{2m}{a}\right)\left(1 - \frac{4m}{3a}\right). \tag{13}$$

The star model in this set-up will be characterized by two independent parameters ρ_a and m/a.

3. General two-parameter family of solutions

The solution of EFEs (3)–(5) given by Finch and Skea is described by the metric

$$\begin{aligned}\mathrm{d}s^2 = {} & [(B - Az)\cos z + (Bz + A)\sin z]^2\,c^2\,\mathrm{d}t^2 \\ & -z^2\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\phi^2,\end{aligned} \tag{14}$$

where $z = z(r)$ and

$$z^2 = \left(1 + \frac{r^2}{R^2}\right).$$

It represents the interior space–time of a spherical distribution of matter characterized by matter density ρ and fluid pressure p respectively with

$$\frac{8\pi G\rho}{c^2} = \left(\frac{1}{R^2}\right)\frac{z^2+2}{z^4}, \tag{15}$$

$$\frac{8\pi G p}{c^4} = \left(\frac{1}{R^2 z^2}\right)\left[\frac{(B-Az)\tan z + (Az+B)}{(A+Bz)\tan z - (Az-B)}\right]. \tag{16}$$

The boundary conditions stipulating the continuity of the metric coefficients of the interior and the exterior metrics and vanishing of pressure at boundary $r = a$ determine the constants of integration A and B as

$$A = \frac{\sqrt{1-(2m/a)}\,[z_a \tan z_a - 1]}{[\{(z_a + \tan z_a) - z_a(z_a \tan z_a - 1)\}\cos z_a + \{(z_a + \tan z_a)z_a + (z_a \tan z_a - 1)\}\sin z_a]}$$

$$B = \frac{z_a + \tan z_a}{z_a \tan z_a - 1}A,$$

where $z_a = z(a)$.

The fluid pressure at any point in the interior is related with the boundary radius through the relation

$$\frac{8\pi G p}{c^4} = \left(\frac{1}{R^2 z^2}\right)\frac{(z_a + \tan z_a)(1 - z\tan z) + (z_a \tan z_a - 1)(z + \tan z)}{(z_a \tan z_a - 1)(\tan z - z) + (z_a + \tan z_a)(1 + z \tan z)}.$$

If ρ_a and m/a are specified, eqs (13), (7) and (11) determine parameters, R and the boundary radius a respectively. All the physical parameters of the models of this class are observed to be easily tractable using numerical procedures.

Finch and Skea have shown that this two-parameter family of solution is not only regular and physically viable for a range of its parameters, but also to a good approximation suitable to describe the interiors of neutron stars in equilibrium by comparing numerical models with neutron star models based on the relativistic mean field theory of Walecka [17]. We shall see in the next section that this family of solutions is also suitable to describe the interiors of other compact stars such as strange stars, hybrid neutron stars etc [18].

4. Discussion

The suitability of the *ansatz* discussed above to describe models of realistic compact stars can be explored by the estimation of their physical parameters such as mass, radius, compactification etc. It is necessary to adopt numerical procedures for such computations. We have examined the behaviour ρ, p and EOS $P = P(\rho)$ for star models obtained by prescribing the 3-paraboloidal geometry for their interior

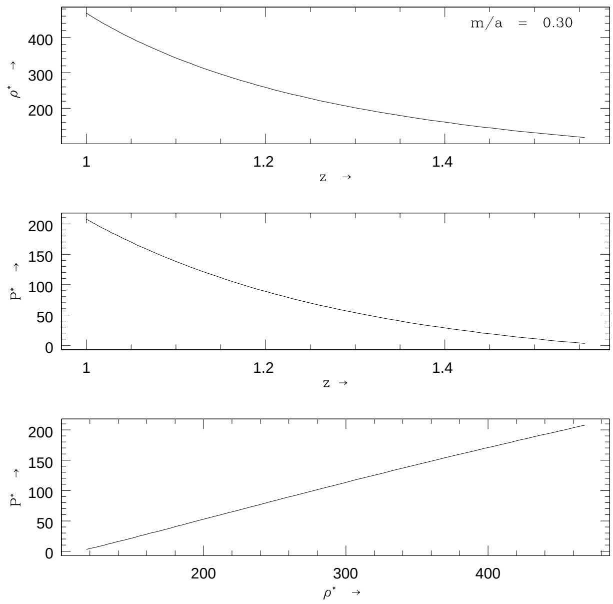

Figure 1. The plots of $\rho^\star \to z$, $P^\star \to z$ and EOS for model $\frac{m}{a} = 0.3$.

Table 1.

m/a	α	a (km)	$M/M_\odot$
0.30	0.24	7.1653	1.4694
0.36	0.1456	9.3808	2.2973
0.39	0.1054	11.0151	2.9224

physical space using numerical procedures. Following Sharma *et al* [12] we have chosen central density $\rho_0 = 4.68\times 10^{15}$ g cm^{-3}. The behaviour of ρ and P for models obtained for different chosen values of the compactification factor are described graphically in figures 1, 2 and 3 after appropriately scaling the dynamical variables as $\rho^\star$, $P^\star$ as in [18] using the relations $\rho(z) = \rho^\star(z) \times 10^{14}$ g cm^{-3} and $P(z) = 7\times 10^{33}\cdot P^\star(z)$ dyn cm^{-2} .

We have examined the implications of the physical conditions $\rho > 0$, $p > 0$, $\rho - \frac{p}{c^2} > 0$ throughout the configuration and the suitability of the *ansatz* for describing the models of relativistic compact stars such as strange stars in equilibrium using numerical procedures. We identify the physical parameters ρ_0 and m/a as the two free parameters. This choice of parameters leads to $a \sim$ 6–10 km. The star radius and the star mass of the three representative models of this family studied for chosen values of compactification parameter m/a have values as displayed in table 1.

It is apparent from figures 1–3 that the conditions $\rho > 0$, $p > 0$, $\rho - \frac{p}{c^2} > 0$ have been fulfilled throughout the interior of the above models. The equations of state for fluid in these models are found to be of approximately linear form. The

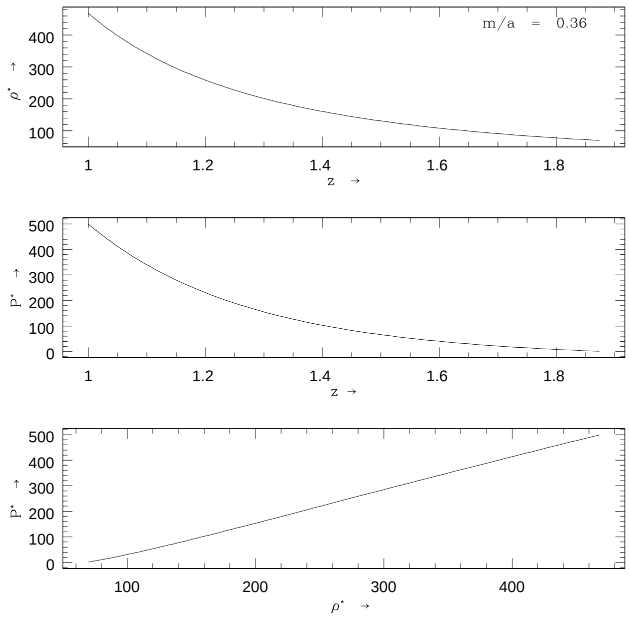

Figure 2. The plots of $\rho^\star \to z$, $P^\star \to z$ and EOS for model $\frac{m}{a} = 0.36$.

ansatz therefore provides an appropriate set-up to describe the interior space–times of strange stars.

The matter content of strange stars is believed to be strange matter in the form of quarks, in hydrostatic equilibrium. The maximum mass of such stellar configurations is estimated to be $1.64M_\odot$, according to Bethe–Jonson model and $2.24M_\odot$, according to Pandharipande–Smith model of neutron stars. It has been further pointed out that strange stars with masses exceeding the maximum mass of neutron stars will not be in hydrostatic equilibrium [19].

A comparative study of the mass–radius relationship of the compact star SAX J1808.4-3658 with the theoretical EOS models of both neutron and strange stars has been done by Li *et al* [4]. The two strange star models discussed in this respect have interior matter with EOS approximating to linear forms. The stars of these two models have compactification factor ∼0.3 and radii 7.07 and 6.53 km respectively. The study indicates that the object SAX J1808.4-3658 is most likely to be a strange star. The information about the central density, the radius and the mass of the compact star in general is necessary to determine whether its interior comprises of *nuclear matter or strange matter.* A compact star with central density of the order of 10^{15} g cm^{-3}, radius of the order of 7–9 km and mass up to $2M_\odot$ is most likely to be a strange star. The *ansatz* prescribing pseudospheroidal geometry for the interior space-time is known to lead to physically viable models of SAX [14]. We point out that the paraboloidal *ansatz* with appropriate parameters also provides other physically viable alternative description for SAX. The interior matter of several observed compact stellar objects has not been conclusively known to be strange matter. Useful information about some of the observed compact stellar objects

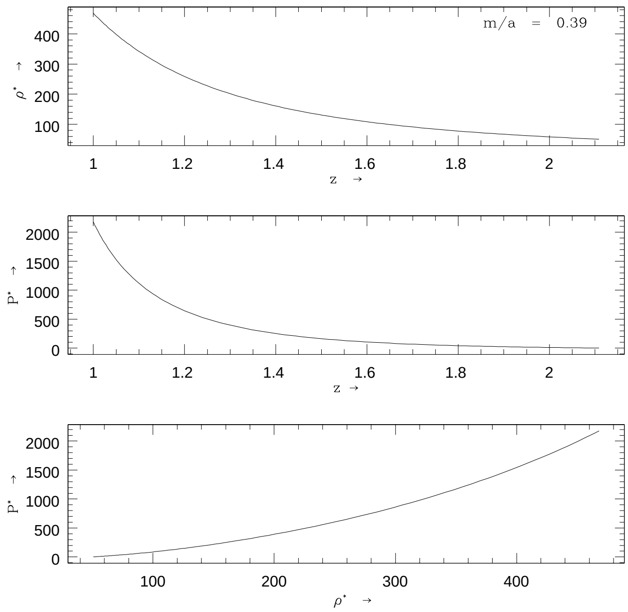

Figure 3. The plots of $\rho^\star \to z$, $P^\star \to z$ and EOS for model $\frac{m}{a} = 0.39$.

Table 2.

Strange star candidate	Mass (M) ($M_\odot$)	Radius (a) (km)	m/a
RXJ 1856-37	0.9	6	0.222
Her. X-1	0.98	6.7	0.216
SAX J1808.4-3658(SS2)	1.323	6.35	0.308
SAX J1808.4-3658(SS1)	1.435	7.07	0.299
4U 1820-30	2.25	< 10.0	0.332
PSR 1937+21	2.1	< 11.5	0.269

relevant in this respect in determining their nature as neutron stars or strange stars is given in table 2.

The $M \to a$ relationship for highly compact stars is expected to provide a vital clue towards their nature. The GR considerations forbid the stellar objects with $m > a/2$ while neutron stars are expected to have $m < 0.3a$. Along with the straight lines $m = 0.5a$ and $m = 0.3a$, we have shown $M \to a$ curve for SAX which follows from their X-ray observations in figure 4. It is apparent from this figure that the objects such as RXJ 1856-37, Her. X-1, SAX J1808.4-3658 are definitely strange stars. The object 4U 1820-30 which is believed to be a neutron star will also be expected to contain matter with density exceeding high nuclear matter densities and possibly will be a strange star. Information about density profile etc. for objects such as PSR 1937+21 will be essential to understand their

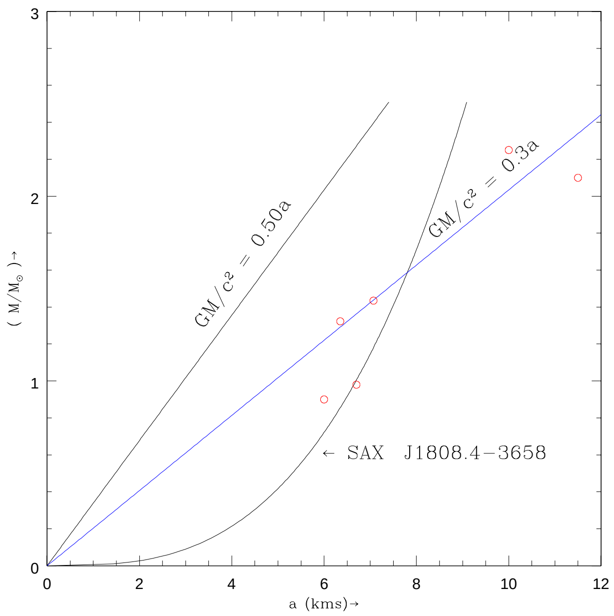

Figure 4. The mass–radius plane. More observational evidence regarding interior content will be necessary to determine the nature of compact stars located to the right of both the SAX and $GM/c^2 = 0.3$ line.

nature. The $M \sim a$ plane for stellar object in figure 4 is confined to region $0 < M < 3M_\odot$, $0 < a < 12$ km in view of the observed or expected mass-size estimates. The object Vela X-1 with a predicted mass of $1.86 \pm 0.33M_\odot$ (which) is likely to be a strange star candidate rather than conventionally claimed neutron star. The compactification factor for a neutron star is not expected to have values exceeding 0.3 (a stringent limit). The strange stars can be divided into two types: Type I: SS with $(m/a) > 0.3$ and Type II: SS with $0.2 < (m/a) < 0.3$. In the case of Type-II: SS, reliable information about the density profile, mass and radius will be essential to distinguish them from their neutron star counterparts and the lower limit may be still lower. The studies of compact star models based on idealized uniform density profiles are not likely to provide highly reliable information about such objects. The theoretical models which stipulate variable density profiles for interior matter of compact stars in hydrostatic equilibrium are definitely more incisive in quantifying astrophysical parameters in this respect. Recent observation of LXMB 2S 0921-30 contains massive compact object of mass $2.9M_\odot$ [20]. This object could be either low-mass black hole or strange star. Because a neutron star having a radius of about 10–12 km and interior matter with nuclear density cannot accomodate the mass mentioned above, the object 2S 0921-30 is expected to be a compact star with compactification parameter $0.33 < m/a < 0.40$. The object, if it is not a black hole, can exist as a strange star only. The upper limit is an extremely high value admissible in the case of theoretical models based on the paraboloidal *ansatz*.

Accordingly, the *ansatz* under consideration will provide a suitable mechanism for studying space-times of compact strange stars.

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References

[1] A R Bodmer, *Phys. Rev.* **D4**, 1601 (1971)
[2] E Witten, *Phys. Rev.* **D30**, 272 (1984)
[3] I Bombaci, *Phys. Rev.* **C55**, 1587 (1997)
[4] X D Li, I Bombaci, Mira Dey and E P J van den Heuvel, *Phys. Rev. Lett.* **83**, 3776 (1999)
[5] P C Vaidya and R Tikekar, *J. Astrophys. Astr.* **3**, 325 (1982)
[6] R Tikekar, *J. Math. Phys.* **31**, 2454 (1990)
[7] H Knutsen, *Mon. Not. R. Astron. Soc.* **232**, 163 (1988)
[8] S D Maharaj and P G L Leach, *J. Math. Phys.* **37**, 430 (1996)
[9] S Mukherjee, N Dadhich and B C Paul, *Class. Quantum Gravitat.* **14**, 3475 (1997)
[10] Y K Gupta and M K Jassim, *Astrophys. Sp. Sci.* **272**, 403 (2000)
[11] R Sharma and S Mukherjee *Mod. Phys. Lett.* **A16**, 049 (2001)
[12] R Sharma, S Mukherjee and S D Maharaj, *Mod. Phys. Lett.* **A15**, 1341 (2000)
[13] R Sharma, S Mukherjee, Mira Dey and Jishnu Dey, *Mod. Phys. Lett.* **A17**, 827 (2002)
[14] Ramesh Tikekar and Kanti Jotania, *Int. J. Mod. Phys.* **D14**, 1037 (2005)
[15] R Tikekar and V O Thomas, *Pramana – J. Phys.* **50**, 95 (1998)
[16] M R Finch and J E K Skea, *Class. Qunantum Gravitat.* **6**, 467 (1989)
[17] J D Walecka, *Phys. Lett.* **B59**, 109 (1975)
[18] Kanti Jotania and Ramesh Tikekar, *Int. J. Mod. Phys.* **D15**, 1175 (2006)
[19] L Shapiro Stuart and A Teukolsky Saul, *Black holes, white dwarfs and neutron stars: The physics of compact objects* (John Wiley, New York, 1983)
[20] T Shahbaz, J Casares, C A Watson, P A Charles, R I Hynes, S C Shih and D Steeghs, *Astrophys. J.* **616**, L123 (2004)