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Unified approach to photo- and electro-production of mesons with arbitrary spins

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Abstract. A new approach to identify the independent amplitudes along with their partial wave multipole expansions, for photo- and electro-production is suggested, which is generally applicable to mesons with arbitrary spin-parity. These amplitudes facilitate direct identification of different resonance contributions.

Keywords. Photo-production of mesons; electro-production of mesons; irreducible tensors; partial wave multipole amplitude.

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1. Introduction

Improved experimental facilities to study photo- and electro-production of pseudoscalar and vector mesons have become available with the advent of the new generation of electron accelerators at JLab, MIT, BNL in USA, ELSA at Bonn, MAMI at Mainz in Germany, ESRF at Grenoble in France and Spring8 at Osaka in Japan [1,2]. With energies going up to 6 GeV, one can anticipate the extension of these studies to include mesons with higher spins $s > 1$, which are either known already [3] or have been predicted theoretically [4]. Since η, ω, ϕ are isoscalars in contrast to π and ρ which are isovectors, photo- and electro-production of the former involve only the nucleon resonances in the intermediate state, whereas the latter involve the contributions from the delta resonances as well. Therefore these experimental studies assume importance in the context of the so-called 'missing resonance problem' [5], which is concerned with the resonances predicted on the basis of various theoretical models [6] but have not been seen experimentally so far. In view of the dramatic violation of the OZI rule [7] observed in $\bar{p}p$ collisions [8], and the measurement [9] of the ratio of ϕ/ω production in NN collisions, a similar ϕ/ω ratio has been investigated recently in the case of photo-production

[10]. Photo-production of exotic mesons and baryons have also attracted attention in recent years [11].

Almost half a century ago a formalism for pion production was presented by Chew *et al* [12] in the context of developing a dispersion theoretical approach to the problem. They expressed the photopion production amplitude in terms of four invariants M_A, M_B, M_C, M_D and their coefficients A, B, C, D (which are essentially dependent on the c.m. energy W and the angle θ in the c.m. frame between the meson and photon momenta denoted by $\boldsymbol{q}$ and $\boldsymbol{k}$ respectively). Using the two componental form for the nucleon spin the differential cross-section for photopion production was expressed in terms of four amplitudes denoted by $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$ and $\mathcal{F}_4$ whose dependence on θ was made explicit through expansions involving the first and second derivatives of Legendre polynomials with respect to $\cos\theta$. The c.m. energy-dependent *electric* and *magnetic* multipole partial wave amplitudes which appear therein were denoted respectively by $E_{l\pm}, M_{l\pm}$ where l denotes the relative angular momentum of the meson with respect to nucleon in the final state and $\pm$ indicate the total angular momentum $j = l \pm 1/2$. The authors themselves mentioned without giving any details that 'the derivation of these formulae is lengthy but can be carried out by straightforward methods'. These formulae apply directly for photo-production of other pseudoscalar mesons like η. In the case of the isovector pion, an earlier isospin analysis by Watson [13] was made use of to express each one of these amplitudes $\mathcal{F}_i (i = 1\text{–}4)$ in terms of three independent nucleon isospin combinations $\mathcal{I}^{(+)}$, $\mathcal{I}^{(-)}$, $\mathcal{I}^{(0)}$, multiplied respectively by $\mathcal{F}_i^{(+)}$, $\mathcal{F}_i^{(-)}$, $\mathcal{F}_i^{(0)}$. Numerical estimates of these twelve amplitudes for pion production may be obtained, for example, from the second of the series of three papers by Berends *et al* [14] over a range of energies going up to 500 MeV. The first of the three papers [14] reviews the extension of the formalism [12] to six amplitudes to include electro-production and describes also the connection with helicity formalism [15]. The helicity approach was used recently [16] to describe the photo-production of vector mesons in terms of twelve independent amplitudes. To our knowledge, no formalism exists as yet for electro-production of vector mesons or for photo- and electro-production of tensor and higher spin mesons.

The purpose of the present paper is to suggest a simple and unified formalism for photo- and electro-production of mesons with arbitrary spin-parity, s^π. It also has a built-in isospin index I along with the total angular momentum j, which makes it ideal for readily identifying different baryon resonance contributions in the intermediate state.

2. New amplitudes for photo- and electro-production of mesons with arbitrary spin-parity s^π

Let us consider photo-production of a meson with spin-parity s^π and isospin $I_{\rm m}$ at c.m. energy W. Let $\boldsymbol{k}$ and $\boldsymbol{q}$ denote respectively the photon and meson momenta in c.m. frame. We use a right-handed frame with z-axis chosen along $\boldsymbol{k}$ and the reaction plane containing $\boldsymbol{k}$ and $\boldsymbol{q}$ as z–x plane. Using natural units with $\hbar, c$ and meson mass as unity, the photon and meson energies in c.m. frame are given in terms of W by

$$k = \frac{1}{2W}[W^2 - M^2], \tag{1}$$

$$\omega = \frac{1}{2W}[W^2 + 1 - M^2] = (q^2 + 1)^{1/2}, \tag{2}$$

where M denotes the nucleon mass $k = |\boldsymbol{k}|$ and $q = |\boldsymbol{q}|$. The differential cross-section for the reaction in c.m. frame may then be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{q}{4k}\left(\frac{M}{4\pi W}\right)^2 \sum_{m_\mathrm{i} m_\mathrm{f} m_\mathrm{s}\, p} |\langle \tfrac{1}{2}\, m_\mathrm{f}; s m_\mathrm{s}; \boldsymbol{q}|T|\boldsymbol{k}, p\,; \tfrac{1}{2}\, m_\mathrm{i}\rangle|^2, \tag{3}$$

where the initial and final nucleon spin projections are denoted by m_i and m_f respectively, the meson spin projection is denoted by m_s, while $p = \pm 1$ denote left and right circular polarization states of photon as defined by [17]. The covariant normalized T-matrix [14,18] is denoted by T. We may introduce the reaction amplitude $\mathcal{F}$ as in [12], through

$$\langle \tfrac{1}{2}\, m_\mathrm{f}; s m_\mathrm{s}; \boldsymbol{q}|\mathcal{F}(p)|\boldsymbol{k}; \tfrac{1}{2}\, m_\mathrm{i}\rangle = \frac{M}{4\pi W}\langle \tfrac{1}{2}\, m_\mathrm{f}; s m_\mathrm{s}; \boldsymbol{q}|T|\boldsymbol{k}, p\,; \tfrac{1}{2}\, m_\mathrm{i}\rangle. \tag{4}$$

In the case of electro-production, the momentum $\boldsymbol{k}$ of the virtual photon is given by $\boldsymbol{k} = \boldsymbol{p}_\mathrm{i} - \boldsymbol{p}_\mathrm{f}$ if $\boldsymbol{p}_\mathrm{i}$ and $\boldsymbol{p}_\mathrm{f}$ denote the initial and final momenta of the electron. The differential cross-section $\mathrm{d}^5\sigma/\mathrm{d}^3 p_\mathrm{f} \mathrm{d}\Omega$ may be expressed following [19] in terms of the differential cross-section $\mathrm{d}\sigma_\mathrm{v}/\mathrm{d}\Omega$, for meson production by a virtual photon in the meson–nucleon c.m. frame. The amplitude for electro-production of mesons is similar to eq. (4), but derives contributions from longitudinal photons, i.e., $p = 0$ as well, in addition to $p = \pm 1$.

Observing that the hadron spins characterizing the entrance and exit channels in the reaction are respectively $s_\mathrm{i} = \frac{1}{2}$ and s_f takes the values $|s - \frac{1}{2}|$ to $(s + \frac{1}{2})$, we may follow [20] and using the same notation introduce operators

$$S^\lambda_\mu(s_\mathrm{f}, \tfrac{1}{2}) = \sum_{n=0}^{1} \frac{[s_\mathrm{f}]^2\,[n]}{\sqrt{2}\,[s]} W(\lambda \tfrac{1}{2} s \tfrac{1}{2}; s_\mathrm{f} n)\left(S^s(s,0) \otimes S^n(\tfrac{1}{2}, \tfrac{1}{2})\right)^\lambda_\mu, \tag{5}$$

which are irreducible tensor operators of rank $\lambda = |s_\mathrm{f} - \frac{1}{2}|$ to $(s_\mathrm{f} + \frac{1}{2})$ in hadron spin space. $W(\lambda \frac{1}{2} s \frac{1}{2}; s_\mathrm{f} n)$ denotes a Racah coefficient [17], $[s] = (2s+1)^{1/2}$ and

$$\left(S^s(s,0) \otimes S^n(\tfrac{1}{2}, \tfrac{1}{2})\right)^\lambda_\mu = \sum_{\mu_\mathrm{s}} C(sn\lambda; \mu_\mathrm{s}\mu_\mathrm{n}\mu) S^s_{\mu_\mathrm{s}}(s,0) S^n_{\mu_\mathrm{n}}(\tfrac{1}{2}, \tfrac{1}{2}) \tag{6}$$

in terms of Clebsch Gordon coefficients C. The reaction amplitude $\mathcal{F}(p)$ given by eq. (4) is then expressed as

$$\mathcal{F}(p) = \sum_{\lambda=|s-n|}^{(s+n)} \sum_{n=0}^{1} \left((S^s(s,0) \otimes S^n(\tfrac{1}{2}, \tfrac{1}{2}))^\lambda \cdot \mathcal{F}^\lambda(n,p)\right), \tag{7}$$

in terms of irreducible tensor amplitudes $\mathcal{F}^\lambda_\mu(n,p)$, which constitute the new basic amplitudes in our formalism.

To obtain formulae for these new amplitudes in terms of the different partial wave multipole amplitudes, we express $\langle \boldsymbol{q}|$ and $|\boldsymbol{k}, p\rangle$ in the right-hand side of eq. (4) in terms of partial waves and multipoles [17] respectively, using

$$\mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}} = 4\pi \sum_{l=0}^{\infty} (-i)^l j_l(qr) \sum_{m_l=-l}^{l} Y_{lm_l}(\hat{\boldsymbol{q}}) Y_{lm_l}(\hat{\boldsymbol{r}})^*, \tag{8}$$

$$\hat{\boldsymbol{u}}_p\, \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sqrt{2\pi} \sum_{L=|p|}^{\infty} i^L [L] [|p| \boldsymbol{A}_{Lp}^{(m)}(\boldsymbol{r}) + ip\, \boldsymbol{A}_{Lp}^{(e)}(\boldsymbol{r}) \\ -i\sqrt{2}\,(1-p^2) \boldsymbol{A}_{Lp}^{(\ell)}(\boldsymbol{r})], \tag{9}$$

where the *magnetic, electric* and *longitudinal* 2^L-pole states of the photon are given respectively by

$$\boldsymbol{A}_{Lp}^{(m)}(\boldsymbol{r}) = j_L(kr)\, \boldsymbol{T}_{LLp}(\hat{\boldsymbol{r}}) \tag{10}$$

$$\boldsymbol{A}_{Lp}^{(e)}(\boldsymbol{r}) = -\sqrt{\frac{L}{2L+1}}\, j_{L+1}(kr) \boldsymbol{T}_{LL+1p}(\hat{\boldsymbol{r}}) \\ +\sqrt{\frac{L+1}{2L+1}}\, j_{L-1}(kr) \boldsymbol{T}_{LL-1p}(\hat{\boldsymbol{r}}) \tag{11}$$

$$\boldsymbol{A}_{Lp}^{(\ell)}(\boldsymbol{r}) = \sqrt{\frac{L+1}{2L+1}}\, j_{L+1}(kr) \boldsymbol{T}_{LL+1p}(\hat{\boldsymbol{r}}) \\ +\sqrt{\frac{L}{2L+1}}\, j_{L-1}(kr) \boldsymbol{T}_{LL-1p}(\hat{\boldsymbol{r}}), \tag{12}$$

in terms of the vector spherical harmonics

$$\boldsymbol{T}_{Ll_\gamma M}(\hat{\boldsymbol{r}}) = \sum_p C(l_\gamma 1 L; m_\gamma p M) Y_{l_\gamma m_\gamma}(\hat{\boldsymbol{r}}) \hat{\boldsymbol{\xi}}_p, \tag{13}$$

which are irreducible tensors of rank L. When the z-axis is chosen along $\boldsymbol{k}$, the summation over p drops to a single term with $M = p$ since m_γ can assume only one value, i.e., $m_\gamma = 0$. The quantum numbers l_γ and L correspond respectively to the orbital and total angular momenta of the photon. If $\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3$ denote mutually orthogonal unit vectors constituting a right-handed system such that $\hat{\boldsymbol{e}}_3$ is along $\boldsymbol{k}$, then $\hat{\boldsymbol{\xi}}_{\pm 1}$, $\hat{\boldsymbol{\xi}}_0$ are defined as

$$\hat{\boldsymbol{\xi}}_{\pm 1} = \mp \frac{1}{\sqrt{2}} (\hat{\boldsymbol{e}}_1 \pm i\, \hat{\boldsymbol{e}}_2), \quad \hat{\boldsymbol{\xi}}_0 = \hat{\boldsymbol{e}}_3, \tag{14}$$

and $\hat{\boldsymbol{u}}_p$ in eq. (9) is given by $\hat{\boldsymbol{u}}_p = -p\, \hat{\boldsymbol{\xi}}_p$, $p = \pm 1$; $\hat{\boldsymbol{u}}_0 = \hat{\boldsymbol{\xi}}_0$, where $\hat{\boldsymbol{u}}_{\pm 1}$ correspond to left and right circular polarization states as defined by Rose [17].

Combining the total angular momentum L of the photon with the initial nucleon spin $\frac{1}{2}$ to yield the total angular momentum j which is conserved in the reaction, it is clear that the same j is obtained on combining the orbital angular momentum l of the meson with channel spin $s_{\rm f}$ in the final state. We thus have

$$\begin{aligned}
&\langle \tfrac{1}{2}\, m_{\rm f}; s m_{\rm s}; \boldsymbol{q}|\mathcal{F}(p)|\boldsymbol{k}; \tfrac{1}{2} m_i\rangle \\
&\quad = \frac{(2\pi)^{1/2} M}{W} \sum_{l=0}^{\infty} \sum_{s_{\rm f}=|s-(1/2)|}^{(s+(1/2))} \sum_{L=|p|}^{\infty} \sum_{j=L-(1/2)}^{L+(1/2)} i^{L-l}[L] \\
&\qquad \times \langle (l(s\tfrac{1}{2})s_{\rm f})j\|T\|(L\tfrac{1}{2})j\rangle C(s\tfrac{1}{2}s_{\rm f}; m_{\rm s} m_{\rm f} m_{s_{\rm f}}) \\
&\qquad \times C(l s_{\rm f} j; m_l m_{s_{\rm f}} m)\, C(L\tfrac{1}{2}j; p m_{\rm i} m)\, Y_{l m_l}(\theta, 0),
\end{aligned} \tag{15}$$

where the reduced matrix elements depend only on the c.m. energy W and the angular dependence is completely taken care of by $Y_{lm_l}(\theta,0)$ where $m_l = p + m_{\rm i} - m_{\rm f} - m_{\rm s}$. We may express

$$\begin{aligned}
&C(L\tfrac{1}{2}j; p m_{\rm i} m) C(l s_{\rm f} j; m_l m_{s_{\rm f}} m) \\
&\quad = \sum_{\lambda} W(L\tfrac{1}{2} l s_{\rm f}; j\lambda)[j]^2[\lambda][s_{\rm f}]^{-1}(-1)^{l+L+\frac{1}{2}-j} \\
&\qquad \times (-1)^{p+\mu} C(\tfrac{1}{2}\lambda s_{\rm f}; m_{\rm i} - \mu m_{s_{\rm f}}) \\
&\qquad \times C(lL\lambda; m_l - p\mu),
\end{aligned} \tag{16}$$

and replace

$$\begin{aligned}
&C(s\tfrac{1}{2}s_{\rm f}; m_{\rm s} m_{\rm f} m_{s_{\rm f}}) C(\tfrac{1}{2}\lambda s_{\rm f}; m_{\rm i} - \mu m_{s_{\rm f}})[\lambda] \\
&\quad = S^{\lambda}_{-\mu}(s_{\rm f}, \tfrac{1}{2}) \\
&\quad = \sum_{n=0}^{1} \frac{[s_{\rm f}]^2\,[n]}{\sqrt{2}\,[s]} W(\lambda\tfrac{1}{2}s\tfrac{1}{2}; s_{\rm f} n) \\
&\qquad \times \left(S^s(s,0) \otimes S^n(\tfrac{1}{2}, \tfrac{1}{2})\right)^{\lambda}_{-\mu},
\end{aligned} \tag{17}$$

so that we have a single elegant formula, viz.

$$\begin{aligned}
\mathcal{F}^{\lambda}_{\mu}(n,p) = [n] &\sum_{l=0}^{\infty} \sum_{s_{\rm f}=|s-\frac{1}{2}|}^{(s+(1/2))} \sum_{L=|p|}^{\infty} \sum_{j=L-(1/2)}^{L+(1/2)} \\
&\times W(\lambda\tfrac{1}{2}s\tfrac{1}{2}; s_{\rm f} n) W(L\tfrac{1}{2} l s_{\rm f}; j\lambda) \mathcal{F}^{j}_{l s_{\rm f};L} \mathcal{A}^{\lambda}_{\mu}(\theta),
\end{aligned} \tag{18}$$

expressing the irreducible tensor amplitudes $\mathcal{F}^{\lambda}_{\mu}(n,p)$ for all allowed values of λ, μ in terms of the partial wave multipole amplitudes $\mathcal{F}^{j}_{ls_{\rm f};L}$ which depend only on the c.m. energy. The above is a new result which is applicable to photo- and electro-production of mesons with arbitrary spin-parity s^{π}. It may perhaps be pointed out in the simplest case of pion production with $s = 0$, the four independent amplitudes $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4$ of CGLN [12] have four different formulae which express them in terms of partial wave multipole amplitudes. One needs two additional formulae [14] to express the six independent amplitudes in terms of the partial wave multipole amplitudes in the case of electro-production amplitudes. Our generalized partial wave multipole amplitudes, valid for photo- or electro-production of mesons with arbitrary spin-parity s^π are related to reduced matrix elements $\langle (l(s\frac{1}{2})s_\mathrm{f})j\|T\|(L\frac{1}{2})j\rangle$ through

$$\mathcal{F}^j_{ls_\mathrm{f};L} = \frac{\sqrt{\pi}M}{W} i^{l+L}(-1)^{L+\frac{1}{2}-j}[L][j]^2[s_\mathrm{f}][s]^{-1}\langle (l(s\frac{1}{2})s_\mathrm{f})j\|T\|(L\frac{1}{2})j\rangle \quad (19)$$

and may also be explicitly written either as *magnetic*, *electric* or *longitudinal* using eq. (9) and taking parity conservation into account. We have

$$\mathcal{F}^j_{ls_\mathrm{f};L} = |p|f_-\mathcal{M}^j_{ls_\mathrm{f};L} + ipf_+\mathcal{E}^j_{ls_\mathrm{f};L} - i\sqrt{2}(1-p^2)f_+\mathcal{L}^j_{ls_\mathrm{f};L}, \quad (20)$$

in terms of the *electric*, *magnetic* and *longitudinal* multipole partial wave amplitudes denoted respectively by $\mathcal{E}^j_{ls_\mathrm{f};L}$, $\mathcal{M}^j_{ls_\mathrm{f};L}$ and $\mathcal{L}^j_{ls_\mathrm{f};L}$. The factors

$$f_\pm = \frac{1}{2}[1 \pm \pi(-1)^{L+l}] \quad (21)$$

assume values either 1 or 0 to ensure parity conservation. The angular dependence of $\mathcal{F}^\lambda_\mu(n,p)$ is fully contained in

$$\mathcal{A}^\lambda_\mu(\theta) = (-1)^p C(lL\lambda; m_l - p\mu) Y_{lm_l}(\theta,0), \quad (22)$$

which are independent of s_f, j and isospin quantum numbers. Isospin considerations lead to

$$\mathcal{F}^j_{ls_\mathrm{f};L} = \sum_{I_\gamma} \sum_{I=|I_\mathrm{m}-\frac{1}{2}|}^{(I_\mathrm{m}+(1/2))} C(\tfrac{1}{2}I_\mathrm{m}I;\nu_\mathrm{f}\nu_\mathrm{m}\nu_\mathrm{i})\, C(\tfrac{1}{2}I_\gamma I;\nu_\mathrm{i}0\nu_\mathrm{i})\, \mathcal{F}^{I_\gamma Ij}_{ls_\mathrm{f};L}, \quad (23)$$

where ν_i and ν_f denote the nucleon isospin projections in the initial and final states respectively, while ν_m denotes the meson isospin projection. The isospin I_γ of the photon [3] takes the values $0,1$. It was suggested [21] that there could also be an isotensor component $I_\gamma = 2$ for the photon, in which case the summation over I_γ may be extended from 0 to 2 and one can look out in experiments for non-zero $\mathcal{F}^{I_\gamma Ij}_{ls_\mathrm{f};L}$ with $I_\gamma = 2$. It is important to note that the superscripts I, j of $\mathcal{F}^{I_\gamma Ij}_{ls_\mathrm{f};L}$ in our formalism may readily be identified with the isospin and spin quantum numbers I, j of the contributing resonances in the intermediate state. It may also be noted that the precise composition of $\mathcal{F}^j_{ls_\mathrm{f};L}$ in terms of $\mathcal{F}^{I_\gamma Ij}_{ls_\mathrm{f};L}$ is known using eq. (23), when the charge states of the hadrons are specified and that the isospin indices I_γ, I are to be attached not only to the amplitudes on the left-hand side but also exactly identically to those on the right-hand side of eq. (20).

It is important to note that $\mathcal{F}^\lambda_\mu(n,p)$ satisfy the symmetry property

$$\mathcal{F}^\lambda_{-\mu}(n,-p) = \pi(-1)^{\lambda-\mu}\mathcal{F}^\lambda_\mu(n,p), \quad (24)$$

which enables us to determine the number of independent amplitudes as $4(2s+1)$ for photo-production where p can take only two values $p=\pm 1$, while it is $6(2s+1)$

for electro-production as p can also take an additional value $p = 0$. It may also be noted that $S^0_0(\frac{1}{2}, \frac{1}{2})$ is a unit 2×2 matrix and

$$S^1_{\pm 1}(\tfrac{1}{2}, \tfrac{1}{2}) = \mp \frac{1}{\sqrt{2}}(\sigma_x \pm i\, \sigma_y); \quad S^1_0(\tfrac{1}{2}, \tfrac{1}{2}) = \sigma_z, \tag{25}$$

where σ_x, σ_y, σ_z denote the Pauli spin matrices for the nucleon.

3. Particular cases

3.1 *Photo- and electro-production of pseudoscalar mesons* ($s^\pi = 0^-$)

The new basic amplitudes in this well-known case are $\mathcal{F}^0_0(0, p)$, $\mathcal{F}^1_\mu(1, p)$ with $\mu = \pm 1, 0$ and $p = \pm 1$, since $s = 0$ implies $\lambda = n = 0, 1$. The use of eq. (24) implies

$$\mathcal{F}^0_0(0, 1) = -\mathcal{F}^0_0(0, -1) \tag{26}$$

$$\mathcal{F}^1_0(1, 1) = +\mathcal{F}^1_0(1, -1) \tag{27}$$

$$\mathcal{F}^1_1(1, 1) = -\mathcal{F}^1_{-1}(1, -1) \tag{28}$$

$$\mathcal{F}^1_{-1}(1, 1) = -\mathcal{F}^1_{+1}(1, -1). \tag{29}$$

Thus, the number of independent amplitudes is the same as in [12], i.e., four. We note moreover that only one amplitude, viz. $\mathcal{F}^0_0(0, 1)$ is spin independent, whereas all the other three are spin dependent, which is the case in [12] as well.

In the case of electro-production, we have two additional independent basic amplitudes, viz.,

$$\mathcal{F}^1_0(1, 0) \tag{30}$$

$$\mathcal{F}^1_{+1}(1, 0) = -\mathcal{F}^1_{-1}(1, 0) \tag{31}$$

thus making the total six, which is consistent with [14]. The two additional amplitudes $\mathcal{F}^1_0(1, 0)$ and $\mathcal{F}^1_{+1}(1, 0)$ are spin dependent as in [14].

3.2 *Photo- and electro-production of vector mesons* ($s^\pi = 1^-$)

The independent basic amplitudes for vector meson photo-production in our formalism are

$$\mathcal{F}^1_0(0, 1) = +\mathcal{F}^1_0(0, -1) \tag{32}$$

$$\mathcal{F}^1_{+1}(0, 1) = -\mathcal{F}^1_{-1}(0, -1) \tag{33}$$

$$\mathcal{F}^1_{-1}(0, 1) = -\mathcal{F}^1_{+1}(0, -1) \tag{34}$$

which are spin independent and

$$\mathcal{F}^0_0(1,1) = -\mathcal{F}^0_0(1,-1) \tag{35}$$
$$\mathcal{F}^1_0(1,1) = +\mathcal{F}^1_0(1,-1) \tag{36}$$
$$\mathcal{F}^1_{+1}(1,1) = -\mathcal{F}^1_{-1}(1,-1) \tag{37}$$
$$\mathcal{F}^1_{-1}(1,1) = -\mathcal{F}^1_{+1}(1,-1) \tag{38}$$
$$\mathcal{F}^2_0(1,1) = -\mathcal{F}^2_0(1,-1) \tag{39}$$
$$\mathcal{F}^2_{+1}(1,1) = +\mathcal{F}^2_{-1}(1,-1) \tag{40}$$
$$\mathcal{F}^2_{-1}(1,1) = +\mathcal{F}^2_{+1}(1,-1) \tag{41}$$
$$\mathcal{F}^2_{+2}(1,1) = -\mathcal{F}^2_{-2}(1,-1) \tag{42}$$
$$\mathcal{F}^2_{-2}(1,1) = -\mathcal{F}^2_{+2}(1,-1) \tag{43}$$

which are spin dependent, taking the total to twelve independent amplitudes, which is in agreement with [16].

Electro-production has not been considered either in [16] or by others. In our formalism, we readily identify the additional independent amplitudes as

$$\mathcal{F}^1_0(0,0) \tag{44}$$
$$\mathcal{F}^1_1(0,0) = -\mathcal{F}^1_{-1}(0,0) \tag{45}$$
$$\mathcal{F}^1_0(1,0) \tag{46}$$
$$\mathcal{F}^1_1(1,0) = -\mathcal{F}^1_{-1}(1,0) \tag{47}$$
$$\mathcal{F}^2_1(1,0) = +\mathcal{F}^2_{-1}(1,0) \tag{48}$$
$$\mathcal{F}^2_2(1,0) = -\mathcal{F}^2_{-2}(1,0), \tag{49}$$

i.e., six in addition to twelve taking the total to eighteen. Of the additional six, two amplitudes $\mathcal{F}^1_0(0,0)$ and $\mathcal{F}^1_1(0,0)$ are spin independent, and the remaining are spin dependent.

3.3 *Photo- and electro-production of tensor mesons* ($s^\pi = 2^+$)

Application of the symmetry relation represented by eq. (24) shows that photo-production of tensor mesons with spin parity $s^\pi = 2^+$ is characterized by a set of twenty independent irreducible tensor amplitudes whereas electro-production of tensor mesons needs an additional ten amplitudes, thus taking the total to thirty.

4. Summary and outlook

A theoretical formalism has been outlined in this paper for photo- and electro-production of mesons with arbitrary spin-parity s^π, where the reaction amplitude $\mathcal{F}$ in each case is expressed in terms of a basic set of independent irreducible tensor amplitudes $\mathcal{F}^\lambda_\mu(n,p)$ of rank λ. Those with $n = 0$ are nucleon spin independent whereas those with $n = 1$ are spin dependent. p can assume only two values $p = \pm 1$ in the case of photo-production whereas it can assume an additional value of $p = 0$

Table 1. The number of independent irreducible tensor amplitudes for photo- and electro-production of mesons with arbitrary spin-parity s^π. Only those amplitudes with $p = \pm 1$ are allowed for photo-production, whereas electro-production includes amplitudes with $p = 0$ as well. The nucleon spin dependent amplitudes are those with $n = 1$ while those with $n = 0$ are nucleon spin independent.

	$p = \pm 1$		$p = 0$		
s^π	$n = 0$	$n = 1$	$n = 0$	$n = 1$	Total
0^-	1	3	–	2	6
0^+	1	3	1	1	6
1^-	3	9	2	4	18
1^+	3	9	1	5	18
2^-	5	15	2	8	30
2^+	5	15	3	7	30
3^-	7	21	4	10	42
3^+	7	21	3	11	42
4^-	9	27	4	14	54
4^+	9	27	5	13	54

corresponding to longitudinal polarization in the case of electro-production. The number of amplitudes increase with spin s of the meson produced as shown in table 1. The number of independent amplitudes in our formalism is in agreement with the number determined in some particular cases employing different arguments by earlier authors. For example, the number of independent irreducible tensor amplitudes is four in the case of photo-production of pseudoscalar mesons. The four different independent amplitudes have been introduced in a different way by Chew *et al* in their famous paper [12]. Each of the four amplitudes of CGLN have different formulae for expressing them in terms of partial wave multipole amplitudes. A highlight of our approach is that a single elegant formula, namely, eq. (18) describes the expansion of the independent irreducible tensor amplitudes in terms of the partial wave multipole amplitudes, irrespective of whether they are four as in the case of photo-production of pseudoscalar mesons or six in the case of electro-production of pseudo-scalar mesons or ten in the case of photo-production of vector mesons or eighteen as in the case of electro-production of vector mesons or twenty in the case of photo-production of tensor mesons or thirty in the case of electro-production of tensor mesons. For the photo-production of isovector mesons, CGLN employ Watson's approach for isospin indexing of each of their amplitudes by three superscripts $(+), (-), (0)$, specific linear combinations of which have to be taken for the reaction when initial and final charge states are given. Instead, our amplitudes carry a specific isospin index I. The explicit superscripts I, j permit us to identify directly the resonance contributions coming from the intermediate states.

As more and more experimental data are forthcoming at higher energies, we hope that the unified and simpler formalism outlined in this paper will be found useful to analyze measurements. We have not explicitly written down the new basic amplitudes of our formalism for mesons with spin $s \geq 2$, as experimental studies are yet to be reported. However, it is clear that the formalism is readily extendable to photo- and electro-production of mesons like $f_2(1270)$ or $a_2(1320)$ or $f_2'(1525)$ with spin parity 2^+ or $\pi_2(1670)$ with spin-parity 2^- and $\omega_3(1670)$ or $f_3(1690)$ with spin-parity 3^- and $a_4(2040)$ or $f_4(2050)$ with spin-parity 4^+ which are known [3] to exist. Since the energy at JLab can go up to 6 GeV, it is clearly possible to reach the thresholds for production of these higher spin mesons.

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