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Odd–even effect in fragment angular momentum in low-energy fission of actinides

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Abstract. Quantitative explanation for the odd–even effect on fragment angular momenta in the low-energy fission of actinides have been provided by taking into account the single particle spin of the odd proton at the fragment's scission point deformation in the case of odd-Z fragments along with the contribution from the population of angular momentum bearing collective vibrations of the fissioning nucleus at scission point. The calculated fragment angular momenta have been found to be in very good agreement with the experimental data for fragments in the mass number region of 130–140. The odd–even effect observed in the fragment angular momenta in the low-energy fission of actinides has been explained quantitatively for the first time.

Keywords. Fission fragment angular momentum; scission point model; fragment deformation; low-energy fission of actinides.

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1. Introduction

Even after nearly six decades of its discovery, nuclear fission process continues to attract the attention of the nuclear scientists owing to the interplay of a large number of degrees of freedom during this process. One of the most fascinating aspects of the low-energy fission of actinides is the generation of a large angular momentum (6–8$\hbar$) in the fission fragments [1] in fission of nuclei having practically no initial angular momentum. This is evident from extensive studies on measurement of fragment angular momenta in the low-energy fission of actinides. The methods include, measurement of (i) independent isomeric yield ratio of fission products [2], (ii) relative intensities of γ-rays corresponding to the ground state rotational bands of even–even nuclei [3], and γ-ray multiplicity [4]. The bending mode oscillation model proposed by Rasmussen *et al* [5] successfully explained the mechanism of the generation of the large fragment angular momentum in terms of the population of the ground state of the bending mode vibration at the scission point. It was also found that the post-scission Columbic torque between the separating fragments also contributes to the fragment angular momentum. Extensive studies on fission fragment angular momentum, however, revealed certain interesting features, namely, (i) fragments having odd-Z showed higher angular momenta (by about $3\hbar$) than their neighboring even-Z fragments (odd–even effect) [6,7], (ii) as the neutron number of the fragment approaches $N = 82$, the fragment angular momentum decreases [8]. The latter observation found explanation in the influence of the spherical shell effects wherein smaller scission point deformation of the fragment was responsible for smaller angular momentum. However, there has not been any satisfactory explanation for the proton odd–even effect on fragment angular momentum, though qualitatively it has been explained in terms of the polarisation of the even-Z core by the odd proton.

In the present paper the proton odd–even effect on fragment angular momentum has been explained in terms of the single particle spin of the odd proton at fragment's scission point deformation. It has been possible to explain the difference between the fragment angular momenta of the neighboring Z fragments quantitatively. The theory is based on the argument that when a proton pair is broken during the fission process to form odd Z fragment pair, the Ω value (quantum number corresponding to j_z) of the odd proton at the scission point deformation of the fragment will add to the fragment spin generated due to the rotational angular momentum.

2. Calculation of fragments' scission point deformation

Deformation (β) of the fission fragments at the scission point was calculated by minimising the scission point potential energy as a function of the deformations (β_1, β_2) of the complementary fragments. The scission point potential energy (V) has been calculated using the formalism of Wilkins *et al* [9],

$$V(Z_1, A_1, \beta_1, Z_2, A_2, \beta_2) = V_1(Z_1, A_1, \beta_1) + V_2(Z_2, A_2, \beta_2) + V_{\rm C} + V_{\rm N}, \quad (1)$$

where V_i represents the deformation energy of the fragment which is given by

$$V_i = V_{\rm LDM} + \Delta U_{\rm s} + \Delta U_{\rm p}. \quad (2)$$

$V_{\rm LDM}$ is the liquid drop deformation energy relative to the spherical shape, and $\Delta U_{\rm s}$ and $\Delta U_{\rm p}$ are the shell and pairing corrections, respectively, to the LDM energy. $V_{\rm LDM}$ was calculated using the prescription of Myers and Swiatecki [10].

$$V_{\rm LDM} = a_2 \frac{[1 - k(N - Z/A)^2]|S|}{4\pi r_{\rm c}^2} + \left(\frac{3}{10}\right)\left(\frac{Z^2 e^2}{r_{\rm c} A^{1/3}}\right) \times \left(\frac{(1-\varepsilon^2)^{1/3}}{\varepsilon}\right) \ln\left(\frac{1+\varepsilon}{1-\varepsilon}\right), \quad (3)$$

where $|S|$ is the area of equivalent sharp density surface, given by

$$|S| = 2\pi x[x + z^2/(z^2 - x^2)^{1/2} \arcsin\{(z^2 - x^2)^{1/2}/z\}]$$
$$\varepsilon^2 = 1 - x^2/z^2; \quad z > x$$

where a_2, k and r_c are constants and x and z are semi-minor and major axes.

$$z = R(\beta)[1 + \sqrt{(5/4\pi)}\beta]$$
$$x = R(\beta)[1 - 0.5\sqrt{(5/4\pi)}\beta]$$
$$R(\beta) = R_0[1 - (15/16\pi)\beta^2 + (1/4)(5/4\pi)^{3/2}\beta^3]^{-1/3}.$$

ΔU_s was calculated following Strutinsky's shell correction method [11]. We used the single particle model of Nilsson *et al* [12] to generate the single particle energy levels for protons and neutrons. Single particle levels up to $N = 8$ were considered for calculating the smooth single particle level density. The deformation parameter (δ) used to generate single particle levels is related to β by the following equation:

$$\delta \approx \frac{3}{2}\sqrt{\frac{5}{4\pi}}\beta. \tag{4}$$

Computer program STRUT for calculating the shell correction was checked by reproducing the shell correction for ^{208}Pb, which was found to be −12.5 MeV. V_C was calculated using the formalism of Nix [13].

A small separation ($d = 2.0$ fm) between the tips of the fragments was assumed which reproduced the total kinetic energy (TKE=190 MeV) released for 132 + 104 fragment split of ^{236}U. The β_1, β_2 values were varied in the range of 0.0 to 0.65 in steps of 0.05 and scission point potential energy was calculated for each combination of β_1 and β_2. The scission potential energy was minimised to obtain the β_1 and β_2 values and the TKE values for mass splits giving heavier mass fragment in the mass region 130–140, where the fragment angular momenta had been measured. Table 1 gives the β and the TKE values for a few fragments in this mass region. The TKE for the mass number 132 agrees quite well with the experimental value [15]. The influence of fragments' nuclear structure on their scission point deformation is clearly seen from the table. The deformation of fragment is found to decrease as its neutron number approaches the spherical shell number of 82 in the case of antimony isotopes. Similar trend is observed in the case of fragment angular momenta of antimony isotopes [16].

3. Single particle spin of odd proton

In the case of deformed nuclei, Ω $(\Lambda + \Sigma)$ becomes a good quantum number instead of $J(l+s)$. The Ω value of the odd proton was obtained from the single particle level scheme at the fragment deformations deduced above. The total energy released in fission is distributed in the form of TKE, deformation energy and intrinsic excitation energy of the fragments. The deformed fragments shed their deformation energy in the form of neutrons and γ-rays to become fission products, which subsequently undergo β^- decay. In order to obtain the spin of the odd proton at the scission point deformation, we took the average of Ω values of six single particle levels around the last occupied level. The average Ω was found to be around 2.0 as shown in table 1. Hence on the average $2\hbar$ of angular momentum can be expected from the odd proton state to contribute to the final angular momentum.

Table 1. Fragment deformations, TKE and Ω values in ^{235}U($n_{\rm th}$,f).

Fragment	β	TKE (MeV)	Average Ω^*
^{132}Sn	0.05	190.0	–
^{130}Sb	0.6	168.8	1.8
^{131}Sb	0.6	174.6	1.8
^{132}Sb	0.1	187.6	1.8
^{133}Sb	0.05	189.6	1.8
^{132}Te	0.6	173.7	–
^{133}Te	0.5	171.8	–
^{134}Te	0.6	168.1	–
^{132}I	0.6	166.9	2.0
^{133}I	0.45	172.6	1.7
^{134}I	0.45	172.7	1.7
^{136}Xe	0.6	167.6	–
^{138}Cs	0.6	168.0	2.0
^{139}Cs	0.6	168.0	2.0

*Average of six levels above the last occupied level.

4. Rotational angular momentum of fission fragments

The rotational angular momentum of the fission fragments was calculated following the prescription of Rasmussen *et al* [5] based on the excitation of the ground state bending mode of the fissioning nucleus at the scission point. According to this model, the average angular momentum of the fission fragment is given by the equation

$$L_{\rm av} = \frac{\sqrt{\pi}}{2\gamma_0} - \frac{1}{2}, \tag{5}$$

where γ_0 is the average angle between the symmetry axis and the fission axis, and is given by

$$\gamma_0^2 = \frac{\hbar}{\sqrt{KB}}. \tag{6}$$

K is the coefficient in the potential energy term dependent on γ,

$$V(\gamma) = (1/2)K\,\sin^2(\gamma). \tag{7}$$

K depends upon the fragment deformation and the center to center distance (σ) between the fragments at the scission point. B is the inertial parameter for the bending motion given by

$$B^{-1} = \Im^{-1} + (M_{\rm r}\sigma^2)^{-1} \tag{8}$$

where $\Im$ is the rotational moment of inertia for the deformed fragment and $M_{\rm r}$ is the reduced mass of the two fragments. In the present calculations, the fragment

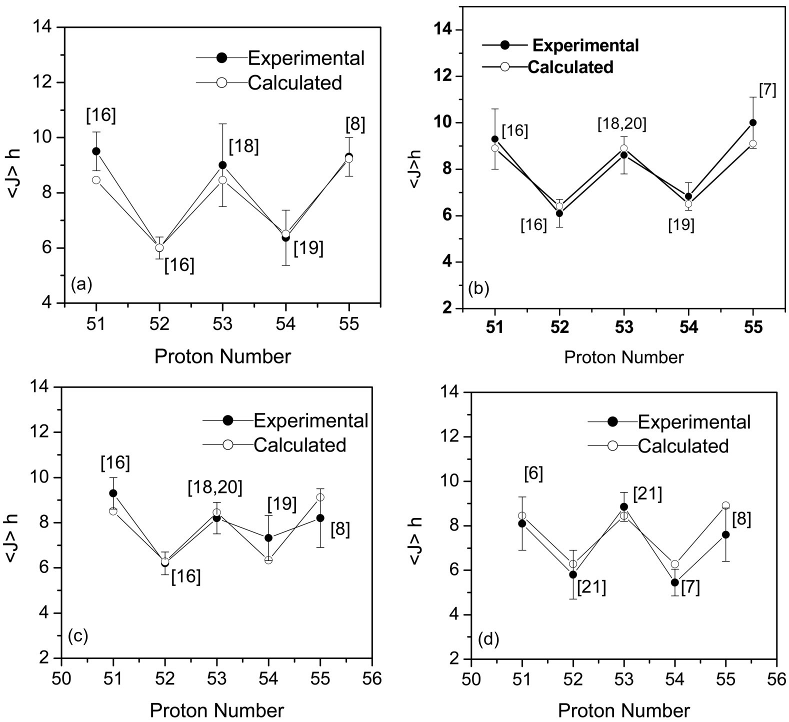

Figure 1. Fragment angular momenta in (**a**) $^{233}\mathrm{U}(n_{\mathrm{th}}, f)$, (**b**) $^{235}\mathrm{U}(n_{\mathrm{th}}, f)$, (**c**) $^{239}\mathrm{Pu}(n_{\mathrm{th}}, f)$ and (**d**) $^{241}\mathrm{Pu}(n_{\mathrm{th}}, f)$. Reference of the data is given in the parentheses.

deformations and TKE obtained by minimising the scission point potential energy, as described above, are used to calculate σ and other deformation parameters of the fragment. The asymmetric deformation (α_3) was taken the same as α_2 as suggested by Rasmussen *et al* [5].

The experimentally obtained fission fragment angular momentum is the resultant of the collective angular momentum of the fragment due to the population of ground state bending mode and the intrinsic spin of the odd proton at its scission point deformation. The coupling of the collective and intrinsic angular momenta of the fragment depends upon the orientation of the two vector quantities, which is not clearly understood as it depends upon the dynamics of the fission process. However, the experimental average angular momenta of odd-Z fission fragments are higher than the neighboring even-Z fragments by about $2\hbar$, which is close to the Ω of odd proton as deduced above. In view of this, the single particle spin of the odd proton was coupled with the rotational angular momentum of the fragment to obtain the total angular momentum of the fragment assuming that the two have the same orientation. Calculations of fragment angular momentum were carried out for fission fragments in the mass region 130–140 produced in the thermal neutron-induced fission of ^{233}U [8,16–19], ^{235}U [7,16,18–20], ^{239}Pu [8,16,18–20] and ^{241}Pu [6–8,21]. The calculated fragment angular momenta are plotted as a function of Z

of the fragment along with the experimental data and are shown in figure 1. It can be seen that the calculated fragment angular momenta are in very good agreement with the experimental data in all the fissioning systems. The proton odd–even effect is well accounted for by the odd proton Ω value, with the odd Z fragments having higher angular momenta than their neighboring even Z fragments.

5. Conclusion

The present work has shown that the higher angular momentum of odd Z fragments in the mass region 130–140 is due to the contribution from the angular momentum state occupied by the odd proton just after the scission.

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