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Diffusion-induced parametric dispersion and amplification in doped semiconductor plasmas

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Abstract. Using the hydrodynamic model of semiconductor plasma, the diffusion-induced nonlinear current density and the consequent second-order effective susceptibility are obtained under off-resonant laser irradiation. The analysis deals with the qualitative behaviour of the anomalous parametric dispersion and the gain profile with respect to the excess doping concentration and pump electric field. The analysis suggests that a proper selection of doping level and pump field may lead to either positive or negative enhanced parametric dispersion, which can be of great use in the generation of sequeezed states. It is found that gain maximizes at moderate doping concentration level, which may drastically reduce the fabrication cost of parametric amplifier based on this interaction.

Keywords. Parametric amplification; diffusion current; electrostriction; doped centro-symmetric semiconductors.

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1. Introduction

Parametric interactions involving the nonlinear mixing of three waves in a medium have proved their importance in the field of nonlinear optics. Parametric amplifiers, parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state generation etc., are some of the important devices and processes whose origins are in the parametric interactions in nonlinear medium. These phenomena can well be explained in terms of bunching of the free carriers present in the medium under the influence of the fields applied externally and those associated with the generated wave [1]. Thus, any mechanism influencing the carrier bunching is expected to modify the linear and nonlinear properties of the medium and hence the dependent phenomena. Since carrier bunching induces a concentration gradient in the medium, their diffusion becomes inevitable and thus can play a crucial role in parametric processes.

Today, semiconductors are used in most of the sophisticated, sensitive and ultra fast optoelectronic devices [2] due to their compactness, provision of control of material relaxation time and highly advanced fabrication technology. Scattering of light beam from free electrons in doped piezoelectric semiconductors was reported by Neogi and Ghosh [3] and by some other workers. Of late, microscopic optical phenomena such as generation of three-wave parametric interactions have attracted keen attention of a large number of workers in the field of quantum optics [4]. Many such investigations are based upon the nonlinear optical response in a semiconducting medium [5,6]. A mechanism for the generation of exciton-induced squeezed state of light is theoretically presented by Nguyen [7]. Coherent tunable terahertz waves are generated successfully by Shikata *et al* [8] using a parametric oscillator made up of $LiNbO_3$ semiconductor. Aghamkar and Suta [9] have reported the parametric excitation of a polaron mode in direct gap III–V weakly polar, doped semiconductor. Improvement in figure of merit by doping for squeezed state generation in GaAs semiconductor is recently reported by Sen [10].

Such critical analysis warrants the appropriate incorporation of various nonlinear processes with considerable impact on the three-wave parametric interactions at excitation intensities well above the threshold. In electrostrictive effect, the nonlinearity is due solely to the motion of free carriers and therefore the effect is nonlocal. The high mobility of excited charge carriers makes diffusion effects particularly relevant in semiconductor technology as they travel significant distances before recombining.

In most of the investigations of nonlinear (NL) interactions, the nonlocal effects such as diffusion of the excitation density responsible for the nonlinear refractive index change has normally been ignored. But inclusion of carrier diffusion in theoretical studies of NL phenomena particularly in high mobility semiconductors seems to be important and thus attracted attention of many workers in the last decade [11–13]. Neogi [14] investigated the possibility of acousto-optic modulation of a laser beam in magnetized diffusive semiconductor. However, it appears from the available literature that no attempt has so far been made on the role of diffusion on the second-order nonlinearity and related phenomena.

In the present note, the authors have analyzed theoretically, the effect of electrostriction on second-order nonlinearity due to carrier diffusion current in an n-type compound semiconductor plasma.

2. Theoretical formulation

In order to study the parametric amplification arising due to three-wave mixing in a doped semiconductor crystal duly shined by a relatively high-power laser, authors have derived analytically the expression for the complex effective second-order optical susceptibility. The model used in the analysis is the well-known hydrodynamic model of the homogenous one-component semiconductor crystal, which simplifies the analysis without diluting the desired informations. Authors have assumed a spatially uniform ($|k_0| \approx 0$) pump represented as

$$\vec{E} = \hat{x}E_0 \exp(-i\omega_0 t) \tag{1}$$

with photon energy ($\hbar\omega_0$) much below the forbidden energy gap ($\hbar\omega_{\mathrm{g}}$) of the crystal which irradiates an n-type diffusive semiconductor medium. The parametric interaction of the pump generated an acoustic wave at ($\omega_{\mathrm{a}}, \vec{k}_{\mathrm{a}}$) and scatters a side band wave at $(\omega_1, \vec{k}_1)$ supported by the lattice and electron plasma in the medium, respectively. This enables one to find the phase matching conditions as $\omega_0 \approx \omega_1 + \omega_{\rm a}$ and $|\vec{k}_1| \approx |\vec{k}_{\rm a}| = k$.

Authors proceed with the following basic equations under simple one-dimensional configuration (along x-axis) for n-type diffusive semiconductor:

$$\frac{\partial \vartheta_{0,1}}{\partial t} + \nu \vartheta_{0,1} = -\frac{e}{m} E_{0,1}, \tag{2}$$

$$\frac{\partial n_1}{\partial t} + \vartheta_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial \vartheta_1}{\partial x} + D \frac{\partial^2 n_1}{\partial x^2} = 0, \tag{3}$$

$$\frac{\partial E_s}{\partial x} = -\frac{n_1 e}{\varepsilon} + \frac{\gamma}{\varepsilon} E_0 \frac{\partial^2 u^*}{\partial x^2}, \tag{4}$$

$$\rho \frac{\partial^2 u}{\partial t^2} + 2\Gamma_{\rm a} \rho \frac{\partial u}{\partial t} + \frac{\gamma}{2} \frac{\partial}{\partial x} (E_0 \cdot E_1^*) = c \frac{\partial^2 u}{\partial x^2}. \tag{5}$$

The subscripts 0 and 1 refer to the physical quantities related to pump and side band wave (SBW), respectively. The acoustic and side band perturbations are assumed to vary as $\exp[i(k_{\rm a,1} x - \omega_{\rm a,1} t)]$. D stands for diffusion coefficient of crystal and other notations used in the above fundamental relations have their usual meanings.

Authors have taken the pump to be an infra-red moderate power nanosecond pulsed laser of frequency $\omega_0 \gg \omega_{\rm a}$ which in turn yields $\omega_1 (= \omega_0 - \omega_{\rm a}) \approx \omega_0$. The pulse duration is considered to be much larger than the acoustic damping time such that the interaction is treated to be of steady state type. In a doped semiconductor the generated low-frequency wave $(\omega_{\rm a})$ while interacting with high-frequency pump electromagnetic wave (ω_0) produces AC density perturbations (n_1) at frequencies $\omega_0 \pm p\omega_{\rm a}, p$ being an integer. Accordingly, the AC components $\vec{\vartheta}_1$ and n_1 are assumed to oscillate at both the frequencies $\omega_{\rm a}$ and ω_1. Authors term them as the low- and high-frequency components, respectively. The perturbations at off-resonance frequencies $p \geq 2$ are neglected.

Following the procedure adopted by Neogi and Ghosh [3] one may obtain the diffusion-induced second-order (DISO) susceptibility $\chi_{\rm d}^{(2)}$ in the coupled mode scheme as

$$\chi_{\rm d}^{(2)} = \frac{e n_0 k^3 D \gamma^2 A}{2\rho\varepsilon\varepsilon_0 \omega_1 (\omega_{\rm a}^2 - k^2 \vartheta_{\rm a}^2 + 2i\Gamma_{\rm a}\omega_{\rm a})}. \tag{6}$$

The above equation infers that diffusion of the carriers induces second-order nonlinearity in the medium that would otherwise be absent or vanishingly small in a non-piezoelectric or centro-symmetric crystal medium.

To incite parametric interactions in the medium, the pump amplitude should exceed certain threshold value $E_{\rm 0th}$ to supply minimum required energy to the medium. This may be obtained from eq. (6) by setting $\chi_{\rm d}^{(2)}$ equals to zero as

$$E_{\rm 0th} = \frac{m}{ek} (\delta_1^2 \delta_{\rm a}^2 + \nu^2 \omega_1 \omega_{\rm a})^{1/2}. \tag{7}$$

The complex DISO susceptibility (eq. (6)) may be rationalized to get its real and imaginary parts as

$$\chi^{(2)}_{\rm dr} = \frac{en_0 k^5 \bar{E}^2 D\gamma^2 \nu(\omega_{\rm a}\delta_1^2 - \omega_1\delta_{\rm a}^2)}{4\varepsilon\varepsilon_0\rho\omega_1\omega_{\rm a}[(k^2\bar{E}^2 - \delta_1^2\delta_{\rm a}^2 - \nu\omega_1\omega_{\rm a})^2 + (\nu\omega_{\rm a}\delta_1^2 - \nu\omega_1\delta_{\rm a}^2)^2]} \tag{8a}$$

and

$$\chi^{(2)}_{\rm di} = \frac{en_0 k^5 \bar{E}^2 D\gamma(k^2\bar{E}^2 - \delta_1^2\delta_{\rm a}^2 - \nu^2\omega_1\omega_{\rm a})}{4\varepsilon\varepsilon_0\rho\omega_1\omega_{\rm a}\left[(k^2\bar{E}^2 - \delta_1^2\delta_{\rm a}^2 - \nu^2\omega_1\omega_{\rm a})^2 + (\nu\omega_{\rm a}\delta_1^2 - \nu\omega_1\delta_{\rm a}^2)^2\right]}. \tag{8b}$$

The above formulation reveals that the crystal susceptibility is influenced strongly by the doping concentration n_0. Equations (8a) and (8b) can be employed to study the parametric dispersion and amplification/attenuation characteristics of the scattered waves respectively.

The amplification of the co-propagating waves in the electrostrictive medium is due to the linear dispersion effects in combination with the NL processes. The steady state gain coefficient (α) of the parametrically excited wave-forms in a doped semiconductor can be obtained through the relation [15]

$$\alpha = -\frac{k}{2\varepsilon_1}\left[\chi^{(2)}_{\rm id}\right] E_0. \tag{9}$$

The nonlinear parametric gain of the signal (ω_1) as well as the idler ($\omega_{\rm a}$) waves can be possible only if α obtainable from eq. (9) is positive or $\chi^{(2)}_{\rm di}$ obtainable from eq. (8b) is negative for pump electric field $|E_0| > |E_{0\rm th}|$.

3. Results and discussion

Authors now address themselves to a detailed numerical analysis of the effective parametric dispersion and absorption/gain in a representative III–V diffusive semiconductor at 77°K duly irradiated by nanosecond pulsed 10.6 μm CO_2 laser. The physical parameters used are: $m = 0.014m_0$ (m_0 being the free electron rest mass), $\varepsilon_1 = 15.8$, $\gamma = 5\times10^{-10}$ MKS units, $\rho = 5.8\times10^3$ kg m^{-3}, $\nu = 3.5\times10^{11}$ s^{-1}, $\omega_0 = 1.78\times10^{14}$ s^{-1}, $\omega_{\rm a} = 2\times10^{11}$ s^{-1}, $\Gamma_{\rm a} = 2\times10^{10}$ s^{-1} and $\vartheta_{\rm a} = 4\times10^3$ m s^{-1}.

Being one of the principal objectives of the present analysis, the nature of the parametric dispersion arising due to the real part of the second-order optical susceptibility, viz. $\chi^{(2)}_{\rm dr}$ of the n-type diffusive semiconductor has been presented in figure 1 as a function of doping concentration n_0 at different $|E_0|$ in the vicinity of $|E_{0\rm th}|$. The anomalous parametric dispersion occurs over a large regime of n_0 and the magnitude increases with pump electric field E_0. The same figure also reveals that the anomalous dispersion regime widens at higher E_0. It is worth mentioning that $\chi^{(2)}_{\rm dr}$ can be both positive and negative under the anomalous regime while for the doping concentration $n_0 \le 1.2\times10^{22}$ and $n_0 \ge 7\times10^{22}$ m^{-3}, $\chi^{(2)}_{\rm dr}$ attains an almost constant value. The change in sign of parametric dispersion may be attributed to the electrostrictive nature of the doped semiconductor. Figure 1 enables one to infer that a proper selection of pump field strength, wave number and doping level can enable one to achieve either positive or negative significantly enhanced parametric dispersion. This result can be appropriately exploited in the generation of squeezed states. It can also be envisaged that a practical demonstration of the

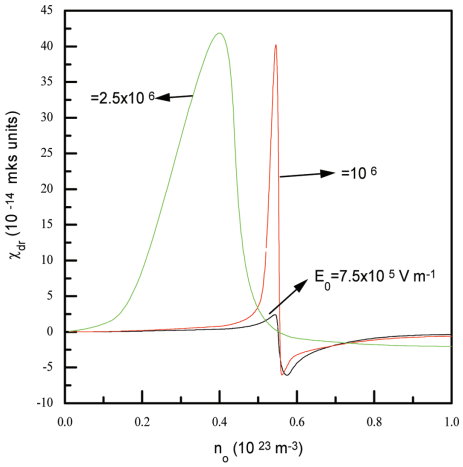

Figure 1. Variation of real part of susceptibility with carrier density using electric field as parameter for $k = 10^8$ m^{-1}.

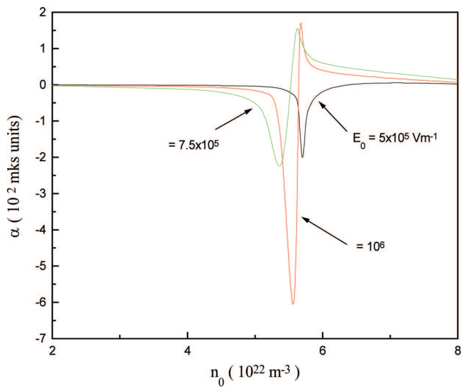

Figure 2. Variation of gain with carrier density using electric field as parameter for $k = 10^8$ m^{-1}.

above kind of parametric dispersion may lead to the possibility of observation of group velocity dispersion in the bulk doped semiconductors.

Figure 2 depicts the dependence of absorption/gain coefficient α on excess doping concentration n_0 for three different values of the pump field E_0. One may notice that the magnitude of gain increases with increase of pump field strength E_0. Moreover, for each value of E_0, there exists a critical doping concentration $n_0 = n_{0\mathrm{cr}}$ above which α becomes positive and one achieves parametric amplification in the diffusive semiconductor. This amplification increases sharply with increase of n_0 and achieves maximum value at $n_0 \approx 5.5 \times 10^{22}$ m^{-3}. If we further increase n_0, gain coefficient starts reducing and then becomes saturated. The gain spectrum is much narrower and shifts to a smaller doping level if the field strength is increased. Thus, one may enhance significantly the parametric amplification by tuning the pump field even for a fixed doping level in the semiconductor.

It is known that resultant perturbed carrier velocity strongly depends on free carrier concentration and applied laser field. The variation in the resultant perturbed velocity due to variations in n_0 and E_0 is responsible for the variation in the modulation of medium conductivity. The parametric acoustic gain is dependent upon the conductivity being modulated; anything, which increases/decreases the conductivity modulation, increases/decreases the parametric gain. Hence the variation in parametric gain illustrated in figure 2 may be attributed to the variation in the modulation of effective conductivity of the medium by the free charge carriers and laser field.

The chief advantage of this analysis lies in the fact that other than the above studies one can also estimate the approximate value of diffusion-induced second-order susceptibility. For n-InSb crystal for $k = 10^8$ m^{-1}, one finds $\chi_d^{(2)} \approx 3.6 \times 10^{-8}$ esu. Thus, one may notice that diffusion current plays an important role in lowering the DISO susceptibility of the crystal, as the experimental value of $\chi^{(2)}$ in InSb is $\approx 3.3 \times 10^{-7}$ esu in the off-resonant regime [16].

4. Conclusion

The above analysis establishes the possibility of diffusion-induced nonlinearity in semiconductor plasma medium. It is found that this nonlinearity maximizes at very moderate doping concentration $n_0 \approx 5.5 \times 10^{22}$ m^{-3}, which drastically reduces the operating cost of parametric amplifiers and other related NL devices based on this interaction. A significant enhancement in the parametric dispersion (both positive as well as negative) can be achieved by a proper selection of doping level, pump field and appropriate wave number. This can be of potentiality in the study of squeezed states generation as well as group velocity dispersion in doped bulk semiconductor. Hence, it is hoped that the diffusion-induced second-order nonlinearity may play an extremely important role in the construction of parametric amplifiers, oscillators, tunable radiation sources etc. from diffusive semiconductor media. Thus, for a clear understanding of the DISO nonlinear interaction and for the experimental verification of the present result, authors propose to initiate a serious laboratory effort using a diffusive semiconductor.

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