PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 66, No. 5 May 2006 pp. 953–958

Properties of B_c meson

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Abstract. The mass spectrum of $c\bar{b}$ meson is investigated with an effective quark–antiquark potential of the form $\frac{-\alpha_c}{r} + Ar^{\nu}$ with ν varying from 0.5 to 2.0. The S and P-wave masses, pseudoscalar decay constant, weak decay partial widths in spectator model and the lifetime of B_c meson are computed. The properties calculated here are found to be in good agreement with other theoretical and experimental values at potential index, $\nu = 1$.

Keywords. Potential; decay constant; weak decay.

PACS No. 14.40.Nd

1. Introduction

The discovery of the B_c meson by the collider detector at Fermilab (CDF Collaboration) [1] in $p\bar{p}$ collision at $\sqrt{s} = 1.8$ TeV has demonstrated the possibility of the experimental study of the charm-beauty system and has created considerable interest in its spectroscopy [2–8]. It is the only meson in the heavy flavour sector with different charge and flavours, due to which its decay properties are expected to be different from that of flavour neutral mesons. Though there exist results on charmed hadrons suggesting the importance of relativistic effects, studies based on nonrelativistic models also provide results close to the experimental values [1–4,9–11].

2. Nonrelativistic treatment for heavy quarks

For a heavy–heavy quark bound system such as $c\bar{b}$ we treat both the quarks c and $\bar{b}$ nonrelativistically. The Hamiltonian for this case is given by [12]

$$H = M + \frac{p^2}{2M_1} + V(r), \tag{1}$$

where

$$M = m_c + m_{\bar{b}} \quad \text{and} \quad M_1 = \frac{m_c m_{\bar{b}}}{m_c + m_{\bar{b}}} \tag{2}$$

m_c and $m_{\bar{b}}$ are the mass parameters of charmed quark and bottom quark respectively, p is the relative momentum of each quark and $V(r)$ is the quark–antiquark potential. We consider here a general power potential with the Coulomb term of the form

$$V(r) = \frac{-\alpha_c}{r} + Ar^{\nu}, \tag{3}$$

where $\alpha_c = \frac{4}{3}\alpha_s$, α_s being the strong running coupling constant, A is a model potential parameter and ν is a general power corresponding to the confining part of the potential. In the present case, we study the system by varying ν from 0.5 to 2.0 and the parameter $A = 0.19$ GeV$^{\nu+1}$, $m_b = 4.66$ GeV, $m_c = 1.31$ GeV, are taken to be the same as used in the study of light–heavy flavour mesons [12].

Within the Ritz variational scheme we assume a trial radial wave function $R(r,\ \mu)$, and compute the expectation value of the Hamiltonian given by eq. (1) ($\langle H\rangle = E(\mu,\ \nu)$) with the potential defined by eq. (3). For the ground state we get

$$E(\mu,\nu) = M + \frac{1}{8}\frac{\mu^2}{M_1} + \frac{1}{2}\left(-\mu\alpha_c + A\frac{\Gamma(\nu+3)}{\mu^{\nu}}\right). \tag{4}$$

The trial wave function is assumed to be of the form

$$R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2}(\mu r)^l \mathrm{e}^{-\mu r/2} L^{2l+1}_{n-l-1}(\mu r). \tag{5}$$

Here, μ is the variational parameter and $L^{2l+1}_{n-l-1}(\mu r)$ is the Laguerre polynomial. For a chosen value of ν, the variational parameter μ is determined for each state using the virial theorem

$$\left\langle \frac{P^2}{2M_1}\right\rangle = \frac{1}{2}\left\langle \frac{r\,\mathrm{d}V}{\mathrm{d}r}\right\rangle. \tag{6}$$

As the interaction potential assumed here does not contain the spin-dependent part, eq. (4) gives the spin average masses of the system in terms of the power index ν. The spin average mass for the ground state is computed and are listed in table 1 for values of ν from 0.5 to 2.0.

For the S- and P-wave mass calculations we consider the spin–spin and spin–orbit interactions as [8]

$$V_{S_c\cdot S_b}(r) = \frac{8}{9}\frac{\alpha_s}{m_c m_b}\vec{S}_c\cdot\vec{S}_b 4\pi\delta(r); \qquad V_{L\cdot S}(r) = \frac{4\alpha_s}{3m_c m_b}\frac{\vec{L}\cdot\vec{S}}{r^3}. \tag{7}$$

The computed masses are compared with other theoretical predictions of Eichten and Quigg [3], Hady [4], Ebert *et al* [5], Davies *et al* [6] and Gershtein *et al* [8] in table 2. Our predicted mass for $B_c(1^1S_0)$ is in good accord with experimental result of 6.40±0.39(stat.) ± 0.13(syst.) GeV/C^2 [1] and the masses obtained for the 2S, 3S, 1P, 2P states are comparable with other theoretical predictions.

Table 1. The variational parameter $\bar{\mu}$, wave function at the origin ($|R(0)|$) S-wave and P-wave (spin average) masses of $c\bar{b}$ meson.

Mesonic systems	State	ν	$\bar{\mu}$ (GeV)	$\|R(0)\|$ (GeV)$^{3/2}$	$E(\bar{\mu})$ (GeV)	EFG [5] (GeV)	ZVR [13] (GeV)
		0.5	1.1920	0.920	6.230		
	$1S$	1.0	1.6020	1.434	6.367	6.317	6.320
		1.5	1.9580	2.358	6.509	6.387*	
		2.0	2.2760	2.428	6.657		
		0.5	1.0910	0.254	6.419		
	$1P$	1.0	1.6960	0.765	6.738	6.736	6.740
		1.5	2.2302	1.516	7.073		
		2.0	2.7040	2.454	7.414		
		0.5	1.1250	0.844	6.457		
$c\bar{b}$	$2S$	1.0	1.7950	1.701	6.846	6.869	6.887
		1.5	2.3965	2.623	7.269		
		2.0	2.9330	3.552	7.699		
		0.5	1.1360	0.459	6.566		
	$2P$	1.0	1.9440	1.756	7.136	7.142	7.150
		1.5	2.7035	4.006	7.799		
		2.0	3.3985	7.097	8.505		
		0.5	1.1528	0.875	6.586		
	$3S$	1.0	1.9930	1.989	7.201	7.224	7.270
		1.5	2.7874	3.291	7.923		
		2.0	3.5174	4.665	8.696		

*AlV [4], $\alpha_s = 0.255$, $m_b = 4.66$ GeV, $m_c = 1.31$ GeV, $A = 0.19$ GeV$^{\nu+1}$.

3. Decay properties of B_c^+ meson

The decay properties of B_c^+ ($\bar{b}c$) meson is of interest as it decays only through weak interactions [1,4,5]. This is due to the fact that its ground state energy lies below the B, D meson production threshold and has nonvanishing flavour. This eliminates the uncertainties encountered due to strong decays and provides a clear decay width and lifetime for B_c^+ meson, which helps to fix more precise value of the weak decay parameters such as the CKM mixing matrix elements (V_{cb}, V_{cs}) and the leptonic decay constant (f_P). Adopting the spectator model for the charm-beauty system [4], the total decay width of B_c^+ meson can be approximated as the sum of the widths of $\bar{b}$-quark decay keeping c-quark as spectator, the c-quark decay with $\bar{b}$-quark as spectator, and the annihilation channel $B_c^+ \to l^+\nu_l(c\bar{s}, u\bar{s})$, $l = e, \mu, \tau$ with no interference assumed between them.

Accordingly, the total width is written as [4]

$$\Gamma(B_c \to X) = \Gamma(b \to X) + \Gamma(c \to X) + \Gamma(\text{Anni}). \tag{8}$$

Neglecting the quark binding effects, we obtain for the b and c inclusive widths in

Table 2. B_c meson mass spectrum (in GeV) with $\nu = 1$.

$n^{2S+1}L_J$	This work	ALV [4]	EQ [3]	EFG [5]	Lattice [6]
1^1S_0	6.349	6.356	6.264	6.270	6.280±30 ± 190
1^3S_1	6.373	6.397	6.337	6.332	6.321±20
1^3P_0	6.715	6.673	6.700	6.699	6.727±30
1^3P_1	6.726	–	6.730	6.734	6.743±30
1^1P_1	6.738	–	6.736	6.749	6.765±30
1^3P_2	6.749	6.751	6.747	6.762	6.783±30
2^1S_0	6.821	6.888	6.856	6.835	6.960±80
2^3S_1	6.855	6.910	6.899	7.072	6.990±80
2^3P_0	7.102	–	7.108	7.091	–
2^3P_1	7.119	–	7.135	7.126	–
2^1P_1	7.136	–	7.142	7.145	–
2^3P_2	7.153	–	7.153	7.156	–
3^1S_0	7.175	–	7.244	7.193	–
3^3S_1	7.210	–	7.280	7.235	–

the spectator approximation [4],

$$\Gamma(b \to X) = \frac{9G_{\rm F}^2 \left|V_{cb}\right|^2 m_b^5}{192\pi^3} = 8.75 \times 10^{-4} \text{ eV}, \tag{9}$$

$$\Gamma(c \to X) = \frac{5G_{\rm F}^2 \left|V_{cs}\right|^2 m_c^5}{192\pi^3} = 4.19 \times 10^{-4} \text{ eV}. \tag{10}$$

Here we have used $|V_{cs}| = 0.975$, $|V_{cb}| = 0.044$ as the upper bound provided by particle data group [10], and the value of m_b, m_c as used in our mass predictions.

In the nonrelativistic limit, the pseudoscalar constant f_P and the ground state wave function at the origin $R(0)$ are related and is given by the Van Royen Weisskopf [3] formula including the color factor, as

$$f_{B_c} = \sqrt{\frac{3}{\pi M_{B_c}}} R_{1s}(0). \tag{11}$$

The f_{B_c} values obtained here are 361 MeV, 556 MeV, 757 MeV and 929 MeV for $\nu = 0.5$, 1.0, 1.5 and 2.0 respectively.

Now, the width of the annihilation channel is computed using the expression given by [4]

$$\begin{aligned}\Gamma(\text{Anni}) &= \frac{G_{\rm F}^2}{8\pi} \left|V_{bc}\right|^2 f_{B_c}^2 M_{B_c} \sum_i m_i^2 \left(1 - \frac{m_i^2}{M_{B_c}^2}\right)^2 \cdot C_i \\ &= 0.923 \times 10^{-4} \text{eV},\end{aligned} \tag{12}$$

where $C_i = 1$ for the $\tau\nu_\tau$ channel and $C_i = 3\left|V_{cs}\right|^2$ for $c\bar{s}$, and m_i is the mass of the heaviest fermions. Our result for f_{B_c} and M_{B_c} obtained with the potential parameter $\nu = 1$ are used in eq. (12).

Table 3. Comparison of the lifetime of B_c meson (in ps) in different models.

This work	Expt. [1]	ALV [4]	GKLT [8]	VVK [14]	SG [15]
0.47	$\tau = 0.46^{+0.18}_{-0.16}$	0.47	0.55±0.15	0.50	0.75

Adding all the three contributions according to eq. (8) yield the total width $\Gamma(\text{total}) = 13.863 \times 10^{-4}$ eV and the lifetime of B_c^+ meson as 0.47 ps, which is in good agreement with the measured value of $\tau = 0.46^{+0.18}_{-0.16}$ ps [1].

4. Conclusion

Based on a simple nonrelativistic potential scheme within variational approach we have been able to predict the S- and P-wave masses and lifetime of B_c^+ meson successfully. Mass spectrum and pseudoscalar decay constants f_{B_c} are computed for the potential index (ν) from 0.5 to 2. Our predictions of the masses and f_{B_c} values are found to be in accordance with other theoretical predictions for $\nu = 1$. It is found that f_{B_c} and M_{B_c} increases as the potential parameter ν increases.

The model parameters such as the charm and beauty quark masses used in our calculations and the pseudoscalar decay constants f_{B_c} obtained here for $\nu \simeq 1$ are found to be appropriate in the calculation of the decay widths. We get about 63% as the branching fractions of b-quark decay, about 30% as that of c-quark decay and about 7% in the annihilation channel. However, the CKM mixing matrix elements V_{cb} and V_{cs} used as free parameters in all the theoretical calculations compared here are different but within the range given in particle data group [10]. The lifetime of B_c^+ predicted by the present calculation is found to be in good agreement with the experimental values as well as that by the Bethe Salpeter method (ALV) (see table 3). The predicted values from relativized model (SG) is found to be far from the experimental values as well as other theoretical models.

In conclusion, a simple nonrelativistic variational method with potential $-\frac{\alpha_c}{r} + Ar^{\nu}$ employed in the present study is found to be quite successful in predicting various properties of B_c^+ meson. The method can be useful to study various hadronic and radiative transitions of the charm-beauty system.

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