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Beyond the New Standard Model in neutrino oscillations

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Abstract. We discuss effects of new physics (NP) in neutrino oscillation experiments. Such effects can modify a production neutrino flux, a detection cross-section and a matter transition. As a result, the NP effects change neutrino oscillations both in vacuum and in matter. A relation between the small effects of NP and the oscillation parameters is discussed. It is shown for which parameters the NP effects are suppressed and when they are potentially large. Oscillations of non-unitary mixed neutrinos are presented in more details.

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1. Introduction

For several years, contrary to the predictions of the orthodox Standard Model (OSM), we know that neutrinos are massive particles [1], and OSM must be extended. In such extension, sometimes called New Standard Model (νSM), two scenarios are possible. In the first one, NP does not appear at low-energy scale – presently observed neutrino masses are related directly to much larger energy scale, GUT or even Planck scale. In the second scenario, νSM and a model of neutrino masses are accompanied by NP at low-energy scale.

The low-energy NP can modify neutrino oscillations both in vacuum and in matter. Many different extensions of OSM are considered in the literature – models with not only three active neutrinos, but also sterile [2] and heavy [3] neutrinos, general models with lepton flavour violation [4], non-standard interaction [5], flavour-changing neutral currents [6], general fermion interactions [7], mass-varying neutrinos [8], violation of Lorentz symmetry [9], violation of principle of general relativity [10] and of the CPT symmetry [11]. Other effects which modify neutrino oscillations were also considered – neutrino wave packet decoherence [12], neutrino's decays [13] or neutrino quantum decoherence [14].

In any real experiment with relativistic neutrino oscillations the measured rate ($\Gamma_{\mathrm{P}\to\mathrm{D}}$) is the product of three terms:

- the flux of the produced (P) initial neutrinos $\mathrm{d}\Gamma_{\mathrm{P}}/L^2\mathrm{d}E_\nu$,
- the probability that, after traveling a distance L, the produced (P) neutrino will oscillate into detected (D) neutrinos, $P_{\mathrm{P}\to\mathrm{D}}(E_\nu, L)$, and
- the cross-section for detection of (D) neutrinos, $\sigma_{\mathrm{D}}(E_\nu)$, all of them integrated over the neutrino energy:

$$\Gamma_{\mathrm{P}\to\mathrm{D}} = \int \mathrm{d}E_\nu \frac{\mathrm{d}\Gamma_{\mathrm{P}}}{L^2\mathrm{d}E_\nu} P_{\mathrm{P}\to\mathrm{D}}(E_\nu, L)\sigma(E_\nu). \tag{1}$$

The NP gives corrections to all three terms, but the feature of these corrections are quite different. Apart from the trivial (L^2) factor which appears in the production flux, both production and detection parts are L independent. Here we would like to concentrate on the NP dependence of the transition probability $P_{\mathrm{P}\to\mathrm{D}}(E_\nu, L)$ on the production ($|\nu_{\mathrm{P}}\rangle$) and detection ($|\nu_{\mathrm{D}}\rangle$) states. The states $|\nu_{\mathrm{P}}\rangle$ produced in the process of a charged lepton (l) scattering on a particle (X)

$$l + X \to \nu_i + Y, \tag{2}$$

are defined in the following way:

$$|\nu_{\mathrm{P}}\rangle = \frac{1}{N}\sum A(l + X \to \nu_i + Y)|\nu_i\rangle, \tag{3}$$

where $A(l + X \to \nu_i + Y)$ is the amplitude for the process (2) in which neutrino in eigenmass state $|\nu_i\rangle$ is produced, and

$$N = \sqrt{\sum_{i=1}^{3} |A(l + X \to \nu_i + Y)|^2}, \tag{4}$$

is the normalization factor.

In a similar way the neutrino states $|\nu_{\mathrm{D}}\rangle$ of detection process are defined. We can see that neutrino states depend on the physics which rules the production and the detection processes. Moreover, for oscillations in the matter NP modifies any coherent neutrino scattering on the background fermions and both scattering amplitudes and neutrino effective masses acquire some corrections. We do not consider any specific extensions of the OSM. Our aim is to parametrize the impact of NP on oscillation data in a model-independent way. Using existing quark and charged lepton data we can bound the NP's parameters. In this way we are able to predict the size of the NP effects in neutrino oscillation experiments.

In the next section we parametrize NP in a model-independent way. The corrections to the production and the detection states as well as to the coherent neutrino scattering inside the matter, are described. We discuss the oscillation transition probabilities. The leading terms in the expansion of transition probabilities including the NP parameters, driven by two factors $\alpha = \delta m^2_{\mathrm{sol}}/\delta m^2_{\mathrm{atm}}$ and $\sin(2\theta_{13})$ are found. Next, in §3, we discuss relations between phenomenological parameters which describe the modification of initial and final states and parameters which change neutrino transitions in matter. We assume that the transition between flavour and mass eigenstates is given by a non-unitary matrix. Such effects could arise as a result of mixing between light or heavy neutrinos, and/or because lepton number is not conserved. Finally §4 gives the conclusion.

2. Where should we look for the new physics effects?

We parametrize NP in a model-independent way as accurate as possible. First, for initial and final neutrino states the mixing between neutrino flavour and mass eigenstates are described by a non-unitary matrix. Then the neutrino production (or detection) states are given by [3]

$$|\nu_{\rm P}\rangle \; = \sum_{i=1}^{3} \mathcal{U}^*_{\rm Pi} \, |\nu_i\rangle \; , \tag{5}$$

where the $\mathcal{U}$ matrix is not unitary and can be identified by (see eq. (3))

$$\mathcal{U}^*_{{\rm P}i} = \frac{A(l + X \to \nu_i + Y)}{N} \; . \tag{6}$$

If we assume that the $\mathbf{U}$ matrix describes the normal unitary transition and is parametrized by traditional mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and the CP-breaking angle δ,

$$|\nu_\alpha\rangle \; = \sum_{i=1}^{3} \mathbf{U}^*_{\alpha i} \, |\nu_i\rangle \; , \tag{7}$$

then the $\mathcal{U}$ matrix can be parametrized using close-to-unity matrix Λ:

$$\mathcal{U} = \Lambda \mathbf{U} \quad \text{with} \quad \Lambda = 1 - \delta\Lambda. \tag{8}$$

Then the production (detection) states can also be decomposed in the orthonormal flavour base [4]:

$$|\nu_{\rm P}\rangle = \sum_{\alpha=1}^{3} \Lambda^*_{{\rm P}\alpha} |\nu_\alpha\rangle \; . \tag{9}$$

The small parameters $\delta\Lambda_{{\rm P}\alpha} \equiv c_{{\rm P}\alpha}$ depends on the production (detection) process and are bounded by existing charged lepton data.

The NP part of the effective Hamiltonian which describe the coherent evolution of neutrino states inside matter can be parametrized in a similar way. Let us assume that in the orthonormal flavour base the full effective Hamiltonian has the SM part and some additional NP part, so

$$\mathbf{H}^{\rm eff} = \mathbf{H}^{\rm SM} + \mathbf{H}^{\rm NP}. \tag{10}$$

The SM part is given by

$$\mathbf{H}^{\rm SM} = \frac{1}{2E_\nu} \left[\mathcal{U} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta m^2_{21} & 0 \\ 0 & 0 & \delta m^2_{31} \end{pmatrix} \mathcal{U}^\dagger + \begin{pmatrix} A_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] , \tag{11}$$

where $A_e = 2\sqrt{2} E_\nu G_{\rm F} N_e$. In the same base,

$$\mathbf{H}^{\rm NP} = \frac{A_e}{2E_\nu}\begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu}{\rm e}^{i\chi_{e\mu}} & \varepsilon_{e\tau}{\rm e}^{i\chi_{e\tau}} \\ \varepsilon_{e\mu}{\rm e}^{-i\chi_{e\mu}} & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau}{\rm e}^{i\chi_{\mu\tau}} \\ \varepsilon_{e\tau}{\rm e}^{-i\chi_{e\tau}} & \varepsilon_{\mu\tau}{\rm e}^{-i\chi_{\mu\tau}} & \varepsilon_{\tau\tau} \end{pmatrix}. \tag{12}$$

Having the effective Hamiltonian (eq. (10)) we can resolve the evolution equation for flavour transition amplitude $A_{\alpha\to\beta}(t)$:

$$i\frac{\rm d}{{\rm d}t}A_{\alpha\to\beta}(t) = \sum_\eta \mathbf{H}^{\rm eff}_{\beta\eta}A_{\eta\to\alpha}(t), \tag{13}$$

and from eq. (7) the full transition between experimentally produced and detected states are given by

$$A_{\rm P\to D} \equiv \langle\nu_{\rm D}|\nu_{\rm P}(t)\rangle = \sum_\beta \langle\nu_{\rm D}|\nu_\beta\rangle\langle\nu_\beta|\nu_{\rm P}(t)\rangle. \tag{14}$$

For uniform matter density distribution ($N_e = {\rm const}(L)$), eq. (13) can be resolved analytically. If $N_e \neq {\rm const.}(L)$ more complicated numerical methods must be used. Here we present analytical formulas for uniform matter density. Numerical results for realistic Earth profile are calculated elsewhere (see e.g. [3]).

For high-energy neutrino beam with $E_\nu \approx$ GeV, two parameters which describe SM neutrino oscillation are small:

$$\alpha = \left|\frac{\delta m^2_{\rm sol}}{\delta m^2_{\rm atm}}\right| \approx 0.03 \text{ and } \sin^2(2\theta_{13}) \leq 0.05. \tag{15}$$

For smaller energy of neutrinos $E_\nu \approx$ MeV and moderate matter density (e.g. like in Earth, $2 \leq \rho[g/{\rm cm}^3] \leq 11$) the third small parameter exists:

$$\widehat{A}_e \equiv \frac{A_e}{|\delta m^2_{31}|} \simeq 0.001. \tag{16}$$

Using the perturbation methods we can calculate the transition amplitudes in the required approximation. $\mathbf{H}^{\rm SM}$ is an initial Hamiltonian and $\mathbf{H}^{\rm NP}$ is a small perturbation. If we denote the three eigenvalues and eigenvectors of $\mathbf{H}^{\rm SM}$ as λ_i and $|\lambda_i\rangle$ (for $i = 1,2,3$), the full eigenvalues and eigenvectors of $\mathbf{H}^{\rm eff}$ (eq. (10)) in the first order, are given by

$$\lambda_i \to \lambda_i^{\rm f} = \lambda_i + \langle\lambda_i|\mathbf{H}^{\rm NP}|\lambda_i\rangle, \tag{17}$$

and

$$|\lambda_i\rangle \to |\lambda_i^{\rm f}\rangle = |\lambda_i\rangle + \sum_{k\neq i}\frac{\langle\lambda_k|\mathbf{H}^{\rm NP}|\lambda_i\rangle|\lambda_k\rangle}{\lambda_i - \lambda_k}. \tag{18}$$

Resolving eq. (13), we obtain the flavour transition amplitude as

$$A_{\alpha\to\beta}(L=t) = \sum_k \mathbf{W}_{\beta k}\mathbf{W}^*_{\alpha k}\;{\rm e}^{-i\frac{\lambda_k^{\rm f}}{2E_\nu}L}, \tag{19}$$

where the matrix $\mathbf{W}_{\beta i} = \langle \nu_\beta | \lambda_i^{\rm f} \rangle$ diagonalizes the full Hamiltonian (10):

$$\mathbf{W}^\dagger H^{\rm eff} \mathbf{W} = {\rm diag}(\lambda_1^{\rm f},\ \lambda_2^{\rm f},\ \lambda_3^{\rm f}). \tag{20}$$

Now the real transition amplitude depends on initial and final states. For states given by eq. (7) we have

$$A_{\rm P \to D}(L) = \sum_k W_{{\rm D}k} W_{{\rm P}k}^* \ {\rm e}^{-i \frac{\lambda_k^{\rm f}}{2E_\nu} L}, \tag{21}$$

where the matrix W is a product of the Λ and $\mathbf{W}$ matrices:

$$W = \Lambda \mathbf{W}. \tag{22}$$

Generally the Λ matrix is non-unitary and the full mixing matrix (22) has the same feature. This property may have important consequences for oscillation phenomena. If lepton number violation in production and detection processes takes place, it must be very small, then the indices P or D are very well identified by flavour numbers α or β. In what follows we will use such flavour indices to identify initial or final neutrino states. The probability for neutrino transition after traveling distance L in a uniform medium cast into the following form:

$$P_{\alpha \to \beta} \equiv |A_{\alpha \to \beta}|^2 = \delta_{\alpha\beta} - 2{\rm Re}(\Lambda_{\alpha\beta}) - 4 \sum_{i>k} R_{\alpha\beta}^{ik} \sin^2 \Delta_{ik} - \sum_{i>k} I_{\alpha\beta}^{ik} \sin 2\Delta_{ik}, \tag{23}$$

where

$$R_{\alpha\beta}^{ik} = {\rm Re}(T_{\alpha\beta}^{ik}), \qquad I_{\alpha\beta}^{ik} = {\rm Im}(T_{\alpha\beta}^{ik}), \tag{24}$$

and

$$T_{\alpha\beta}^{ik} = W_{\alpha i} W_{\beta k} W_{\alpha k}^* W_{\beta i}^*, \qquad \Delta_{ik} = (\lambda_i^{\rm f} - \lambda_k^{\rm f}) \frac{L}{4E_\nu}. \tag{25}$$

Each element of the W matrix can decompose into the SM term and a small part linear in the NP, $\delta\Lambda \equiv c$ and ε parameters:

$$W = \Lambda \mathbf{W} = (1 - \delta\Lambda)(W^{\rm SM} + \delta W^\varepsilon) = W^{\rm SM} + \delta W^{\rm NP}, \tag{26}$$

where

$$\delta W^{\rm NP} = \delta W^\varepsilon + \delta W^c \quad \text{and} \quad \delta W^c = -\delta\Lambda W^{\rm SM}. \tag{27}$$

The same can be done for Δ mass differences:

$$\Delta_{ik} = \Delta_{ik}^{\rm SM} + \delta\Delta_{ik}^{\rm NP}. \tag{28}$$

For the neutrino transition probability we obtain similar expansion:

$$P_{\alpha\to\beta}(L) = P^{\rm SM}_{\alpha\to\beta}(L) + \delta P^{\rm NP}_{\alpha\to\beta}(L), \tag{29}$$

where the NP part is once more decomposed into three parts, which have different physical origin:

$$\delta P^{\rm NP}_{\alpha\to\beta}(L) = \delta P^{c}_{\alpha\to\beta}(L) + \delta P^{\varepsilon}_{\alpha\to\beta}(L), \tag{30}$$

with

$$\delta P^{\varepsilon}_{\alpha\to\beta}(L) = \delta P^{\rm int}_{\alpha\to\beta}(L) + \delta P^{\rm mass}_{\alpha\to\beta}(L). \tag{31}$$

The first term in eq. (29) describes the typical SM neutrino transition. The three pieces in eqs (30) and (31) describe the impact of NP on probability coming from (i) production and detection states ($\delta P^{c}_{\alpha\to\beta}(L)$), (ii) additional NP neutrino interaction inside matter ($\delta P^{\rm int}_{\alpha\to\beta}(L)$), and (iii) an effective neutrino mass change ($\delta P^{\rm mass}_{\alpha\to\beta}(L)$). By appropriate decomposition of the probability eq. (29), all these contributions can be calculated:

$$\begin{aligned}\delta P^{c}_{\alpha\to\beta}(L) &= (c_{\alpha\alpha} + c_{\beta\beta}) P^{\rm SM}_{\alpha\to\beta}(L) - 2{\rm Re}(c_{\alpha\beta}) \\ &\quad -4\sum_{i>k} (\delta R^{c})^{ik}_{\alpha\beta} \sin^2 \Delta^{\rm SM}_{ik} - 2\sum_{i>k} (\delta I^{c})^{ik}_{\alpha\beta} \sin 2\Delta^{\rm SM}_{ik},\end{aligned} \tag{32}$$

$$\delta P^{\rm int}_{\alpha\to\beta}(L) = -4\sum_{i>k} (\delta R^{\varepsilon})^{ik}_{\alpha\beta} \sin^2 \Delta^{\rm SM}_{ik} - 2\sum_{i>k} (\delta I^{\varepsilon})^{ik}_{\alpha\beta} \sin 2\Delta^{\rm SM}_{ik}, \tag{33}$$

and finally the mass term

$$\delta P^{\rm mass}_{\alpha\to\beta}(L) = -\sum_{i>k} [4(R^{\rm SM})^{ik}_{\alpha\beta} \sin 2\Delta^{\rm SM}_{ik} + 2(I^{\rm SM})^{ik}_{\alpha\beta} \cos 2\Delta^{\rm SM}_{ik}] \delta\Delta^{\rm NP}_{ik}. \tag{34}$$

In eqs (32) and (33) tensors δR and δI are respectively real and imaginary parts of the full tensor $T^{i,k}_{\alpha\beta}$ linear in δW^c or δW^ε.

In the case of the non-unitary neutrino mixing the probability is not conserved and for any initial neutrino state α:

$$\sum_{{\rm all}\ \beta} P_{\alpha\to\beta} \neq 1. \tag{35}$$

The effect is due to the non-vanishing c parameters $c_{\alpha\beta} \neq 0$ (eqs (8) and (32)) which change the initial and final states and modify the oscillation probabilities $P_{\alpha\to\beta}(L)$, even in vacuum. Now we will consider the new effect for neutrino oscillation in matter, so we assume for a moment that $c_{\alpha\beta} = 0$ but $H^{\rm NP}$ in eq. (12) is non-vanishing. This is an unusual case – normally the c and ε parameters are connected, as we will see in the next section. Neglecting the c parameters, the $H^{\rm NP}$ Hamiltonian alone is hermitian, the probability is conserved and for all initial α flavour neutrino states:

$$\sum_{\text{all }\beta} P_{\alpha\to\beta} = 1. \tag{36}$$

The $P_{\alpha\to\beta}(L)$ probabilities are the sum of three terms:

$$P_{\alpha\to\beta}(L) = P^{\text{SM}}_{\alpha\to\beta} + \delta P^{\text{int}}_{\alpha\to\beta} + \delta P^{\text{mass}}_{\alpha\to\beta} \tag{37}$$

and conservation of probability means that

$$\sum_{\text{all }\beta} P^{\text{SM}}_{\alpha\to\beta} = 1, \qquad \sum_{\text{all }\beta} \delta P^{\text{int}}_{\alpha\to\beta} = 0, \qquad \sum_{\text{all }\beta} \delta P^{\text{mass}}_{\alpha\to\beta} = 0. \tag{38}$$

All terms in eq. (37) can be decomposed as power series of small parameters, α and $\sin(2\theta_{13})$. In the SM part we keep terms up to $0(\alpha^2)$ and $0(\sin^2(2\theta_{13})$, so all probabilities will have the form

$$\begin{aligned} P^{\text{SM}}_{\beta\to\gamma} = {} & \delta_{\beta\gamma} + A^0_{\beta\gamma} + \alpha A^\alpha_{\beta\gamma} + \alpha^2 A^{\alpha\alpha}_{\beta\gamma} \\ & + \sin^2(2\theta_{13}) A^s_{\beta\gamma} + \alpha\sin(2\theta_{13}) A^{\alpha s}_{\beta\gamma}. \end{aligned} \tag{39}$$

We will not reproduce all terms $A^x_{\beta\gamma}$ for $x = \alpha, \alpha\alpha, s, \alpha s$, as they can be found elsewhere (see e.g. [15]). As an example we only present the amplitudes A's for $\nu_\mu \to \nu_e$ transition:

$$A^0_{\mu e} = 0, \tag{40}$$

$$A^\alpha_{\mu e} = 0, \tag{41}$$

$$A^{\alpha\alpha}_{\mu e} = \frac{\cos^2(\theta_{23})\sin^2(2\theta_{12})\sin^2(\widehat{A}_e\Delta)}{\widehat{A}_e^2}, \tag{42}$$

$$A^s_{\mu e} = \frac{\sin^2(\theta_{23})\sin^2(\Delta(1-\widehat{A}_e))}{(1-\widehat{A}_e)^2}, \tag{43}$$

$$\begin{aligned} A^{\alpha s}_{\mu e} = {} & \frac{\sin(2\theta_{12})\sin(2\theta_{23})\sin(\widehat{A}_e\Delta)\sin(\Delta(1-\widehat{A}_e))}{\widehat{A}_e(1-\widehat{A}_e)} \\ & \times (\cos\delta\cos\Delta - \sin\delta\sin\Delta), \end{aligned} \tag{44}$$

where the θ_{ij} and δ angles define the MNS mixing matrix (eq. (7)), and

$$\Delta = \frac{\delta m^2_{31} L}{2E_\nu}. \tag{45}$$

We would like to underline that for any β and x,

$$\sum_{\text{all }\gamma} A^x_{\beta\gamma} = 0. \tag{46}$$

For processes with electron neutrino the $A^0_{\mu e}$ term that is not suppressed does not appear, for channels with muon and tau neutrinos they are equal up to the sign

$$A^0_{\beta\gamma} = \pm\sin^2(2\theta)\sin^2(\Delta). \tag{47}$$

The CP-violating terms (with opposite signs for neutrino and antineutrino oscillations) exist only in appearance channels and in the case of neutrino oscillations in vacuum they are equal up to the sign:

$$A^{\alpha s}_{\beta\gamma} = \pm \sin\delta \sin(2\theta_{12}) \, \frac{\sin\Delta \sin(\widehat{A}_e\Delta)\sin((1-\widehat{A}_e)\widehat{A}_e)}{\widehat{A}_e(1-\widehat{A}_e)}. \tag{48}$$

We can see that the CP-violation effects are strongly suppressed, i.e. in each channel they appear in terms multiplied by the product of $\alpha\sin(2\theta_{13})$.

The NP terms in eq. (37) are decomposed only up to $0(\alpha)$ and $0(\sin(2\theta_{13}))$, so their general structures have the form:

$$\delta P^{\rm int}_{\beta\to\gamma} = B^0_{\beta\gamma} + \alpha B^\alpha_{\beta\gamma} + \sin(2\theta_{13})B^s_{\beta\gamma} \tag{49}$$

and similarly

$$\delta P^{\rm mass}_{\beta\to\gamma} = C^0_{\beta\gamma} + \alpha C^\alpha_{\beta\gamma} + \sin(2\theta_{13})B^s_{\beta\gamma}. \tag{50}$$

Again, we will not present here the formulas for all channels. As an example, for $\nu_\mu \to \nu_e$ transition the $\delta P^{\rm mass}_{\mu\to e}$ and $B^0_{\mu e}$ parts equal zero. The two non-vanishing elements are given by

$$\begin{aligned} B^0_{\mu e} = {} & \frac{1}{(\widehat{A}_e-1)\widehat{A}_e^2}\cos\theta_{23}\sin(2\theta_{12})\{\varepsilon_{e\tau}\cos\theta_{23}\sin\theta_{23} \\ & \times[\cos\chi_{e\tau}[\widehat{A}_e\cos(2\Delta) - \widehat{A}_e\cos(2(\widehat{A}_e-1)\Delta) \\ & -2(\widehat{A}_e-2)\sin^2(\widehat{A}_e\Delta)] + \sin\chi_{e\tau}\widehat{A}_e \\ & \times[\sin(2\Delta) - \sin(2(1-\widehat{A}_e)\Delta) - \sin(2\widehat{A}_e\Delta)]] \\ & +\varepsilon_{e\mu}[2\cos\chi_{e\mu}\sin(\widehat{A}_e\Delta)[2(\widehat{A}_e-1)\cos^2\theta_{23}\sin(\widehat{A}_e\Delta) \\ & +\widehat{A}_e(\sin((\widehat{A}_e-2)\Delta)+\sin(\widehat{A}_e\Delta))\sin^2\theta_{23}] \\ & +\sin\chi_{e\mu}\widehat{A}_e\sin^2\theta_{23}[\sin(2\Delta) - \sin(2(1-\widehat{A}_e)\Delta) - \sin(2\widehat{A}_e\Delta)]]\} \end{aligned} \tag{51}$$

and

$$\begin{aligned} B^s_{\mu e} = {} & \frac{1}{(\widehat{A}_e-1)\widehat{A}_e^2}\sin\theta_{23}\{\varepsilon_{e\tau}\sin\theta_{23}\cos\theta_{23} \\ & \times[\cos(\delta+\chi_{e\tau})[1+\widehat{A}_e - (\widehat{A}_e-1)\cos(2\Delta) - \cos(2(\widehat{A}_e-1)\Delta) \\ & -(1-\widehat{A}_e)\cos(2\widehat{A}_e\Delta)] + \sin(\delta+\chi_{e\tau})(\widehat{A}_e-1) \\ & \times[\sin(2\Delta) - \sin(2(1-\widehat{A}_e)\Delta) - \sin(2\widehat{A}_e\Delta)]] \\ & +\varepsilon_{e\mu}[\cos(\delta+\chi_{e\mu})[(\widehat{A}_e-1)\cos^2\theta_{23}(\cos(2\Delta) - \cos(2(\widehat{A}_e-1)\Delta) \\ & +2\sin^2(\widehat{A}_e\Delta)) + 4\widehat{A}_e\sin^2((\widehat{A}_e-1)\Delta)\sin^2\theta_{23}] \\ & +\sin(\delta+\chi_{e\mu})(\widehat{A}_e-1)\cos^2\theta_{23}[\sin(2(1-\widehat{A}_e)\Delta) \\ & -\sin(2\Delta) + \sin(2\widehat{A}_e\Delta)]]\}. \end{aligned} \tag{52}$$

The NP terms which are not suppressed by α and $\sin(2\theta_{13})$ do not appear in common muon and electron neutrino channels. The $B^0_{\beta\gamma}$ and $C^0_{\beta\gamma}$ terms give contribution only to (ν_μ, ν_μ), (ν_μ, ν_e) and (ν_e, ν_e) channels. They have the form

$$B^0_{\beta\gamma} = \pm\sin(4\theta_{23})\{\sin(2\theta_{23})(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) + 2\cos(2\theta_{23})\cos[\chi_{\mu\tau}]\varepsilon_{\mu\tau}\}\sin^2\Delta \quad (53)$$

and

$$C^0_{\beta\gamma} = \pm\Delta\sin^2(2\theta_{23})\{\cos(2\theta_{23})(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) + 2\sin(2\theta_{23})\cos[\chi_{\mu\tau}]\varepsilon_{\mu\tau}\}\sin(2\Delta). \quad (54)$$

It is worth to stress that the interaction term (eq. (53)) and the mass term (eq. (54)) have different dependence on the baseline L. The C term is linear in L while B is oscillating. The SM CP-violating parts, with the Dirac δ phase, appear only in B^s and vanish for $\delta \to 0$ or π. Any NP CP-violating terms, proportional to $(\varepsilon_{e\mu}\sin\chi_{e\mu})$ or to $(\varepsilon_{e\tau}\sin\chi_{e\tau})$ give contribution both to α and $\sin(2\theta_{13})$ suppressing factors. They do not contribute to B^0 and are absent also in δP^{mass} probabilities. And final remark, in the approximation which we consider, the NP effects proportional to ε_{ee} end CP-violation given by the $\chi_{\mu\tau}$ phase do not appear.

3. Relations between c and ε parameters

The $c_{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$ parameters are connected in a model-dependent way. This means that to find a relation among them, specific NP models must be considered. As an example we consider two such models. At first, let us take the model with heavy neutrinos and with their non-negligible couplings to the light ones, a left-handed current interaction is assumed. To be more precise we assume the charge current Lagrangian in the form ($n > 3$):

$$\mathcal{L}_{\text{CC}} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{n} \psi_\alpha \gamma^\mu (1-\gamma_5)(\mathcal{U}_\nu)_{\alpha i}\nu_i W^-_\mu + \text{h.c.}, \quad (55)$$

and similarly the neutral current Lagrangian:

$$\mathcal{L} = \frac{e}{2\sin(2\theta_W)} \sum_{i,j} \bar{\nu}_i \gamma^\mu (1-\gamma_5)\Omega_{ij}\nu_j Z_\mu, \quad (56)$$

where

$$\Omega_{ij} = \sum_{\alpha=e,\mu,\tau} (\mathcal{U}_\nu)^*_{\alpha i}\, (\mathcal{U}_\nu)_{\alpha j}. \quad (57)$$

If we define the matrix $\mathcal{U}_\nu$ as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_e \\ N_\mu \\ N_\tau \end{pmatrix} = \begin{pmatrix} \mathcal{U}_{\alpha i} & V_{\alpha I} \\ V'_{Ai} & \mathcal{U}'_{AI} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3 \end{pmatrix}, \tag{58}$$

then from eq. (8) and unitary condition for full $\mathcal{U}_\nu$ matrix we get

$$\Lambda\Lambda^\dagger = I - VV^\dagger. \tag{59}$$

We can parametrize:

$$\delta\Lambda = \frac{1}{2} \begin{pmatrix} c_{ee} & 0 & 0 \\ 2c_{\mu e} & c_{\mu\mu} & 0 \\ 2c_{\tau e} & 2c_{\tau\mu} & c_{\tau\tau} \end{pmatrix}, \tag{60}$$

where

$$c_{\alpha\beta} = (VV^\dagger)_{\alpha\beta}. \tag{61}$$

Then we are able to calculate the effective NP matter Hamiltonian as

$$\mathbf{H}^{\mathrm{NP}} = \frac{1}{2E_\nu} \left(-A_e[\delta\Lambda^\dagger E(1) + E(1)\delta\Lambda] + \frac{A_n}{2}[\delta\Lambda + \delta\Lambda^\dagger] \right), \tag{62}$$

where $A_n = 2\sqrt{2}G_{\mathrm{F}}E_\nu N_n \varrho$ depends on neutron matter density N_n, $\varrho = \frac{M_W^2}{\cos^2\theta_W M_Z^2}$. Now we can easily find relations between c and ε parameters:

$$\varepsilon_{ee} = c_{ee}\left(-1 + \frac{A_n}{2A_e}\right) \quad \text{and} \quad \varepsilon_{\alpha\beta} = \frac{A_n}{2A_e} c_{\alpha\beta} \quad \text{for } \alpha, \beta \neq e. \tag{63}$$

In the case of the left–right symmetric model a situation is more complicated. The right-handed currents change the oscillation of Dirac and Majorana neutrinos. For Dirac neutrinos the effective NP Hamiltonian is the same as in the previous example (eq. (62)) where the ϱ parameter is given by

$$\varrho \to \varrho' = 1 + \gamma \frac{1 - 2\sin^2\theta_W}{\cos(2\theta_W)}, \quad \text{with} \quad \gamma = \frac{M_{Z_1}^2}{M_{Z_2}^2}. \tag{64}$$

In the case of Majorana neutrinos, $\mathbf{H}^{\mathrm{NP}}$ is modified by the right-handed currents giving:

$$\mathbf{H}^{\mathrm{NP}}(\text{Majorana}) = \mathbf{H}^{\mathrm{NP}}(\text{Dirac}) - A_e\beta(\bar{V}^\dagger E(1)\bar{V}) - A_n\gamma \frac{\cos^4\theta_W}{\cos(2\theta_W)}(\delta\Lambda^\dagger + \delta\Lambda), \tag{65}$$

where

$$\beta = \frac{M_{W_1}^2}{M_{W_2}^2} \quad \text{and} \quad \bar{V} = V'\mathcal{U}^\dagger. \tag{66}$$

This gives a connection between the c and ε parameters. For Dirac neutrinos these are the same as before (eq. (63)). In the case of Majorana neutrinos we get

$$\varepsilon_{\alpha\beta} = c_{\alpha\beta}\frac{A_n}{A_e}\varrho\left(\frac{\varrho'}{2} - \gamma\frac{\cos^4\theta_W}{\cos(2\theta_W)}\right) - \beta(\bar{V}^*_{N_e\alpha}\bar{V}_{N_e\beta}), \tag{67}$$

with

$$\varrho = \frac{M_{W_1}^2}{\cos^2\theta_W M_{W_2}^2}. \tag{68}$$

From experimental estimations we know that the additional charged and neutral bosons are very heavy, parameters β and γ are small and practically in the L–R model, Dirac and Majorana neutrinos oscillate in the same way.

4. Conclusions

We have investigated where is the best place to look for any visible new physics effects in the future neutrino oscillation GeV scale experiments. The influence of NP on production and detection states has been parametrized in a model-independent way. A modification of the neutrino states changes their oscillation behaviour both in vacuum and in matter. Oscillations in matter are additionally modified by new neutrino interactions with the background matter particles. The calculation was performed for neutrino oscillations in matter of constant density by expansions of appropriate small parameters. The full neutrino transition probabilities have been decomposed to the standard model transitions and corrections coming from the NP changes of the initial and final states (generated by the modified neutrino interactions with matter's particles and by changing the effective neutrino masses in matter). Connections between parameters which describe non-unitary mixing of neutrino states and new matter interactions have been presented in a frame of two models, the model with light–heavy neutrino mixings and the model with left–right symmetry.

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