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Ion-acoustic solitons in multispecies spatially inhomogeneous plasmas

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Abstract. Ion-acoustic solitons are investigated in the spatially inhomogeneous plasma having electrons–positrons and ions. The soliton characteristics are described by Korteweg–de Vries equation which has an additional term. The density and temperature of different species play an important role for the amplitude and width of the solitons. Numerical calculations show only the possibility of compressive solitons. Further, analytical results predict that the peak amplitude of soliton decreases with the decrease of density gradient. Soliton characteristics like peak amplitude and width are substantially different from those based on KdV theory for homogeneous plasmas.

Keywords. Ion-acoustic solitons; compressive; multispecies; electron–positron.

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1. Introduction

In a historical current of nonlinear wave studies of plasmas, Korteweg–de Vries (KdV) solitons have been extensively investigated both theoretically as well as experimentally. Propagation of solitary waves/solitons is also important as it describes the characteristics of the interaction between the waves and the plasmas. Solitons are amazing and a beautiful manifestation of nature arising out of a delicate balance of nonlinearity and dispersive properties of plasma. The observation of the soliton propagation extends from laboratory [1–4,6–9] to on-board satellite based measurements [10–14]. However, most of the theoretical results of the heuristic features on the formation of solitons have been limited to homogeneous plasmas. But, we invariably encounter inhomogeneous plasmas under actual conditions both in the laboratory as well as in space. Inhomogeneity may stem from density gradient or from temperature or it could be due to magnetic field in space and they are more pronounced closer to the edges and the boundaries of the system. Under such conditions, characteristics like propagation, collision and reflection of solitons etc., are significantly influenced and our knowledge is not adequate to account for some observations of ion-acoustic solitons. Moreover, the observations tend to show that the KdV theory does not exhibit the solitary wave behavior exactly as reported in the laboratory plasmas.

The electron–positron plasmas which supposedly appeared in the early universe [15] and was frequently encountered in galactic nuclei and pulsar magnetosphere [16], have attracted a great deal of interest in the study of linear and nonlinear wave motions. However, most of the astrophysical plasmas usually contain ions, in addition to the electrons and positrons. For example, outflow of electron–positron plasmas from pulsars entering an interstellar cloud, low density electron–ion plasma form two temperature electron–positron–ion plasmas. The characteristics of the wave motion in the electron–positron–ion plasma should be quite different from those of the two-component electron–positron plasma [17–19]. Such three component plasmas have been studied in the context of pulsar magnetosphere [20–22]. Investigations in such plasmas have been reported recently in a number of cases [23–35]. Most of these studies considered magnetic field in electron–positron–ion plasmas because of their potential relevance to astrophysical background. Berezhiani and Mahajan [36,37] and Mahajan *et al* [38] have analytically shown that large amplitude localized structure can be generated in an unmagnetized electron–positron plasma with a small fraction of ions. Popel *et al* [19] have also reported large amplitude solitons in unmagnetized electron–positron–ion plasma. Ion-acoustic envelope solitons in unmagnetized electron–positron–ion plasma were studied by Salahuddin *et al* [39]. However, most of these investigations are confined to uniform plasmas. To the best of our knowledge, no research work for inhomogeneous electron–positron–ion plasma has been undertaken. Extension to magnetized plasma is worth taking and is under progress. Further, the effect of inhomogeneity on the soliton propagation is an important aspect. Only a few investigations in different context, have attracted the attention of researchers to study ion-acoustic solitons (IAS) in an inhomogeneous plasma [40–44]. Thus, a thought should be given to relate the soliton behavior in inhomogeneous plasmas. We present here an investigation of nonlinear ion-acoustic waves in an inhomogeneous plasma consisting of cold ions and hot electrons and positrons. We have derived the KdV equation, with additional terms, that governs the propagation of ion-acoustic solitons in the presence of weak density gradient in such a plasma. The modified KdV equation is also solved for constant density gradient to obtain characteristics of the solitary waves. In §2, we have set up the basic equations of plasma dynamics. Section 3 deals with the small amplitude solitary waves, and we conclude the paper in §4.

2. Basic equations

We consider a spatially inhomogeneous and collisionless plasma having electrons, positrons and ions. The continuity and momentum equations for ions are as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x}. \tag{2}$$

We assume the Boltzmann's distribution for the electrons and positrons which are given as

$$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right), \tag{3}$$

$$n_p = n_{p0} \exp\left(\frac{-e\phi}{T_p}\right), \tag{4}$$

and Poisson's equation as

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_e - n_i - n_p). \tag{5}$$

In the above equations n_i, n_e and n_p are the number densities of ions, electrons and positrons respectively. Also v_i is the ion velocity, e is the magnitude of the electron charge and m_i is the mass of the ion. T_e and T_p are electron and positron temperatures respectively. Further, ϕ is the electrostatic potential. n_{e0} and n_{p0} are the unperturbed electron and positron number densities.

The normalized form of continuity, momentum and Poisson's equations are as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0, \tag{6}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{7}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (n_e - n_i - n_p). \tag{8}$$

We have normalized n_i, n_e and n_p by the zeroth order positive ion density at an arbitrary reference point in the plasma, which we choose to be at $x = 0$ (say n_0). Also x and t are normalized by the Debye length $\lambda_{\mathrm{D}d} = \sqrt{(T_e/4\pi e^2 n_0)}$ and ion inverse plasma frequency $\omega_{pi}^{-1} = \sqrt{m_i/4\pi e^2 n_0}$ respectively. Velocity and electrostatic potential are normalized by $c_{si} = \sqrt{T_e/m_i}$ and T_e/e respectively.

3. Small amplitude solitary wave

We study the propagation of nonlinear ion-acoustic wave in an inhomogeneous plasma by using the reductive perturbation method. The purpose of this work is to determine the soliton behaviour by carrying out a perturbation expansion based on the assumption that soliton width is small compared with the scale length of the plasma inhomogeneity. Under that condition, the soliton retains its identity and further its amplitude, width and speed are slowly varying functions of the position. In this analysis, we use a set of stretched coordinates, which is appropriate for spatially inhomogeneous plasma, along with the zeroth order fluid velocities. The suitable set of stretched coordinates for the inhomogeneous plasma is defined by [43,44]

$$\xi = \epsilon^{1/2}\left(\int^x \frac{\mathrm{d}x'}{\lambda_0(x')} - t\right) \tag{9}$$

and

$$\eta = \epsilon^{3/2} x, \tag{10}$$

where λ_0 is the phase velocity. In an uniform plasma λ_0 is a constant. However, in a nonuniform plasma λ_0 is a function of the slow variable η. Here ϵ is a small expansion parameter which is a measure of inhomogeneity and $\epsilon^{-3/2}$ is a measure of the scale length on which unperturbed quantities like n_{i0}, n_{e0}, n_{p0}, etc. depend. Since the basis for the perturbation expansion is that the scale length be sufficiently large, it follows that ϵ can be taken as a formal expansion parameter. The condition $\epsilon \ll 1$ implies that the plasma dimension must be much larger than the Debye length, which is satisified in most cases of interest. Since λ_0, n_{i0}, n_{p0} and n_{e0} are functions of x only, we have the following:

$$\frac{\partial \lambda_0}{\partial \xi} = 0, \quad \frac{\partial n_{i0}}{\partial \xi} = 0, \quad \frac{\partial n_{p0}}{\partial \xi} = 0, \quad \frac{\partial n_{e0}}{\partial \xi} = 0. \tag{11}$$

Now we carry out the reductive perturbation analysis of equations (6)–(8). For this, dependent variables like quantities n, v and ϕ are expanded as

$$\begin{aligned} n_e &= n_{e0} + \epsilon n_{e1} + \epsilon^2 n_{e2} + \cdots, \\ n_i &= n_{i0} + \epsilon n_{i1} + \epsilon^2 n_{i2} + \cdots, \\ n_p &= n_{p0} + \epsilon n_{p1} + \epsilon^2 n_{p2} + \cdots, \\ v_i &= v_{i0} + \epsilon v_{i1} + \epsilon^2 v_{i2} + \cdots, \\ \phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \cdots. \end{aligned} \tag{12}$$

Here v_{i0}, n_{i0}, n_{p0} and n_{e0} are the zeroth-order velocity, ion, positron and electron densities respectively, existing in the presence of the spatial inhomogeneity. The charge neutrality condition implies absence of the electric field. Therefore, equilibrium potential is taken as zero. Further, from the zeroth-order equations, we obtain the following:

$$\frac{\partial v_{i0}}{\partial \xi} = 0. \tag{13}$$

To the next higher order equations in ϵ, we have

$$-\frac{\partial n_{i1}}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(n_{i0} v_{i1} + n_{i1} v_{i0}) + \frac{\partial}{\partial \eta}(n_{i0} v_{i0}) = 0, \tag{14}$$

$$-\frac{\partial v_{i1}}{\partial \xi} + \frac{v_{i0}}{\lambda_0}\frac{\partial v_{i1}}{\partial \xi} + v_{i0}\frac{\partial v_{i0}}{\partial \eta} = -\frac{1}{\lambda_0}\frac{\partial \phi_1}{\partial \xi}, \tag{15}$$

$$-n_{i1} + \phi_1 \left(n_{e0} + \frac{T_e}{T_p} n_{p0} \right) = 0. \tag{16}$$

Integrating these equations and using boundary conditions $(n_{p1}, n_1, v_{p1}, v_{i1}, \phi_1) \to 0$ as $|\xi| \to \infty$, we have

$$v_{i1} = Pn_{i1} - \xi T, \quad \phi_1 = Pn_{i0}v_{i1} - R\xi, \tag{17}$$

where

$$P(\eta) = \frac{\lambda_0 - v_{i0}}{n_{i0}}, \quad T(\eta) = \frac{\lambda_0}{n_{i0}} \frac{\partial}{\partial \eta}(n_{i0}v_{i0}) \quad \text{and} \quad R(\eta) = \lambda_0 (v_{i0} \frac{\partial v_{i0}}{\partial \eta}).$$

Using eq. (17) and following simple algebraic manipulations leads to the following equation:

$$\phi_1 = \xi \frac{Pn_{i0}T + R}{P^2 n_{i0} Q - 1}, \tag{18}$$

where $Q = (n_{e0} + \frac{T_e}{T_p} n_{p0})$. Note that the left-hand side of equation (18) is a first-order perturbation while the right-hand side contains only zeroth-order quantities. Thus, in order to obtain nonsecular solution of ϕ_1, numerator and denominator of equation (18) must be equal to zero [43]. This yields

$$(\lambda_0 - v_{i0})^2 = \frac{n_{i0}}{Q} = S, \tag{19}$$

$$\lambda_0 \frac{\partial v_0}{\partial \eta} + \left(\frac{\lambda_0 - v_{i0}}{n_{i0}} \right) v_{i0} \frac{\partial n_{i0}}{\partial \eta} = 0. \tag{20}$$

Thus, we get a self-consistent relation between n_{i0} and v_{i0}.

To the next order in ϵ, we obtain the following equations:

$$-\frac{\partial n_{i2}}{\partial \xi} + \frac{1}{\lambda_0} \frac{\partial}{\partial \xi}(n_{i1}v_{i1} + n_{i0}v_{i2} + n_{i2}v_{i0}) + \frac{\partial}{\partial \eta}(n_{i0}v_{i1} + n_{i1}v_{i0}) = 0, \tag{21}$$

$$-\frac{\partial v_{i2}}{\partial \xi} + \frac{1}{\lambda_0} \left(v_{i0} \frac{\partial v_{i2}}{\partial \xi} + v_{i1} \frac{\partial v_{i1}}{\partial \xi} \right) + \left(v_{i0} \frac{\partial v_{i1}}{\partial \eta} + v_{i1} \frac{\partial v_{i0}}{\partial \eta} \right) + \frac{1}{\lambda_0} \frac{\partial \phi_2}{\partial \xi} + \frac{\partial \phi_1}{\partial \eta} = 0, \tag{22}$$

$$\frac{1}{\lambda_0^2} \frac{\partial^3 \phi_1}{\partial \xi^3} - \frac{n_{i0}}{S} \frac{\partial \phi_2}{\partial \xi} - \left(n_{e0} - \frac{T_e^2}{T_p^2} n_{p0} \right) \phi_1 \frac{\partial \phi_1}{\partial \xi} + \frac{\partial n_{i2}}{\partial \xi} = 0. \tag{23}$$

Using eq. (19), we are able to eliminate all the second-order quantities exactly. Substituting for v_{i1} and n_{i1} in terms of ϕ_1 from eq. (17) into eqs (21)–(23), we get the following modified KdV equation:

$$\frac{\partial \phi_1}{\partial \eta} - \frac{\xi}{\lambda_0} \frac{\partial \lambda_0}{\partial \eta} \frac{\partial \phi_1}{\partial \xi} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} + C\phi_1 = 0, \tag{24}$$

where

$$A = \frac{S^{3/2}}{2\lambda_0^2 n_{i0}}\left[\frac{3n_{i0}}{S^2} - \left(n_{e0} - \frac{T_e^2}{T_p^2}n_{p0}\right)\right], \quad B = \frac{S^{3/2}}{2\lambda_0^4 n_{i0}}, \text{ and}$$

$$C = \frac{1}{2n_{i0}}\frac{\partial n_{i0}}{\partial \eta} - \frac{1}{2\lambda_0 S}\left(S^{1/2} + \frac{\lambda_0}{2} + v_{i0}\right)\frac{\partial S}{\partial \eta}.$$

In order to obtain the solitary wave solution, we have to simplify the equation to a standard form. For this, we use a new transformation

$$\phi_1 = g(\eta)\Phi_1, \tag{25}$$

where

$$g(\eta) = n_{i0}^{-1/2} S^{3/4}.$$

Using this transformation in (24), we get the well-known form of modified-KdV equation

$$B\frac{\partial^3 \Phi_1}{\partial \xi^3} + Ag\Phi_1\frac{\partial \Phi_1}{\partial \xi} - \frac{\xi}{\lambda_0}\frac{\partial \Phi_1}{\partial \xi}\frac{\partial \lambda_0}{\partial \eta} + \frac{\partial \Phi_1}{\partial \eta} = 0. \tag{26}$$

Since coefficients have spatial dependence, it is not so simple to derive the soliton propagation as was done earlier. Thus, the problem becomes intractable. For the sake of simplicity of mathematical development, the variation of nonlinear coefficients are assumed to be negligibly small as compared to the scale length or we can assume parameters to be locally constant. The presumptions, though not actually feasible, in no way differ from getting the main profiles of soliton propagation in the plasma systems.

In order to obtain the solution of (26), we introduce a new variable $\chi = \xi - U\eta$, where U is a constant velocity. Finally, at constant density gradient, we obtain the solution of (26) as

$$\Phi_1(\xi) = \Phi_m \operatorname{sech}^2\left(\frac{\xi - U\eta}{w}\right), \tag{27}$$

where the peak amplitude Φ_m and width w of the solitons are given by

$$\Phi_m = \frac{3}{gA}\left(\frac{\xi}{\lambda_0}\frac{\partial \lambda_0}{\partial \eta} + U\right), \tag{28}$$

$$w = 2\sqrt{\frac{B}{\frac{\xi}{\lambda_0}\frac{\partial \lambda_0}{\partial \eta} + U}}. \tag{29}$$

We first observe explicitly how the product of the peak amplitude and the square width depends upon the ratio of B and A. When these entities are expressed in terms of physical quantities like densities, temperatures, phase velocity etc., it is noticed that the q = (amplitude)(width)^2 is inversely proportional to (i) the square of the phase velocity and (ii) the densities which are spatially dependent parameters. Thus, the relationship q presumed to be a constant in homogeneous plasma, does not hold in inhomogeneous case. It is further observed that for the various values of n_{p0}/n_{e0} and T_e/T_p taken from Salahuddin *et al* [33] and Kakati and Goswami [34], the peak amplitude is always positive and consquently, only compressive solitons are possible. It is clear from the above equations (28) and (29) that the peak amplitude depends upon the density, temperature of different species as well as the density gradient. The peak amplitude of the soliton decreases as density decreases, a fact earlier observed by Chang *et al* [45]. For constant drift velocity, the peak amplitude increases and the width decreases with the increase in density gradient.

4. Conclusion

In this paper, we have studied the role of density and temperature of electron and positron on ion-acoustic solitons in collisionless inhomogeneous plasma consisting of electrons, positrons and ions. We have derived the modified KdV equation using reductive perturbation method. Results are summarized as follows:

(i) Only compressive solitons are obtained.

(ii) For a given density, density gradient and other given parameters of different species (electron and positron), it is always possible to have a large peak amplitude in an inhomogeneous plasma than that predicted by KdV theory for the homogeneous plasma.

(iii) In laboratory plasmas, measured amplitude is observed to have larger value than that calculated theoretically for homogeneous plasma. This discrepancy can be accounted for only when inhomogeneous plasmas are treated analytically. In that case, density, density gradient and other parameters appearing in the soliton amplitude determine the expression for the peak amplitude. On the other hand, the width of the soliton exclusively depends on the density, propagation distance as well as density gradient. Thus, the present theoretical results may be more relevant for astrophysical plasmas.

(iv) The relationship for the soliton q = (amplitude)$\cdot$(width)^2 = constant, observed for solitons in homogeneous plasmas is only approximately one.

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