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Beta decay rates for nuclei with $115 < A < 140$ for r-process nucleosynthesis

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Abstract. For r-process nucleosynthesis the β-decay rates for a number of neutron-rich intermediate heavy nuclei are calculated. The model for the β-strength function is able to reproduce the observed half-lives quite well.

Keywords. r-process; β-decay half-life; Gamow–Teller strength; waiting point nuclei.

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More than half of the nuclei heavier than iron are produced through r-process nucleosynthesis [1]. Though one knows that high neutron density and fast neutron capture time-scales compared to β-decay are required for this process, astrophysical sites for them are not clearly identified. As the r-process path often reaches highly neutron-rich nuclei, collecting experimental information about them sometimes becomes prohibitively difficult. On the other hand, near the neutron shell magic numbers depending on the shell gap [2] and β-decay half-lives the chain often has to wait long enough before proceeding to the next heavier nucleus [3]. These are called waiting point nuclei and and can be identified by their observed relative abundances. With the lack of experimental information, one often has to fall back on the predictions of theory for these neutron-rich nuclei. In this letter we consider such nuclei near $N = 82$ magic shell for $115 < A < 140$. The shell closure at $N = 82$ for n-rich nuclei in the r-process path is receiving a lot of attention lately and realistic models for β-decay half-lives and rates in this region are badly needed. We construct here a model for the β-decay strength function based on the experience with lighter nuclei in the fp shell. We deal with nuclei for which a few low-lying log fts are known, to test the model and then also apply it to cases where not much is known experimentally beyond the Q-value and half-life. The actual β-decay rates are also calculated for stellar density and temperatures which are thought to be typical for the r-process.

Calculation of β-decay and electron capture rates have been pursued vigorously during the last 20 years for pre-supernova evolution and gravitational collapse stage for core collapse supernovae [4]. Whereas for the sd-shell nuclei the rates are given by Oda *et al* [5], for the fp-shell nuclei for quite sometime the works of Fuller *et al* [6] were used as the standard reference. Aufderheide *et al* [7] indicated the usefulness of interacting shell model for the rates. They also calculated approximate abundances in stellar conditions appropriate for the pre-supernova and collapse stage and identified the nuclei which are the most important in the network for each density–temperature grid point. On the other hand, statistical methods for the β-decay strength distribution were used to calculate the rates for fp-shell nuclei [8]. This was followed by large space and detailed shell model calculations of the allowed β-decay strength to calculate the weak interaction rates for nuclei with $A < 65$ [4,9]. The effect of the new shell model rates on the later stage of evolution of massive stars has also been seen [10]. Recently, Pruet and Fuller [11] used FFN-type ideas supplemented by experimental information wherever available to calculate the rates for nuclei relevant for supernova collapse. However, for much lower densities and temperatures appropriate for s- and r-process nucleosynthesis the work of Takahashi and Yokoi [12] is still being widely used. As large number of nuclei are involved in the r-process path the estimates of half-lives use a combination of global mass models and the quasi-particle random-phase approximation (QRPA). These are FRDM/QRPA [13] and ETSI/QRPA [14]. Self-consistent Hartree–Fock–Bogoliubov plus QRPA model has also been used [15].

The β-decay rate for a nucleus in stellar conditions at temperature T is given by

$$\lambda = \sum_i \mathrm{e}^{-E_i/kT} \sum_j \lambda_{ij}/Z, \tag{1}$$

where E_i is the energy of the state of the mother nucleus and Z is the partition function of the nucleus. 'j' sums over states of the daughter nucleus to which transitions are allowed. The rate from the parent nuclear state i to the daughter nuclear state j is given by

$$\lambda_{ij} = \frac{\ln 2}{(ft)_{ij}} f_{ij}, \tag{2}$$

where f_{ij} is the phase-space factor for β-decay with an electron gas outside [8]. For the Coulomb correction factor we use the Schenter–Vogel expressions [8,16]. The allowed ft values have two components

$$\frac{1}{(ft)_{ij}} = \frac{1}{(ft)_{ij}^{\mathrm{GT}}} + \frac{1}{(ft)_{ij}^{\mathrm{F}}} = \frac{10^{3.59}}{|M_{\mathrm{GT}}|_{ij}^2} + \frac{10^{3.79}}{|M_{\mathrm{F}}|_{ij}^2}. \tag{3}$$

For calculation of half-lives the decay is from the ground state, i.e., there is only one mother state with the Boltzmann factor equal to 1 in eq. (1). In eq. (3) the first part is the contribution from the allowed Gamow–Teller operator and the second from the allowed Fermi operator. Normally the GT strength distributes among three different types of final states of the daughter:

(a) a discrete set of low-lying states for which log fts are known experimentally,
(b) another set of discrete states above them for which the strengths are not known,
(c) a part of the GT giant resonance.

Usually one sees only the tail of the giant resonance as the isobaric analogue state (IAS) for β^- transitions is pushed up by the Coulomb interaction. In our treatment we include in λ of eq. (1) as well as in the half-life the states of type (a), in particular all the low-lying states connected by allowed transitions wherever known experimentally. For the contribution from (c), i.e., the giant resonance region, we adopt the procedure detailed below.

The Fermi resonance is easy to take care of. In the absence of Coulomb interaction, the total Fermi strength goes to IAS. Coulomb interaction spreads this strength into a sharp resonance around the IAS and we take the resonance width as $\sigma_c = 0.157ZA^{-1/3}$ [8,17]. As the spreading width is small this resonance cannot be reached by Q-value and so Fermi makes very little contribution to half-lives and rates. But the situation for GT is quite different. As the spin–isospin-dependent nuclear Hamiltonian does not commute with the GT operator, the GT resonance is a broad one. Detailed shell model results [18] for fp shell nuclei and use of spectral distribution theory [19] indicate that the GT strength distribution can be taken as a Gaussian to a good approximation when the initial state is somewhat high in excitation energy (a few MeV) and as a result in the chaotic regime. This point has been studied at length in the last few years. A good understanding of the Gaussian forms follows by using Hamiltonians from the two-body random matrix ensembles which model chaos in many-particle quantum systems very well [20]. Shell model results for lighter nuclei show that even with specific two-body interactions the Gaussian results persist. However for the ground state region there is some departure. In this letter we do not take into account such departure and take the GT resonance for each nucleus given by a Gaussian. Hence the problem comes down to fixing the centroid and width of the resonance. For the GT centroid, Bertsch and Esbensen [21], using the Tamm–Dancoff approximation give the following expression [11]:

$$E_{\mathrm{GT}^-} = E_{\mathrm{IAS}} + \Delta E_{\mathrm{s.o.}} + 2[k_{\sigma\tau} S_{\mathrm{GT}^-}/3 - (N-Z)k_\tau], \tag{4}$$

where $\Delta E_{\mathrm{s.o.}}$ is the contribution coming from the spin-orbit force. For $\Delta E_{\mathrm{s.o.}}$ we use the orbit averaged value of 3.0 MeV [21]. The last two terms come from the spin–isospin- and isospin-dependent nuclear forces. Here for the sum rule strength S_{GT^-} we use the expression $3(N-Z)$. Following Pruet and Fuller [11] we use the values $k_\tau = 28.5/A$ and $k_{\sigma\tau} = 23/A$. For the overall normalisation of the sum rule Gaussian we use the quenched $3(N-Z)$ value minus the experimental strength already included in (a). For the quenching factor for the β-decay we use the value 0.6 [8]. Finally to include the contribution from (b), i.e., the discrete states below the GT resonance region, we artificially broaden the resonance by using the width larger than the normal giant resonance widths. This is open to improvement. Perhaps partitioning of the space into subspaces [22] and taking into account a number of Gaussian strengths instead of a single Gaussian is a better way of handling this. But then the width of each Gaussian either has to be calculated theoretically or used as a parameter. We first use this model to calculate half-lives of nuclei around the $N = 82$ shell closure. We include in this paper nuclei in the range $115 < A < 140$ and also use the selection criterion that the Q-value is greater than 5 MeV for the statistical treatment of the high-lying states. We separate the nuclei into two groups – one set with known experimental ft values and the other with no knowledge about low-lying fts.

We also mention here that the spectral distribution result for the strength distribution gives it as a bivariate Gaussian in both the initial and final energies. However, for the r-process nuclei the stellar temperature (in MeV) is quite low compared to the energies of the excited states of the mother. As a result the decay takes place only from the ground state of the mother and hence the full bivariate GT distribution does not come into play.

In table 1 we present the calculated half-lives for 13 nuclei compared to their experimental values. These are obtained by taking the width of the GT giant resonance as a free parameter and minimising the summed $[\log(\tau_{1/2}^{\text{cal}}/\tau_{1/2}^{\text{expt}})]^2$. The calculated values show reasonably good agreement with the observed values and $\Sigma[\log(\tau_{1/2}^{\text{cal}}/\tau_{1/2}^{\text{expt}})]^2/N$ is 0.06 where N stands for the number of nuclei. The optimised width is 5.0 MeV.

Table 2 shows a similar comparison for nuclei for which no low-lying log fts are known. Here the minimised $\Sigma[\log(\tau_{1/2}^{\text{cal}}/\tau_{1/2}^{\text{expt}})]^2/N$ is 0.07 for the width 7.7 MeV. The fact that the width for this case is larger than the earlier one is expected as here the resonance has to be extended to the ground state region.

Table 1. Half-lives of nuclei for which $\log ft$ values are available and were used.

Mother nucleus	Daughter nucleus	Q-value (MeV)	No. of low-lying log fts taken	$\tau_{1/2}$ Exp. (s)	$\tau_{1/2}$ Calc. (s)
${}^{138}_{53}\text{I}_{85}$	${}^{138}\text{Xe}$	7.820	3	6.49	12.99
${}^{137}_{53}\text{I}_{84}$	${}^{137}\text{Xe}$	5.880	2	24.5	33.10
${}^{136}_{53}\text{I}_{83}$	${}^{136}\text{Xe}$	6.930	3	83.4	69.50
${}^{134}_{51}\text{Sb}_{83}$	${}^{134}\text{Te}$	8.420	2	10.43	7.72
${}^{133}_{50}\text{Sn}_{83}$	${}^{133}\text{Sb}$	7.830	4	1.44	1.19
${}^{132}_{51}\text{Sb}_{81}$	${}^{132}\text{Te}$	5.290	2	167.4	141.30
${}^{131}_{49}\text{In}_{82}$	${}^{131}\text{Sn}$	6.746	2	0.282	0.280
${}^{130}_{49}\text{In}_{81}$	${}^{130}\text{Sn}$	10.250	3	0.32	0.25
${}^{128}_{49}\text{In}_{79}$	${}^{128}\text{Sn}$	8.980	3	0.84	2.30
${}^{125}_{48}\text{Cd}_{77}$	${}^{125}\text{In}$	7.160	3	0.65	2.12
${}^{120}_{47}\text{Ag}_{73}$	${}^{120}\text{Cd}$	8.200	4	1.23	0.987
${}^{118}_{47}\text{Ag}_{71}$	${}^{118}\text{Cd}$	7.060	3	3.76	7.23
${}^{116}_{45}\text{Rh}_{71}$	${}^{116}\text{Pd}$	8.900	3	0.68	0.49

Table 2. Half-lives of nuclei for which no log ft values are available.

Mother nucleus	Daughter nucleus	Q-value (MeV)	$\tau_{1/2}$ Exp. (s)	$\tau_{1/2}$ Calc. (s)
$^{138}_{52}Te_{85}$	^{138}I	6.370	1.400	1.860
$^{137}_{52}Te_{84}$	^{137}I	6.940	2.490	1.410
$^{136}_{51}Sb_{85}$	^{136}Te	9.300	0.201	0.208
$^{132}_{49}In_{83}$	^{132}Sn	13.60	0.820	0.507
$^{117}_{45}Rh_{72}$	^{117}Pd	7.000	0.440	1.350

Table 3. Decay rates in s^{-1} for ^{133}Sn and ^{132}In.

Nucleus	Density (g/cm^3)	Temperature T_9 (in 10^9 K) 0.5	1.0	2.0	3.0
	10^8	0.543	0.542	0.493	0.450
	10^7	0.581	0.558	0.498	0.452
^{133}Sn	10^6	0.584	0.559	0.498	0.452
	10^5	0.584	0.560	0.498	0.452
	10^4	0.584	0.560	0.498	0.452
	10^3	0.584	0.560	0.498	0.452
	10^8	1.70	1.75	1.59	1.46
	10^7	1.93	1.83	1.61	1.47
^{132}In	10^6	1.95	1.83	1.61	1.47
	10^5	1.95	1.84	1.61	1.47
	10^4	1.95	1.84	1.61	1.47
	10^3	1.95	1.84	1.61	1.47

In table 3 we calculate the β-decay rates for ^{133}Sn and ^{132}In for the purpose of illustration over a range of density and temperature. The chemical potential of the electrons changes from 1.32 MeV for the density 10^8 g/cc to zero for 10^3 g/cc for the temperature 5×10^8 K. At higher temperatures the chemical potential is less. For 3×10^9 K it is only 0.062 MeV for density 10^8 g/cc. We have considered the density range far beyond the r-process to examine the dependence on density. In this range this dependence is found to be very mild but the temperature dependence is seen to be a bit stronger. Of course the rates can be calculated for other nuclei and extended to other densities and temperatures. As the values of kT are small compared to the excitation energies of even the first excited state of the mother nucleus we have given only the ground state contribution. For similar reasons the rates coming from 'back resonances' of some special excited states with large overlap with the daughter ground state region are also unimportant here and hence are neglected. In future we plan to extend these calculations to more neutron-rich nuclei for which very little experimental information is available. We shall also consider heavier nuclei.

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