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Resonance-like tunneling across a barrier with adjacent wells

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Abstract. We examine the behavior of transmission coefficient T across the rectangular barrier when attractive potential well is present on one or both sides and also the same is studied for a smoother barrier with smooth adjacent wells having Woods–Saxon shape. We find that presence of well with suitable width and depth can substantially alter T at energies below the barrier height leading to resonant-like structures. In a sense, this work is complementary to the resonant tunneling of particles across two rectangular barriers, which is being studied in detail in recent years with possible applications in mind. We interpret our results as due to resonant-like positive energy states generated by the adjacent wells. We describe in detail the possible potential application of these results in electronic devices using n-type oxygen-doped gallium arsenide and silicon dioxide. It is envisaged that these results will have applications in the design of tunneling devices.

Keywords. Potential barrier with wells; tunneling; resonance.

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1. Introduction

The mechanism of quantum tunneling (T) across potential barriers plays a crucial role in physics. The theory of alpha decay in some nuclei, fusion of colliding nuclei, tunneling mechanism in semiconducting devices, Josephson effect and scanning tunneling electron microscope are some of the major fields where quantal tunneling has a critical role. Furthermore, with the advent of nanotechnology it is envisaged that quantal tunneling phenomena will have a crucial role in various devices, which may be developed using this technology at micron level. Therefore, it is of much interest to explore the rich physics that still remains unexplored in the area of quantal tunneling. The recent book by Razavy [1] gives an exhaustive survey of different formulations for tunneling mechanism. In recent years, resonant tunneling through successive barriers have been studied in considerable detail, for example, by Lyngdoh *et al* [2], Crankshaw [3], Olkhovsky *et al* [4] and Barbero *et al* [5].

In particular, this problem of resonant tunneling is useful in the study of time delay by the cold neutrons through the so-called neutron interference filter [6,7]. In this paper, we examine the role of the presence of attractive wells adjacent to the potential barrier in the behaviour of T below barrier energies. To the best of our knowledge, such a study and its significance are not reported in literature elsewhere.

In a sense, the problem we are considering is the counterpart of the problem of transmission across two potential barriers. In the latter case one studies T, resonance behaviour associated with it and tunneling time for a particle moving across the potential

$$U(x) = \begin{cases} 0, & -\infty < x < 0 \\ U_1, & 0 < x < a; U_1 > 0 \\ U_2, & a < x < b; U_2 \leq 0 \\ U_3, & b < x < c; U_3 > 0 \\ 0, & \infty > x > c. \end{cases} \quad (1)$$

This is a well-studied case [2]. The potential pocket in the region $a < x < b$ generates resonance-like tunneling at specific energies corresponding to resonance states of the pocket when T becomes close to unity. On the contrary, in this paper we examine the behaviour of the transmission across the potential given by

$$U(x) = \begin{cases} 0, & -\infty < x < 0 \\ U_1, & 0 < x < a; U_1 < 0 \\ U_2, & a < x < b; U_2 > 0 \\ U_3, & b < x < c; U_3 < 0 \\ 0, & \infty > x > c \end{cases} \quad (2)$$

Clearly this has a barrier of height U_2 having width $(b-a)$ and has wells of depth U_1 and U_3 on either side. Naively speaking, if one takes into consideration only WKB type formula for T at energy E below the barrier height, T smoothly increases with E and T has dependence only on the barrier height and width. However, we consider it necessary to examine this problem more carefully by solving it exactly so that the role of attractive potential on one or both sides of the barrier can be determined. Then, we repeat a similar calculation for the case where barrier with adjacent wells has smooth continuous behaviour so that the general validity of our results can be ascertained. For this latter case we construct a potential having the form

$$U(x) = \begin{cases} \frac{V_0}{1+\exp((|x|-x_0)/a)} - V_1, & |x| < 2x_0 \\ \frac{-V_2}{1+\exp((|x|-x_2)/b)}, & |x| > 2x_0, \end{cases} \quad (3)$$

where $V_0 > 0$, $V_1 > 0$, $V_0 > V_1$ and $x_2 > 2x_0$. Here V_0, V_1, x_0, x_2, a and b are adjustable parameters. Parameter V_2 is fixed such that $U(x)$ is continuous at $x = 2x_0$. This gives

$$V_2 = \left[V_1 - \frac{V_0}{1+\exp(x_0/a)}\right][1 + \exp((2x_0 - x_2)/b)]. \quad (4)$$

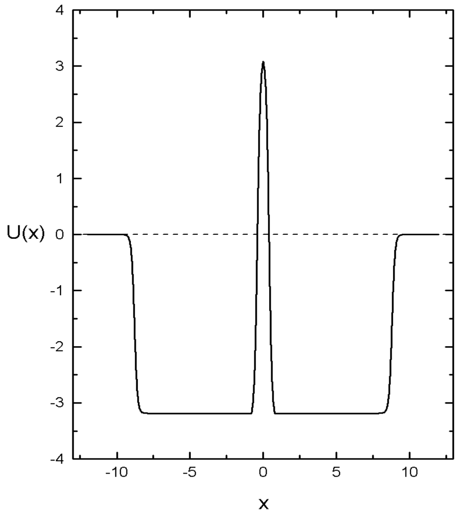

Figure 1. Plot of the smooth potential given by expression (3) with parameters V_0=6.5, V_1=3.3, x_0=0.4, x_2=22x_0, a=0.1 and b=0.1.

In the above potential (3), V_0 and x_0 control the height and width of the barrier respectively. Similarly V_2 and x_2 are the depth and width parameters of the well. Slopes of the barrier and wells are adjusted by the parameters a and b. Also the half-width $\bar{x}$ at the base of the barrier is given by

$$|\ \bar{x}\ | = x_0 + a \ln \frac{(V_0 - V_1)}{V_1}. \tag{5}$$

In figure 1, the behaviour of the potential is schematically shown. In order to study the transmission coefficient T for this smooth potential (3), we solve the time-independent Schroedinger equation numerically. The variation of T with the energy for both these potentials given by (2) and (3) gives interesting results below the barrier height.

We believe that these results would have potential applications. In this paper we describe the potential application of the concept of resonant tunneling in the design of novel electronic devices using the electrical properties of gallium arsenide and SiO_2. In the emerging era of nanoscience these concepts will have wider applications.

In §2, we give details of the formulation of the problem for the calculation of T and also for tunneling time. In §3, we discuss the results of numerical calculations and conclusions based on them. In §4, we describe the application of the concept in the possible design of electronic device based on GaAs and SiO_2.

2. Formalism

The time-independent Schroedinger equation for potential (2) in different regions and its general solutions are

$$\text{Region I:} \quad \frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + k^2\Psi = 0, \quad \text{with } \Psi = A\mathrm{e}^{ikx} + B\mathrm{e}^{-ikx}. \tag{6}$$

$$\text{Region II:} \quad \frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + (k^2 - V_1)\Psi = 0, \quad \text{with } \Psi = C\mathrm{e}^{i\alpha x} + D\mathrm{e}^{-i\alpha x}. \tag{7}$$

$$\text{Region III:} \quad \frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + (k^2 - V_2)\Psi = 0, \quad \text{with } \Psi = F\mathrm{e}^{i\beta x} + G\mathrm{e}^{-i\beta x}. \tag{8}$$

$$\text{Region IV:} \quad \frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + (k^2 - V_3)\Psi = 0, \quad \text{with } \Psi = H\mathrm{e}^{i\gamma x} + I\mathrm{e}^{-i\gamma x}. \tag{9}$$

$$\text{Region V:} \quad \frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + k^2\Psi = 0, \quad \text{with } \Psi = L\mathrm{e}^{ikx}. \tag{10}$$

In the above, $V_i = \frac{2mU_i}{\hbar^2}$, $k^2 = \frac{2mE}{\hbar^2}$, $\alpha^2 = k^2 - V_1$, $\beta^2 = k^2 - V_2$ and $\gamma^2 = k^2 - V_3$. β^2 is negative when $E < U_2$.

We apply the continuity condition for ψ and its derivative at boundaries $x = 0$, $x = a$, $x = b$, $x = c$ and setting $A = 1$, we solve the resulting equations to get the following expressions for transmission amplitude L and reflection amplitude B. The results are

$$L = [M_1D_1 + M_2D_2]^{-1}, \tag{11}$$

$$B = [N_1D_1 + N_2D_2]/[M_1D_1 + M_2D_2]. \tag{12}$$

Here

$$D_1 = \mathrm{e}^{-i\beta b}\mathrm{e}^{ikc}[(\beta+\gamma)(\gamma+k)\mathrm{e}^{i\gamma(b-c)} + (\beta-\gamma)(\gamma-k)\mathrm{e}^{-i\gamma(b-c)}]/(4\beta\gamma),$$
$$D_2 = \mathrm{e}^{i\beta b}\mathrm{e}^{ikc}[(\beta-\gamma)(\gamma+k)\mathrm{e}^{i\gamma(b-c)} + (\beta+\gamma)(\gamma-k)\mathrm{e}^{-i\gamma(b-c)}]/(4\beta\gamma),$$
$$M_1 = \mathrm{e}^{i\beta a}[(k+\alpha)(\alpha+\beta)\mathrm{e}^{-i\alpha a} + (k-\alpha)(\alpha-\beta)\mathrm{e}^{i\alpha a}]/(4\alpha k),$$
$$M_2 = \mathrm{e}^{-i\beta a}[(k+\alpha)(\alpha-\beta)\mathrm{e}^{-i\alpha a} + (k-\alpha)(\alpha+\beta)\mathrm{e}^{i\alpha a}]/(4\alpha k),$$
$$N_1 = \mathrm{e}^{i\beta a}[(k-\alpha)(\alpha+\beta)\mathrm{e}^{-i\alpha a} + (k+\alpha)(\alpha-\beta)\mathrm{e}^{i\alpha a}]/(4\alpha k),$$
$$N_2 = \mathrm{e}^{-i\beta a}[(k+\alpha)(\alpha+\beta)\mathrm{e}^{i\alpha a} + (k-\alpha)(\alpha-\beta)\mathrm{e}^{-i\alpha a}]/(4\alpha k).$$

Now the transmission and reflection probabilities are given by

$$T = |L|^2, \quad R = |B|^2 \quad \text{and} \quad T + R = 1.$$

The average quantal time delay is calculated using the formula given in p. 355 of ref. [1]

$$T_q = \frac{m}{\hbar k}\left[R\frac{\mathrm{d}\phi_\mathrm{r}}{\mathrm{d}k} + T\frac{\mathrm{d}\phi_\mathrm{t}}{\mathrm{d}k}\right] = T_{q\mathrm{r}} + T_{q\mathrm{t}} \tag{13}$$

where

$$\frac{\mathrm{d}\phi_\mathrm{r}}{\mathrm{d}k} = \frac{P_x\dot{P}_y - \dot{P}_x P_y}{|P|^2} \quad \text{and}$$
$$P = B = P_x + iP_y = (P_x^2 + P_y^2)^{\frac{1}{2}}\mathrm{e}^{i\phi_\mathrm{r}}, \quad \phi_\mathrm{r} = \tan^{-1}(P_y/P_x), \tag{14}$$
$$\frac{\mathrm{d}\phi_\mathrm{t}}{dk} = \frac{J_x\dot{J}_y - \dot{J}_x J_y}{|P|^2} \quad \text{and}$$
$$J = L = J_x + iJ_y = (J_x^2 + J_y^2)^{\frac{1}{2}}\mathrm{e}^{i\phi_\mathrm{t}}, \quad \phi_\mathrm{t} = \tan^{-1}(J_y/J_x). \tag{15}$$

$T_{q\mathrm{r}}$ and $T_{q\mathrm{t}}$ are time delay associated with reflection and transmission.

3. Calculations and discussions

We study the variation of T with energy for barrier with and without adjacent wells. In the calculations, we have set $2m$=1, $\hbar^2$=1 and k is expressed in Å^{-1}, energy and potential strength U_i are expressed in Å^{-2} and lengths are expressed in Å. We have done the calculations for the following cases:

(A) $U_1 = 0.0$, $U_2 = 3.0$, $U_3 = 0.0$, $a = 7$, $b = 7.5$, $c = 14.5$ (barrier only),
(B) $U_1 = -3.0$, $U_2 = 3.0$, $U_3 = 0.0$, $a = 7$, $b = 7.5, c = 14.5$ (barrier with one adjacent well),
(C) $U_1 = -3.0$, $U_2 = 3.0$, $U_3 = -3.0$, $a = 7$, $b = 7.5$, $c = 14.5$ (barrier with one well each on either side).

It may be noted that if these cases are treated as tunneling of electrons across barrier with adjacent wells, the height of the barrier $U_2 = 3$ Å^{-2} corresponds to 11.4 eV. The conversion factor from Å^{-2} unit to eV unit is 3.816. In figure 2, we show the variation of T as a function of energy $E = k^2$. When $E < U_2$ the presence of one well generates two peaks in T (B), which is significantly larger as compared to the T when no adjacent wells are present (A). When a well is added to either side of the barrier (C) these peaks rise still higher, the one at lower energy approaching close to unity. This is a peculiar result, which establishes that it is possible to substantially enhance T at below barrier energies by setting up attractive wells on one or both sides of the barrier. This should be compared with the behaviour of T across two successive rectangular barrier [2] given by (1). There, it is known that T gets high peaks corresponding to the resonant states of the pocket trapped in between two barriers. Now we have to visualize the reason behind the somewhat unexpected sharp rise in T when attractive potentials on one or both sides of the barrier are present. We offer the following explanations for this:

Let us treat the wells on either sides as a kind of 1D box of widths a and $(c-b)$. We know that in the case of infinite 1D box of width $L = a = (c-b) = 7.0$ and depth $U = -3.0$ the energy levels are $E = (n^2\pi^2/L^2) + U$ in the system of units chosen. First few levels are at positions $E = -2.798$, -2.195, -1.188, 0.222, 2.035, 4.25,.... In the case of finite wells that we study the corresponding negative energy states, and resonance-like low energy positive energy states can be expected

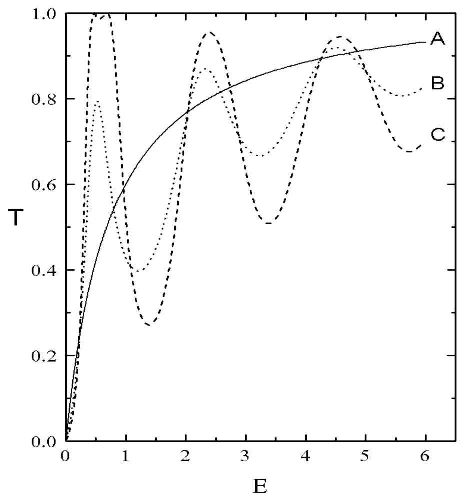

Figure 2. The transmission co-efficient T with energy E (Å^{-2}) is plotted in A, B and C for a rectangular barrier with parameters $U_2 = 3$ Å^{-2} and $b-a = 0.5$ Å, for a rectangular barrier with one adjacent well with parameters $U_1 = -3$ Å^{-2}, $U_2 = 3$ Å^{-2}, $a = 7$ and $b - a = 0.5$, and for a rectangular barrier with two adjacent wells with parameters $U_1 = -3$ Å^{-2}, $U_2 = 3$ Å^{-2}, $U_3 = -3$ Å^{-2}, $a = 7$, $b = 7.5$ and $c = 14.5$, respectively.

to be close to these values. The first two positive energy states of infinite well are at 0.222 and 2.035, and therefore, these correspond approximately to the position of the peaks in figure 2 at $E = 0.50$ and 2.3, respectively. Hence, we can state that the peaks in T below barrier are related to the positive energy resonance-like states of the wells. This means the peaks in T roughly correspond to resonant-like positive energy states generated by the two wells. The presence of two identical wells on either side of the barrier further enhances the importance of these states in the tunneling process. Thus, T across a barrier at below barrier energy shows peaks corresponding to resonant-like positive energy states generated by the wells. Therefore, we conclude that it is possible to significantly alter and control T below the barrier by suitably adjusting the well parameters. This feature, we believe, should be of importance in the design of novel tunneling devices.

In figure 3, we examine the effect of variation of T when the width of the barrier is increased. In this figure we show the variation of T when width of the barrier is increased from 0.5 (A) to 1.0 (B) and we find that there is a substantial reduction in peak heights when width of the barrier is increased. This, however, is quite as expected.

In order to further substantiate the fact that it is indeed the states associated with wells that generate sub-barrier peaks in T, we examine the problem of transmission across an attractive rectangular well with width $a = 7$ Å and depth $V_0 = 3$ Å^{-2}. Transmission co-efficient T for an attractive rectangular well is given as

$$T = \left[1 + \frac{(k_1^2 - k^2)^2 \sin^2 k_1 a}{4k_1^2 k^2}\right]^{-1}, \tag{16}$$

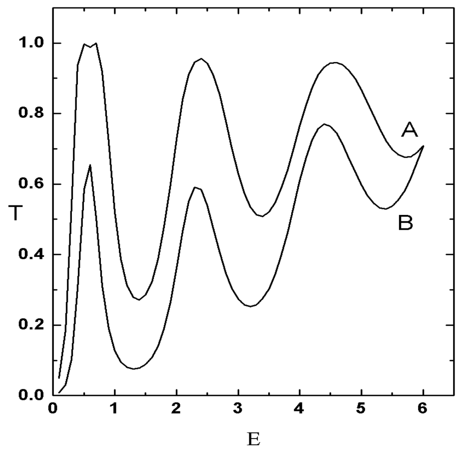

Figure 3. Plot of transmission co-efficient T with energy E (Å^{-2}) for a rectangular barrier with different widths having two similar adjacent attractive wells.

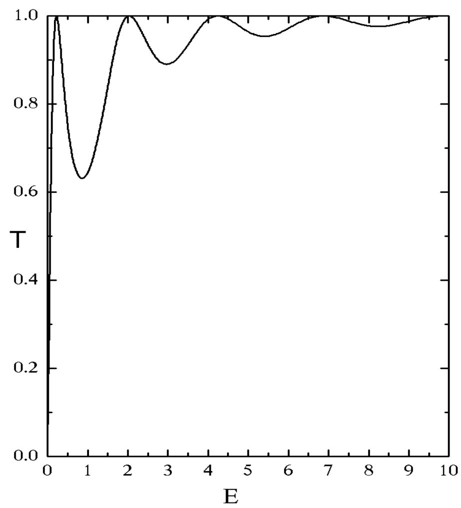

Figure 4. Plot of transmission co-efficient T with energy E (Å^{-2}) for an attractive rectangular well.

where $k_1 = \sqrt{(E + V_0)}$. In figure 4, we show the variation of this T as a function of energy. The corresponding infinite well of this width and depth 3.0 generates states at energies $(\frac{n^2\pi^2}{L^2} - 3)$ and these are given by -2.7986, -2.1944, -1.1874, 0.2224, 2.035, 4.2504 and 6.868 etc. It is clear that the positive energy states here are very close to the peak positions of T. Figures 2 and 3 should indeed be associated with the positive energy resonance-like states generated by the attractive adjacent wells.

In order to confirm that the above pattern of variation of T at sub-barrier energy is present for continuous barrier with smoother adjacent wells, we repeat the

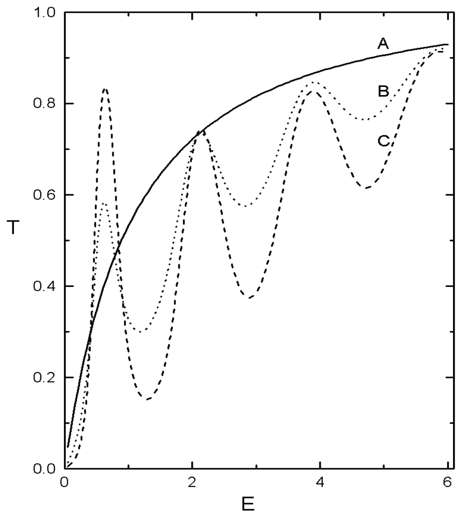

Figure 5. The transmission co-efficient T with energy E (Å^{-2}) is plotted in A, B and C for a smooth barrier, a smooth barrier with one adjacent well and a smooth barrier with two adjacent wells with parameters $V_0 = 6.5$, $V_1 = 3.3$, $x_0 = 0.4$, $x_2 = 22x_0$, $a = 0.1$ and $b = 0.1$ respectively.

calculations of T for the potential given by (3) with the parameters $V_0 = 6.5$, $V_1 = 3.3$, $x_0 = 0.4$, $x_2 = 22x_0$, $a = 0.1$ and $b = 0.1$ (figure 1). The transmission co-efficient T is calculated numerically by solving the Schroedinger equation with the required boundary conditions. The variation of T as a function of energy is shown in figure 5 with label C. This figure also includes corresponding results for the smooth single barrier without adjacent wells and a smooth barrier with one adjacent well denoted by A and B, respectively. We observe that the below barrier enhancement as well as oscillations in T due to the presence of smooth well on one or both sides of the smooth barrier is quite consistent with the results of the rectangular barrier case shown in figure 2. The sub-barrier peak in this case is very prominent in figure 5 at energy around $E = 0.6$ and the second peak at around $E = 2.0$ is not higher than the corresponding results of barrier without wells. However, it should be stressed that by adjusting well and barrier parameters the second peak also can be made higher as in the corresponding case in potential (2).

A resonant-like state giving rise to peaks in T is also characterized by the concept of time delay T_q (13) in the transmitting wave. From eq. (13) it is clear that the sign of $T_{q\mathrm{r}}$ and $T_{q\mathrm{t}}$ depend on the derivatives, $\mathrm{d}\phi_\mathrm{r}/\mathrm{d}k$ and $\mathrm{d}\phi_t/\mathrm{d}k$ respectively. It is also known that for an attractive well time delay can be negative [8]. One may term it as time advance. Such a feature is present in three-dimensional scattering also [9]. In figure 6 we depict the variation of T_q, $T_{q\mathrm{r}}$ and $T_{q\mathrm{t}}$ in the relevant energy range for the case $U_1 = -3.0$, $U_2 = 3.0$, $a = 7$, $b = 7.5$, $c = 14.5$. In this case $T_{q\mathrm{t}} < 0$, $T_{q\mathrm{r}} > 0$, $T_q > 0$. The tunneling particle is assumed to be electron. The broad peak, $T_{q\mathrm{t}}$ corresponds to the peak in T. The total time T_q has a peak at $E = 0.312$. This time delay estimate is important in designing the potential application of the concept of resonance tunneling to devices.

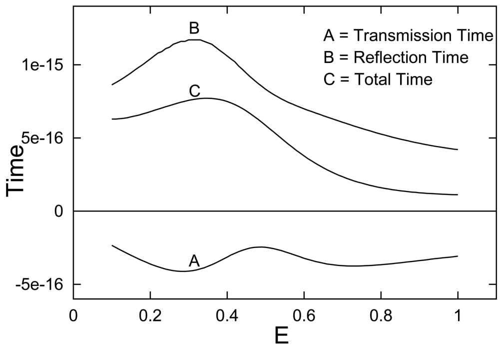

Figure 6. Variation of time in second with energy (E) in Å^{-2} for the potential parameters $U_1 = -3.0$, $U_2 = 3.0$, $U_3 = 0.0$, $a = 7$, $b = 7.5$ and $c = 14.5$.

Based on the above set of calculations we conclude that barrier penetration at sub-barrier energies can be substantially enhanced and controlled selectively in specific energy regions by the presence of adjacent wells giving rise to resonance-like peaks. We believe that such a phenomena should be of use in the design of novel tunneling devices.

4. Applications of resonance tunneling

The above calculations clearly show that for a tunnel structure with adjacent well on both sides of the barrier (A, B in figure 3), at resonance, all the electrons possessing certain values of energy can be transported across the barrier to produce a current flow. An external electric field applied across the structure accomplishes supply of the energy required for electrons to tunnel through the barrier. The electric field can be adjusted to obtain maximum T (resonance peak) or minimum T; thus a tunnel device can effectively perform a logic operation or can be used to function as an electronic on/off switch. The external electric field can be a power supply or a variable signal source such as a thermal or optical sensor whose signal can be employed to pump electrons across the barrier to the desired load. The goal of this section is geared towards assessing the possibility of utilizing semiconductor–insulator structure for the tunnel device. A novel approach is to take advantage of the deep levels (which are not recombination center) inherent to some doped semiconductors and use them as potential wells because of their attractive property to electrons.

In order to design and construct such a tunnel device employing the above principle, one needs to utilize a material which provides with electron states that can perform the function of potential well that is attractive to electrons. We have chosen semi-insulating high resistivity (1.0×10^8 Ω-cm) n-type oxygen-doped gallium arsenide (GaAs(O)) for this purpose. This material is known to have [10] a deep level located at energy (E_{t}) of 0.75 eV below the bottom of the conduction band (E_{c}). It is well-known to exhibit trapping effects in devices such as transistors, photo-conductors and Schottky-barrier diodes made using GaAs(O), all attributed to oxygen. Since traps attract electrons they function as potential wells. Thin films of oxygen-doped GaAs with precise thickness can be prepared easily with *in-situ* doping employing molecular epitaxy. In the conceptual design, the device consists of high resistivity (1.0×10^{14} Ω-cm) silicon dioxide (SiO_2) sandwiched between two layers of GaAs(O). For this structure we first examine if one can achieve resonance with such shallow potential wells. Evaluation of the band parameters of GaAs(O) and (SiO_2) enables one to locate zero energy reference for the structure. For gallium arsenide [11], using electron mobility value of 8.5×10^3 cm^2/(Vs) and electron effective mass ratio (m^*/m_0) of 0.07, the electron density in the conduction band is calculated to be 7.6×10^6 cm^{-3} and the Fermi level (E_f) is located at 0.62 eV below E_c. In the case of silicon dioxide [12], using an electron mobility value of 20 cm^2/(Vs), and (m^*/m_0) ratio of 1, the electron density is calculated to be 3.1×10^3 cm^{-3}. Therefore E_f is located at 0.92 eV below E_c of SiO_2. When GaAs(O) is brought into intimate contact with SiO_2 on both sides, the resulting electron energy levels are similar to the case in which a potential well exists on both sides of the barrier. For simplicity, we will consider an ideal abrupt junction on both sides of the barrier with rectangular wells. In thermal equilibrium, E_f in both materials should coincide. Under this circumstance, taking the position of E_f as the reference, the deep level or potential well depth is $(E_\mathrm{f} - E_\mathrm{t}) = 0.62 - 0.75 = -0.13$ eV and the height of the barrier for electrons in SiO_2 is 0.92 eV with respect to E_f. In the proposed structure, the electrons on the side of GaAs(O) above E_f experience an attractive force towards the well of depth 0.13 eV and also face a barrier of height 0.92 eV for the transport into SiO_2. Calculations are performed for the specific case of wells on both sides of the barrier taking $U_1 = -0.13$ eV, $U_2 = 0.92$ eV and $U_3 = -0.13$ eV. One may notice that potential well depth and barrier height is one percentage and eight percentage respectively, of the values used in §3. In order to determine if resonance occurs when practically attainable minimum widths are used for well and barrier, we examined the behaviour of the structure using different barrier and well widths. The calculations are confined to the case of equal well width on both sides of the barrier.

Now we discuss numerical results of figures 7 and 8. Table 1 contains relevant data pertaining to these figures. Figure 7 shows T as a function of electron energy E for three values of barrier widths and fixed value of well width (5 Å). Curve A corresponds to barrier width of 0.5 Å. A single resonance peak with $T = 1$ occurs at $E = 110$ MeV. At higher E values T exhibits a minimum before it increases. The curve B corresponds to barrier thickness of 1 Å and the peak value of $T = 1$ occurs at $E = 49.5$ MeV. Curve C shows the result for the case with a barrier width of 5 Å. One can notice a small resonance type peak at 20.8 MeV with $T < 0.10$. For E beyond this value the curve behaves similar to the case of a barrier without an adjacent well (curve A of figure 2). This is expected because the thermal energy of electrons is 25.8 MeV. This influences the electron density, which in turn results in electron collisions. This modifies the number and energy of electrons that participate to produce resonance. It is possible to obtain the value of $T = 1$ if the device is cooled to 100 K or lower. In all the cases the E values at which peak occurs are lower than U_1.

The effect of varying the well widths is also evaluated to estimate the minimum width required for resonance. Figure 8 shows the results with well width as parameter for barrier width of 0.5 Å and 1 Å. Curves A and B correspond to well width

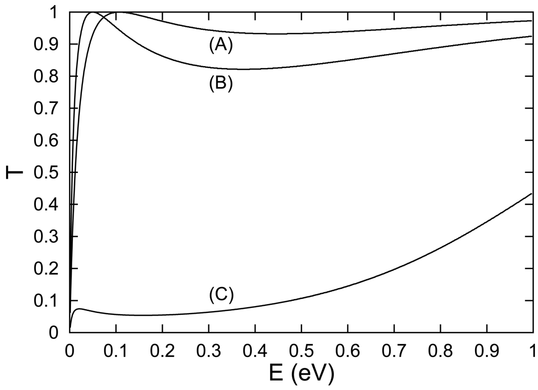

Figure 7. Variation of transmission coefficient (T) with energy to various potential parameters (see also table 1) (A) $U_1 = -0.13, U_2 = 0.92$, $U_3 = -0.13$, $a = 5, b = 5.5$ and $c = 10.5$; (B) $U_1 = -0.13, U_2 = 0.92$, $U_3 = -0.13$, $a = 5$, $b = 6$ and $c = 11$; (C) $U_1 = -0.13, U_2 = 0.92$, $U_3 = -0.13$, $a = 5$, $b = 10$ and $c = 15$.

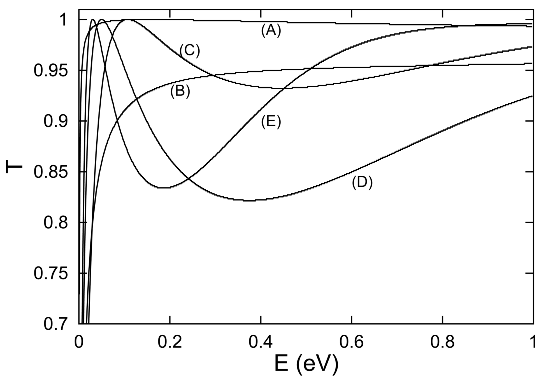

Figure 8. Variation of transmission coefficient (T) with energy to various potential parameters (see also table 1) (A) $U_1 = -0.13$, $U_2 = 0.92$, $U_3 = -0.13$, $a = 2$, $b = 2.5$ and $c = 4.5$; (B) $U_1 = -0.13$, $U_2 = 0.92, U_3 = -0.13$, $a = 2$, $b = 3$ and $c = 5$; (C) $U_1 = -0.13, U_2 = 0.92$, $U_3 = -0.13$, $a = 5$, $b = 5.5$ and $c = 10.5$; (D) $U_1 = -0.13, U_2 = 0.92$, $U_3 = -0.13$, $a = 5$, $b = 6$ and $c = 11$; (E) $U_1 = -0.13$, $U_2 = 0.92$, $U_3 = -0.13$, $a = 7$, $b = 7.5$ and $c = 14.5$.

of 2 Å. No sharp resonance is observed in these. These behave similar to figure 2 as if no wells are present which is a typical tunneling behaviour of a metal–insulator type structure. This is somewhat expected because of the thinness of well width. Curves C and D correspond to a well width of 5 Å and barrier width 0.5 Å and 1 Å, respectively. In this case resonance was found to occur at E values of 110 and 49.5 MeV, respectively with $T = 1$ in both cases. Curve E shows the result when the well width is increased to 7 Å and keeping the barrier width at 0.5 Å. A sharp resonance was found to occur at 30 MeV with $T = 0.999$. These results show that a minimum well width is required to generate resonance.

Table 1. Transmission coefficient (T), reflection coefficient (R), reflection time (R_t), transmission time (T_t) and total time delay (T_T) for different potential parameters.

Potential parameters $U_1 = -0.13$ eV $U_2 = 0.92$ eV $U_3 = -0.13$ eV	Resonance energy (eV)	T	R	T_t	R_t (s)	T_T (s)	Remarks
$a = 5$, $b = 5.5$ $c = 10.5$	0.110	1.00	<0.4989e-04	−0.234e-14	0.2301e-21	−0.234e-14	Figure 7 (Curve A)
$a = 5$, $b = 6$ $c = 11$	0.0495	1.00	<0.112e-04	−0.419e-14	0.188e-18	−0.419e-14	Figure 7 (Curve B)
$a = 5$, $b = 10$ $c = 15$	0.0208	0.074	0.9259	0.9427e-15	0.4335e-13	0.4430e-13	Figure 7 (Curve C)
$a = 2$, $b = 2.5$ $c = 4.5$	0.1524 No sharp resonance observed	1.0	<0.1525e-7	−0.2002e-15	0.8294e-23	0.2002e-15	Figure 8 (Curve A)
$a = 2$, $b = 3$ $c = 5$	No resonance observed	–	–	–	–	–	Figure 8 (Curve B)
$a = 5$, $b = 5.5$ $c = 10.5$	0.110	1.0	<0.4989e-4	−0.234e-14	0.2301e-21	−0.234e-14	Figure 8 (Curve C)
$a = 5$, $b = 6$ $c = 11$	0.0495	1.0	<0.112e-04	−0.419e-14	0.188e-18	−0.419e-14	Figure 8 (Curve D)
$a = 7$, $b = 7.5$ $c = 14.5$	0.03	0.999	<0.6613e-04	−0.1045e-13	0.2377e-20	−0.1045e-13	Figure 8 (Curve E)

Finally, these calculations show that it is possible to generate resonance using shallow wells also although the number of resonance peaks are much less than that for deep wells. One may notice that the E values at which resonance occurs are low, much less than 1.0 eV. The tunneling time is found to be 1.0×10^{-14} or less. The low values of E at the resonance peak suggest that only very low applied field is needed to supply the energy to electrons to generate resonance. Such low energy and fast tunnel time are the added benefits of this device for fast switching and signal detection applications including daylight activated lighting system, when used in conjunction with a photocell. Judging from this analysis, it is possible that any semiconductor with single deep level, which is not a recombination center, can be utilized as an efficient potential well.

We believe that the currently emerging nanofilm technology can be effectively employed to fabricate these devices.

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