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A new 'hidden colour hypothesis' in hadron physics

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Abstract. A new 'hidden colour hypothesis' within the framework of QCD, as an extension of and in keeping with the spirit of the 'colour singlet hypothesis' is hereby proposed. As such it should play a role in a consistent description of exotic hadrons, such as diquonia, pentaquarks, dibaryons etc. How these exotic hadrons are affected by this new hypothesis is discussed here. This new hypothesis suggests that the experimentalists may not be looking for single exotics but for composites of two or more of the same.

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As per the colour confinement hypothesis of QCD, only the colour singlet part of the quark configurations would manifest themselves as physically observed particles. All the other representations like the colour octet, though they arise in qqq and $q\bar{q}$ systems, do not show up as physical particles. These forbidden configurations though show up in larger number multiquark systems to give colour singlet objects themselves. So such components though missing in simple qqq and $q\bar{q}$ systems actually play a prominent role in a large number of quark colour singlet objects. That is, they are hidden inside larger colour singlet objects. As such these hidden colour parts (to be clearly defined below) are actually as significant as the colour singlet part. No one has been able to prove the colour singlet hypothesis from the fundamentals of the QCD. It may become a 'law' only after it can satisfactorily be shown to arise from QCD itself. So though no violation of it has been found (remember that actually the colour singlet idea is first an empirical fact then later only a theoretical concept) it still retains the prefix of 'hypothesis'.

Now, one has to define the 'hidden colour' concept. The baryons, made up of three quarks, besides the colour singlet part which shows up in nature explicitly as per the colour singlet hypothesis, have other forbidden parts like the colour octet. When one goes to the six quarks then the total composite system can be colour singlet in the following manner:

$$|6q\rangle = \frac{1}{\sqrt{5}}|SS\rangle + \frac{2}{\sqrt{5}}|CC\rangle, \tag{1}$$

where S represents a 3-quark cluster which is singlet in colour space and C represents the same as octet in colour space. Hence $|CC\rangle$ is overall colour singlet.

Table 1. Hidden colour components in multiquark systems.

Multiquark configurations	Percentage of hidden colour
$qq\bar{q}\bar{q}$ (qq in [3])	33
$qq\bar{q}\bar{q}$ (qq in [5])	66
Pentaquark $qqqq\bar{q}$	66
$A = 2q^6$	80
$A = 3q^9$	97.6
$A = 4q^{12}$	99.8

This part is called the hidden colour because as per confinement ideas of QCD these octets cannot be separated out asymptotically and so manifest themselves only within the 6-q colour-singlet system [1]. The relevant group theory is given in the Appendix and the hidden colour parts of the wave function for a few typical multiquark systems are given in table 1.

If colour singlet is so basic to nature, the author feels that the allied concept of the hidden colour should not be any less important. Hence the author would like to propose here a new hypothesis. This actually very simply arises as an extension of the colour singlet hypothesis and in a manner with physical manifestations opposite to those of the colour singlet hypothesis. Let us call it the 'hidden colour hypothesis'. As per this hypothesis, in the ground state or at low excitations, the creation as an independent entity of a hadron which has a large hidden colour part in its wave function would be appropriately suppressed. Hence if in a particular configuration in space–time a particular hadron has large hidden colour component in the state in which it is expected to exist, then it will be difficult to produce it in the laboratory. Also deep inside the colour singlet objects, it will manifest as repulsive core in the appropriate region.

As an application of this hypothesis let us look at deuteron. Most of the time the two nucleons in deuteron are isolated colour singlet objects. However, when two such nucleons come together to form a bound system like deuteron, why do they not have configuration where the two nucleons overlap strongly in regions of size $\leq$1 fm to form 6-quark bags? Why is deuteron such a big and loose system? The reason has to do with the structure of the 6-q bags formed had the two nucleons overlapped strongly. At the centre of mass of the bound system, clearly the 6-q configuration should be important. Hence this 80% hidden colour part (see table 1) would prevent the two nucleons to come together and overlap strongly. Therefore the hidden colour would manifest itself as a short-range repulsion in the region $\leq$1 fm in deuteron. So the two nucleons though bound, stay considerably away from each other. Hence this new hypothesis is able to explain the basic property of short-range repulsion in nuclear physics.

The hidden colour concept in spite of being successful has had a rough start. There have been some claims that hidden colour may not be a useful concept as these hidden colour states can be rearranged in terms of asymptotic colour singlet states. But as discussed in [3] the hidden colour concept is not unique only when the two clusters do not overlap fully or when the two clusters can be separated out asymptotically. However, when the clusters of 3-q each overlap strongly so that the relative distance between them goes to zero, then the hidden colour concept becomes relevant and unique [2]. We would like to point out that indeed, this necessarily is the situation for deuteron discussed above. Also note that for the ground state the quark configuration is s^6 given by configuration space representation [6] while s^4p^2 given by [4] does not come into play as there is not enough energy to put two quarks into the p-orbital.

Group theoretically the author had earlier obtained the hidden colour components in 9- and 12-quark systems [4,5]. For the ground state and low-energy description of nucleons we assume that $SU(2)_F$ with u- and d-quarks is required. Hence we assume that 9- and 12-quarks belong to the totally antisymmetric representation of the group $SU(12) \supset SU(4)_{SF} \otimes SU(3)_c$ where $SU(3)_c$ is the QCD group and $SU(4)_{SF} \supset SU(2)_F \otimes SU(2)_S$ where S denotes spin. Note that up to 12 quarks can sit in the s-state in the group $SU(12)$. The calculation of the hidden colour components for 9- and 12-quark systems requires the determination of the coefficients of fractional parentage for the group $SU(12) \supset SU(4) \otimes SU(3)$ [4,5] which becomes quite complicated for large number of quarks (see Appendix). The author found that the hidden colour component [4,5] of the 9-q system is 97.6% while the 12-q system is 99.8%, i.e. practically all coloured (see table 1).

Where would these 9- and 12-quark configurations be relevant in nuclear physics? The $A = 3, 4$ nuclei ^{3}H, ^{3}He and ^{4}He have sizes of 1.7 fm, 1.88 fm and 1.674 fm respectively. Given the fact that each nucleon is itself a rather diffuse object, quite clearly in a size $\leq$1 fm at the centre of these nuclei the three or four nucleons would overlap strongly. As the corresponding 9- and 12-q are predominantly hidden colour, there would be an effective repulsion at the centre keeping the three or four nucleons away from the centre as per the hidden colour hypothesis above. Hence it was predicted by the author [4,5] that there should be a hole at the centre of ^{3}H, ^{3}He and ^{4}He. And indeed, this is what is found through electron scattering. Hence the hole, i.e. significant depression in the central density of ^{3}H, ^{3}He and ^{4}He is a signature of the hidden colour hypothesis in this ground state property.

The author has used this concept of the role of hidden colour in $A = 3,4$ nuclei to predict triton clustering in nuclei [6]. Equally significant is the suggestion of a new symmetry in the same neutron-rich nuclei [3]. Herein triton ('t') $^3_1\mathrm{H}_2$ and helion ('h') $^3_2\mathrm{He}_1$ are treated as fundamental representations of a new symmetry group $SU(2)_{\mathcal{A}}$ [3] which has been christened as 'nusospin symmetry'. Note that basically this nusospin symmetry arises due to the application of the hidden colour hypothesis. Clearly the fact that $^{3Z}_{Z}A_{2Z}$ nuclei are made up of Z number of tritons leads to new stability for them. The author has found extra-ordinary stability manifested by the separation energy data for the proton and neutron pairs (Z, N): (6,12), (8,16), (10,20), (11,22) and (12,24) [7]. This also hints at the possibility of neutron stars.

Now let us look at the significance of hidden colour for the exotic hadrons such as the diquonia, pentaquarks, dibaryons etc. The recent claims of the discovery of the pentaquark has been much in the news. However, even the discoverers of this exotic states ask for caution in the acceptance of this claim. This should be seen in the light of the lack of exotic states in spite of intense searches for these in laboratories all over the world in the last few decades. Once in a while a claim is made. But mostly it does not withstand closer scrutiny as none of these are starred objects in the *Particle Data Tables*! This is by and large the story of the exotic baryons. So one may ask as why, if QCD demands their existence, is it that still we do not have a definite candidate for the exotic hadron as of now.

The answer is provided by the new hidden colour hypothesis proposed here. We have already seen that inside nuclei these hidden colour components can manifest themselves over a short range near the centre of mass of the nucleus and there they provide for short-range repulsion in two-, three- and four-nucleon systems. From table 1 we notice that these exotic hadrons also have large hidden colour parts and so if we try to to create these in these configurations, it will be resisted. Also in the ground state we feel that these configuration should be significant. Hence, in short, the hidden colour hypothesis says that it will be difficult to create these exotic hadrons thereby explaining their reluctance to show up as of now.

It should be pointed out that a major support for the colour singlet hypothesis comes from the fact that as of now we have not found any coloured object in free state. So, the fact that as of now we have not found a definite candidate for exotic hadrons, should be taken as a proof or strong motivation for the hidden colour hypothesis. One exception may be the pentaquark. Here, as shown in table 1, in one particular case the hidden colour part is only 33%. The small resistance may be bypassed by some ingenious experimentation. However, one possible prediction of this hypothesis is that as the multiquarks have large hidden colour parts within them, two or more multiquarks of the same kind may like to form a composite as then they may have many more ways of forming a colour singlet hadron. So perhaps the experimantalists should look for these objects as clusters of two or more of the same kind of multiquarks.

Appendix

Let $S(f_1)$ and $S(f_2)$ be the permutation groups for the particles $1, 2, \ldots, f_1$ and $f_1 + 1,\ f_2 + 2, \ldots, f_1 + f_2$, respectively, with $f = f_1 + f_2$. We use the following notation [8] to designate the irreducible representations (irreps) of the nine groups $S^x(f_1), \ldots, S^q(f)$ and the irreps of $SU(m)$, $SU(n)$ and $SU(mn)$ (of the group chain $SU(mn) \supset SU(m) \times SU(n)$)

$$\begin{pmatrix} \sigma' & \mu' & \nu' \\ \sigma'' & \mu'' & \nu'' \\ \sigma & \mu & \nu \end{pmatrix} \begin{pmatrix} S^x(f_1) & S^y(f_1) & S^q(f_1) \\ S^x(f_2) & S^y(f_2) & S^q(f_2) \\ S^x(f) & S^y(f) & S^q(f) \end{pmatrix} \begin{pmatrix} SU(m) & SU(n) & SU(mn) \\ SU(m) & SU(n) & SU(mn) \\ SU(m) & SU(n) & SU(mn) \end{pmatrix}. \quad (2)$$

For example, $\{\mu''\}$ and $\{\sigma\}$ are the irreps of $S^y(f_2)$ and $S^x(f)$ respectively. Let

$$\left| \begin{matrix} \{\nu'\} \\ \beta'\{\sigma'\}W_1'\{\mu'\}W_2' \end{matrix} \right\rangle, \left| \begin{matrix} \{\nu''\} \\ \beta''\{\sigma''\}W_1''\{\mu''\}W_2'' \end{matrix} \right\rangle, \left| \begin{matrix} \{\nu\} \\ \beta\{\sigma\}W_1\{\mu\}W_2 \end{matrix} \right\rangle \quad (3)$$

be the $SU(mn) \supset SU(m) \times SU(n)$ irreducible bases in the q-space of the particles $(1, 2, \ldots, f_1)$, $(f + 1, \ldots, f)$ and $(1, 2, \ldots, f)$ respectively; $\{\sigma\}$, $\{\mu\}$ and $\{\nu\}$ are partition labels for the irreducible representations of $SU(m)$, $SU(n)$ and $SU(mn)$ respectively; $W_1(W_2)$ is the component index for the irrep $\{\sigma\}$ ($\{\mu\}$); and β', β'' and

β are inner multiplicity labels: $\beta' = 1, 2, \ldots, (\sigma'\mu'\nu')$, $\beta'' = 1, 2, \ldots, (\sigma''\mu''\nu'')$, $\beta = 1, 2, \ldots, (\sigma\mu\nu)$. The integers $(\sigma'\mu'\nu')$, $(\sigma''\mu''\nu'')$ and $(\sigma\mu\nu)$ are determined by the CG series of the permutation group. The $SU(mn) \supset SU(m) \times SU(n)$ coefficient of fractional parentage (CFP) are defined as the coefficients in the following expansion:

$$\left| \begin{matrix} \{\nu\}\tau \\ \beta\sigma W_1 \mu W_2 \end{matrix} \right\rangle = \sum_{\beta'\beta''\sigma'\sigma''\mu'\mu''\theta\phi} C^{\{\nu\}\tau,\beta\{\sigma\}\theta\{\mu\}\phi}_{\nu'\beta'\sigma'\mu',\nu''\beta''\sigma''\mu''} \times \left[\left| \begin{matrix} \{\nu'\} \\ \beta'\sigma'\mu' \end{matrix} \right\rangle \left| \begin{matrix} \{\nu''\} \\ \beta''\sigma''\mu'' \end{matrix} \right\rangle \right]^{\{\sigma\}\theta\{\mu\}\phi}_{W_1 W_2}, \tag{4}$$

where τ, θ and ϕ are the outer multiplicity labels. $\tau = 1, 2, \ldots, \{\nu'\nu''\nu\}$, $\theta = 1, 2, \ldots, \{\sigma'\sigma''\sigma\}$, $\phi = 1, 2, \ldots, \{\mu'\mu''\mu\}$.

The integers $\{\nu'\nu''\nu\}$, $\{\sigma'\sigma''\sigma\}$ and $\{\mu'\mu''\mu\}$ are decided by the Littlewood rule. The bases are finally combined into the irreducible basis $\{\sigma\}W_1$ and $\{\mu\}W_2$ in terms of the CG coefficients of $SU(m)$ and $SU(n)$ respectively.

Next the irreducible bases classified according to the irreps of the group chain $S(f) \supset S(f_1) \times S(f_2)$ in the x, y and $q(x, y)$ spaces be respectively denoted by

$$\left| \begin{matrix} \{\sigma\} \\ \theta\{\sigma'\}m_1'\{\sigma''\}m_1'' \end{matrix} \right\rangle, \left| \begin{matrix} \{\mu\} \\ \phi\{\mu'\}m_2'\{\mu''\}m_2'' \end{matrix} \right\rangle, \left| \begin{matrix} \{\nu\} \\ \tau\{\nu'\}m'\{\nu''\}m'' \end{matrix} \right\rangle, \tag{5}$$

where m_1', m_2' etc. are the Yamanouchi symbols. The $S(f) \supset S(f_1) \times S(f_2)$ isoscalar factor (ISF) are defined as the expansion coefficients in the following:

$$\left| \begin{matrix} \{\nu\}\beta \\ \tau\nu'm'\nu''m'' \end{matrix} \right\rangle = \sum_{\beta'\beta''\sigma'\sigma''\mu'\mu''\theta\phi} C^{\{\nu\}\beta,\tau\{\nu'\}\beta'\{\nu''\}\beta''}_{\{\sigma\}\theta\sigma'\sigma'',\{\mu\}\phi\mu'\mu''} \times \left[\left| \begin{matrix} \{\sigma\} \\ \theta\sigma'\sigma'' \end{matrix} \right\rangle \left| \begin{matrix} \{\mu\} \\ \phi\mu'\mu'' \end{matrix} \right\rangle \right]^{\{\nu'\}\beta'\{\nu''\}\beta''}_{m'm''}, \tag{6}$$

where the coupling in terms of the CG coefficients is indicated in square bracket. It can be shown that the $SU(mn) \supset SU(m) \times SU(n)$ CFP are identical with the $S(f) \supset S(f_1) \times SU(f_2)$ ISF, i.e.

$$C^{\{\nu\}\tau,\beta\{\sigma\}\theta\{\mu\}\phi}_{\{\nu'\}\beta'\sigma'\mu',\{\nu''\}\beta''\sigma''\mu''} = C^{\{\nu\}\beta,\tau\{\nu'\}\beta'\{\nu''\}\beta''}_{\{\sigma\}\theta\sigma'\sigma'',\{\mu\}\phi\mu'\mu''}. \tag{7}$$

Therefore these CFPs are independent of m and n.

Transforming the Yamanouchi basis of $S(f)$ into the $S(f) \supset S(f_1) \times SU(f_2)$ basis by

$$\left| \begin{matrix} \{\nu\} \\ m, \beta\{\sigma\}W_1\{\mu\}W_2 \end{matrix} \right\rangle = \sum_{\nu''m''\tau} \begin{pmatrix} \{\nu\} \\ m \end{pmatrix} \left| \{\nu\}, \begin{matrix} \tau\{\nu'\} & \{\nu''\} \\ m' & m'' \end{matrix} \right) \times \left| \begin{matrix} \{\nu\} \\ \tau\{\nu'\}m'\{\nu''\}m'', \beta\{\sigma\}W_1\{\mu\}W_2 \end{matrix} \right\rangle \tag{8}$$

and after some algebra one obtains the $SU(mn) \supset SU(m) \times SU(n)$ CFP as

$$C^{\{\nu\}\tau,\beta\{\sigma\}\theta\{\mu\}\phi}_{\{\nu'\}\beta'\sigma'\mu',\{\nu''\}\beta''\sigma''\mu''} = \sum_{m}^{fix.m'} \sum_{m_1 m_2 m_1'' m_2''} C^{\{\nu\}\beta,m}_{\sigma m_1,\mu m_2} C^{\{\nu'\}\beta',m'}_{\sigma' m_1',\mu' m_2'} C^{\{\nu''\}\beta'',m''}_{\sigma'' m_1'',\mu'' m_2''}$$
$$\times \left(\begin{array}{c} \{\nu\} \\ m \end{array} \middle| \{\nu\}, \begin{array}{cc} \tau\{\nu'\} & \{\nu''\} \\ m' & m'' \end{array} \right) \left(\begin{array}{c} \{\sigma\} \\ m_1 \end{array} \middle| \{\sigma\}, \begin{array}{cc} \theta\{\sigma'\} & \{\sigma''\} \\ m_1' & m_1'' \end{array} \right)$$
$$\times \left(\begin{array}{c} \{\mu\} \\ m_2 \end{array} \middle| \{\mu\}, \begin{array}{cc} \phi\{\mu'\} & \{\mu''\} \\ m_2' & m_2'' \end{array} \right), \qquad (9)$$

where the C on the RHS are the Clebsch–Gordan coefficients.

This is rather a complex looking expression. It however simplifies considerably for the case of totally antisymmetric irreps $\{\nu'\} = \{1^{f_1}\}, \{\nu''\} = \{1^{f_2}\}, \{\nu\} = \{1^f\}$ whence

$$C^{\{1^f\},\{\sigma\}\theta\{\mu\}\phi}_{\{1^{f_1}\}\{\sigma'\}\{\mu'\},\{1^{f_2}\}\{\sigma''\}\{\mu''\}} = \delta_{\tilde{\sigma}\mu}\delta_{\tilde{\sigma}'\mu'}\delta_{\tilde{\sigma}''\mu''}\delta_{\theta\sigma}\left(\frac{h_{\sigma'}h_{\sigma''}}{h_\sigma}\right)^{1/2}, \qquad (10)$$

where $h_{\sigma'}$, $h_{\sigma''}$ and h_σ, are the dimensions of the irreps $\{\sigma'\}$, $\{\sigma''\}$ and $\{\sigma\}$ of the permutation groups $S(f_1)$, $S(f_2)$ and $S(f)$ respectively. We need the last expression to calculate the hidden colour components of multiquark systems.

The results for the colourless channel are given in table 1. For example C for the 9-quark state for the state $(56,1)\otimes(490,1) \to (980,1)$ of $SU(12) \supset SU(6)\otimes SU(3)_c$ is given as

$$C^{[1^9][333][\tilde{3}\tilde{3}\tilde{3}]}_{[1^3][3][3],[1^6][33][\tilde{3}\tilde{3}]} = \left(\frac{h_{[3]}h_{[33]}}{h_{[333]}}\right)^{1/2} = \sqrt{\frac{5}{42}}. \qquad (11)$$

Multiplying this with $\sqrt{\frac{1}{5}}$ of $(490,1)$ we get $\sqrt{\frac{1}{42}}$ for the colourless channel. Therefore the 9-quark state consists of 97.6% hidden colour components (see table 1).

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