

Hadrons in compact stars

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Abstract. We discuss β -equilibrated and charge neutral matter involving hyperons and \bar{K} condensates within relativistic models. It is observed that populations of baryons are strongly affected by the presence of antikaon condensates. Also, the equation of state including \bar{K} condensates becomes softer resulting in a smaller maximum mass neutron star.

Keywords. Hadrons; dense matter; composition; equation of state; compact stars.

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1. Introduction

There is a growing interplay between the physics of dense matter in relativistic heavy-ion collisions and neutron stars [1]. Though quantum chromodynamics predicts a very rich phase structure of dense matter, we can only probe a small region of it in the laboratories. Relativistic heavy-ion experiments at CERN and BNL produce a hot (a few hundreds of MeV) and dense matter (a few times the normal nuclear matter density) whereas the cold and dense matter relevant to neutron stars cannot be produced in a laboratory. The study of dense matter in heavy-ion collisions reveals many new and interesting results such as the modifications of hadron properties in dense medium, the properties of strange matter including hyperons and (anti)kaons and the formation of quark-gluon plasma. These empirical information from heavy-ion collisions may be useful in understanding dense matter in neutron star interior. In addition, satellite-based observatories such as Hubble space telescope and Chandra X-ray observatory are pouring in very exciting data on compact stars. Measurements of masses and radii of compact stars from various observations might constrain the composition and equation of state (EoS) of neutron star matter.

Neutron star matter encompasses a wide range of densities, from the density of iron nucleus at the surface of the star to several times normal nuclear matter density in the core. The temperature of a neutron star is a few MeV whereas the baryon chemical potential in its interior is a few hundreds of MeV. That is why neutron star matter is called the cold and dense matter. As chemical potentials of baryons and leptons increase with density in the core, exotic forms of matter such

as hyperons, Bose–Einstein condensate of antikaons and quarks may appear there [2].

In this article, we discuss the composition and EoS of neutron star matter involving Bose–Einstein condensates of antikaons within relativistic models. Also, the structures of non-rotating neutron stars are calculated using this EoS.

2. Hadrons in cold and dense medium

At normal nuclear matter density, neutron star matter mainly consists of neutrons, protons and electrons. The particle population is so arranged as to attain a minimum energy configuration maintaining electrical charge neutrality and chemical equilibrium. At higher baryon density, hyperon formation becomes energetically favorable in neutron star interior as the total energy and pressure of the system are lowered by sharing baryon number among several baryon species.

In compact star interior, hyperons maintain chemical equilibrium through weak processes. The generalised β -decay processes may be written in the form $B_1 \rightarrow B_2 + l + \bar{\nu}_l$ and $B_2 + l \rightarrow B_1 + \nu_l$ where B_1 and B_2 are baryons and l is a lepton. Therefore, the generic equation for chemical equilibrium condition is

$$\mu_i = b_i \mu_n - q_i \mu_e, \quad (1)$$

where μ_n , μ_e and μ_i are respectively the chemical potentials of neutrons, electrons and i th baryon and b_i and q_i are baryon and electric charge of i th baryon respectively. The above equation implies that there are two independent chemical potentials μ_n and μ_e corresponding to two conserved charges, i.e. baryon number and electric charge.

We adopt a relativistic field theoretical model to describe the pure hadronic matter [3]. The constituents of matter are $n, p, \Lambda, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^-, \Xi^0$ of the baryon octet and electrons and muons. In this model, baryon–baryon interaction is mediated by the exchange of scalar and vector mesons and for hyperon–hyperon interaction, two additional hidden-strangeness mesons – scalar meson $f_0(975)$ (denoted hereafter as σ^*) and the vector meson $\phi(1020)$ [4] are incorporated. Therefore, the Lagrangian density for the pure hadronic phase is given by

$$\begin{aligned} \mathcal{L}_B = & \sum_B \bar{\Psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - g_{\rho B} \gamma_\mu t_B \cdot \boldsymbol{\rho}^\mu) \Psi_B \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) \\ & - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu + \mathcal{L}_{YY}. \end{aligned} \quad (2)$$

The isospin multiplets for baryons $B = N, \Lambda, \Sigma$ and Ξ are represented by the Dirac spinor Ψ_B with vacuum baryon mass m_B , isospin operator t_B and $\omega_{\mu\nu}$ and $\rho_{\mu\nu}$ are field strength tensors. The scalar self-interaction term [5]

$$U(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4. \quad (3)$$

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The Lagrangian density for hyperon–hyperon interaction (\mathcal{L}_{YY}) is given by

$$\begin{aligned} \mathcal{L}_{YY} = & \sum_B \bar{\Psi}_B (g_{\sigma^* B} \sigma^* - g_{\phi B} \gamma_\mu \phi^\mu) \Psi_B \\ & + \frac{1}{2} (\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}) - \frac{1}{4} \phi_{\mu\nu} \phi^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu. \end{aligned} \quad (4)$$

We perform this calculation in the mean-field approximation [6]. The mean values for the corresponding meson fields are denoted by σ , σ^* , ω_0 , ρ_{03} and ϕ_0 . Therefore, we replace meson fields with their expectation values and meson field equations become

$$m_\sigma^2 \sigma = -\frac{\partial U}{\partial \sigma} + \sum_B g_{\sigma B} n_B^s, \quad (5)$$

$$m_{\sigma^*}^2 \sigma^* = \sum_B g_{\sigma^* B} n_B^s, \quad (6)$$

$$m_\omega^2 \omega_0 = \sum_B g_{\omega B} n_B, \quad (7)$$

$$m_\phi^2 \phi_0 = \sum_B g_{\phi B} n_B, \quad (8)$$

$$m_\rho^2 \rho_{03} = \sum_B g_{\rho B} I_{3B} n_B. \quad (9)$$

The scalar density

$$n_B^S = \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} \frac{m_B^*}{(k^2 + m_B^{*2})^{1/2}} k^2 dk, \quad (10)$$

and the baryon number density

$$n_B = (2J_B + 1) \frac{k_{FB}^3}{6\pi^2}, \quad (11)$$

where k_{FB} is the Fermi momentum, J_B is the spin, and I_{3B} is the isospin projection.

Effective mass and chemical potential of baryon B are $m_B^* = m_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^*$ and $\mu_B = (k_{FB}^2 + m_B^{*2})^{1/2} + g_{\omega B} \omega_0 + g_{\phi B} \phi_0 + I_{3B} g_{\rho B} \rho_{03}$, respectively. In the pure hadronic phase, the total charge density

$$Q^h = \sum_B q_B n_B^h - n_e - n_\mu = 0, \quad (12)$$

where n_B^h is the number density of baryon B in the pure hadronic phase and n_e and n_μ are charge densities of electrons and muons respectively.

Solving the equations of motion in the mean-field approximation along with effective baryon masses (m_i^*) and equilibrium conditions we immediately compute

the equation of state in the pure hadronic phase. The energy density (ε^h) is related to the pressure (P^h) in this phase through the Gibbs–Duhem relation

$$P^h = \sum_i \mu_i n_i - \varepsilon^h. \quad (13)$$

Here μ_i and n_i are the chemical potential and number density for the i th species.

Kaplan and Nelson [7] first showed in a chiral $SU(3)_L \times SU(3)_R$ model that Bose–Einstein condensation of K^- mesons could be possible in dense hadronic matter. They argued that the strongly attractive K^- –nucleon interaction might lower the effective mass and the in-medium energy of K^- mesons in dense matter. The s -wave condensation sets in when the effective energy of K^- mesons equals its chemical potential. Recently, strongly attractive antikaon–nucleon interaction in-medium has been extracted from the flow data of antikaons in heavy-ion collisions [8].

In neutron star interior, strangeness changing processes such as

$$N \rightleftharpoons N + \bar{K}, \quad (14)$$

$$e^- \rightleftharpoons K^- + \nu_e, \quad (15)$$

may occur. Here $N \equiv (n, p)$ and $\bar{K} \equiv (K^-, \bar{K}^0)$ denote the isospin doublets for nucleons and antikaons, respectively. The threshold conditions for \bar{K} condensation

$$\begin{aligned} \mu_{K^-} &= \mu_n - \mu_p = \mu_e, \\ \mu_{\bar{K}^0} &= 0. \end{aligned} \quad (16)$$

When the effective energy of K^- mesons (ω_{K^-}) equals its chemical potential (μ_{K^-}) which, in turn, is μ_e , K^- condensation sets in. Similarly, \bar{K}^0 condensation occurs when $\omega_{\bar{K}^0} = \mu_{\bar{K}^0} = 0$.

The pure antikaon condensed phase is composed of baryons of the baryon octet, leptons and antikaons. In this phase, baryons are embedded in Bose–Einstein condensates. The (anti)kaon–baryon interaction is treated on the same footing as baryon–baryon interaction. The Lagrangian density for (anti)kaons in the minimal coupling scheme [3,9]

$$\mathcal{L}_K = D_\mu^* \bar{K} D^\mu K - m_K^{*2} \bar{K} K, \quad (17)$$

where the covariant derivative

$$D_\mu = \partial_\mu + ig_{\omega K} \omega_\mu + ig_{\phi K} \phi_\mu + ig_{\rho K} \boldsymbol{\tau}_K \cdot \boldsymbol{\rho}_\mu. \quad (18)$$

The isospin doublet for kaons is denoted by $K \equiv (K^+, K^0)$ and that for antikaons is denoted by $\bar{K} \equiv (K^-, \bar{K}^0)$. The effective mass of (anti)kaons in this minimal coupling scheme is given by

$$m_K^* = m_K - g_{\sigma K} \sigma - g_{\sigma^* K} \sigma^*. \quad (19)$$

The equation of motion for kaons is given by

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$$(D_\mu D^\mu + m_K^{*2}) K = 0. \quad (20)$$

The s -wave ($\mathbf{p} = 0$) dispersion relation for antikaons is

$$\omega_{\bar{K}} = m_{\bar{K}}^* - g_{\omega K} \omega_0 - g_{\phi K} \phi_0 + I_{3\bar{K}} g_{\rho K} \rho_{03}, \quad (21)$$

where the isospin projection $I_{3K^-} = -1/2$ for K^- and $I_{3\bar{K}^0} = 1/2$ for \bar{K}^0 .

The conserved current associated with (anti)kaons is derived by using

$$J_\mu^K = \left(\bar{K} \frac{\partial \mathcal{L}}{\partial \mu \bar{K}} - \frac{\partial \mathcal{L}}{\partial \mu K} K \right). \quad (22)$$

The density of antikaons is given by

$$\begin{aligned} n_{\bar{K}} &= -J_0^K \\ &= 2(\omega_{\bar{K}} + g_{\omega K} \omega_0 + g_{\phi K} \phi_0 - I_{3\bar{K}} g_{\rho K} \rho_{03}) K \bar{K} \\ &= 2m_{\bar{K}}^* K \bar{K}. \end{aligned} \quad (23)$$

In the mean-field approximation, the meson field equations in the presence of antikaons are

$$m_\sigma^2 \sigma = -\frac{\partial U}{\partial \sigma} + \sum_B g_{\sigma B} n_B^s + g_{\sigma K} \sum_{\bar{K}} n_{\bar{K}}, \quad (24)$$

$$m_{\sigma^*}^2 \sigma^* = \sum_B g_{\sigma^* B} n_B^s + g_{\sigma^* K} \sum_{\bar{K}} n_{\bar{K}}, \quad (25)$$

$$m_\omega^2 \omega_0 = \sum_B g_{\omega B} n_B - g_{\omega K} \sum_{\bar{K}} n_{\bar{K}}, \quad (26)$$

$$m_\phi^2 \phi_0 = \sum_B g_{\phi B} n_B - g_{\phi K} \sum_{\bar{K}} n_{\bar{K}}, \quad (27)$$

$$m_\rho^2 \rho_{03} = \sum_B g_{\rho B} I_{3B} n_B + g_{\rho K} \sum_{\bar{K}} I_{3\bar{K}} n_{\bar{K}}. \quad (28)$$

The total charge density in the antikaon condensed phase is

$$Q^{\bar{K}} = \sum_B q_B n_B^{\bar{K}} - n_{\bar{K}} - n_e - n_\mu = 0, \quad (29)$$

where $n_B^{\bar{K}}$ is the baryon number density in antikaon condensed phase.

The total energy density in the \bar{K} condensed phase consists of three terms, $\varepsilon = \varepsilon_B + \varepsilon_l + \varepsilon_{\bar{K}}$,

$$\begin{aligned} \varepsilon &= \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} \\ &\quad + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\phi^2 \phi_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\ &\quad + \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} (k^2 + m_B^{*2})^{1/2} k^2 dk \\ &\quad + \sum_l \frac{1}{\pi^2} \int_0^{k_{Fl}} (k^2 + m_l^2)^{1/2} k^2 dk + \frac{\mu_{\nu_e}^4}{8\pi^2} \\ &\quad + m_{\bar{K}}^* (n_{K^-} + n_{\bar{K}^0}). \end{aligned} \quad (30)$$

And the pressure is given by

$$\begin{aligned}
 P = & -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
 & -\frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
 & + \frac{1}{3}\sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} \frac{k^4 dk}{(k^2 + m_B^{*2})^{1/2}} \\
 & + \frac{1}{3}\sum_l \frac{1}{\pi^2} \int_0^{K_{Fl}} \frac{k^4 dk}{(k^2 + m_l^2)^{1/2}} + \frac{\mu_{\nu_e}^4}{24\pi^2}.
 \end{aligned} \tag{31}$$

The mixed phase of pure hadronic and antikaon condensed matter is described by Gibbs phase equilibrium rules. Also, neutron star matter has two conserved charges denoted by μ_n and μ_e . Therefore, the conditions of global charge neutrality and baryon number conservation are imposed through the relations [10]

$$(1 - \chi)Q^h + \chi Q^{\bar{K}} = 0, \tag{32}$$

$$n_B = (1 - \chi)n_B^h + \chi n_B^{\bar{K}}, \tag{33}$$

where χ is the volume fraction of K^- condensed phase in the mixed phase. The total energy density in the mixed phase is

$$\varepsilon = (1 - \chi)\varepsilon^h + \chi\varepsilon^{\bar{K}}. \tag{34}$$

3. Composition and EoS of dense matter

In this model calculation, three distinct sets of coupling constants, meson–nucleon, meson–hyperon and meson–kaon, are required. Meson–nucleon coupling constants are generated by reproducing normal nuclear matter properties – saturation density ($n_0 = 0.153 \text{ fm}^{-3}$), binding energy, incompressibility and symmetry energy. Here we exploit GM1 set for meson–nucleon coupling constants [3]. It is to be noted that nucleons do not couple with σ^* and ϕ .

Meson–hyperon coupling constants are determined from hypernuclei data and quark model. The vector coupling constants for hyperons are obtained from $SU(6)$ symmetry as

$$\begin{aligned}
 \frac{1}{2}g_{\omega\Lambda} &= \frac{1}{2}g_{\omega\Sigma} = g_{\omega\Xi} = \frac{1}{3}g_{\omega N}, \\
 \frac{1}{2}g_{\rho\Sigma} &= g_{\rho\Xi} = g_{\rho N}; \quad g_{\rho\Lambda} = 0, \\
 2g_{\phi\Lambda} &= 2g_{\phi\Sigma} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3}g_{\phi N}.
 \end{aligned} \tag{35}$$

The scalar meson (σ) coupling to hyperons

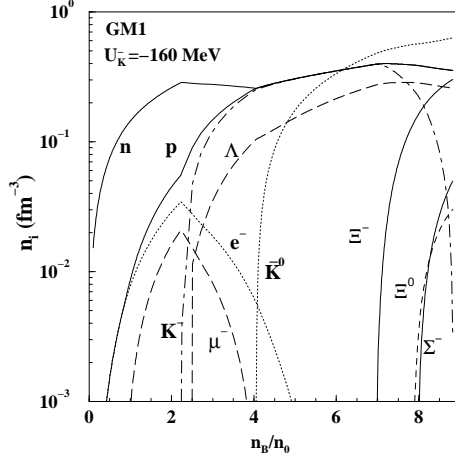


Figure 1. The number densities n_i of various particles in β -equilibrated hyperon matter including both K^- and \bar{K}^0 condensates for GM1 set and antikaon optical potential depth at normal nuclear matter density $U_{\bar{K}} = -160$ MeV are plotted with baryon number density.

$$U_Y^N(n_0) = -g_{\sigma Y}\sigma + g_{\omega Y}\omega_0, \quad (36)$$

is determined from the knowledge of the potential depth of a hyperon in symmetric nuclear matter using hypernuclei data. For Λ hyperon, this potential depth is -30 MeV and it is -18 MeV for Ξ hyperon. However, Σ hyperon potential depth is $+30$ MeV.

The σ^* - Y coupling constants are calculated by fitting them to a well depth for a hyperon in hyperon (Y) matter at normal nuclear matter density, $U_Y^{(Y')}(n_0)$ [11],

$$U_{\Xi}^{(\Xi)}(n_0) = U_{\Lambda}^{(\Xi)}(n_0) = 2U_{\Xi}^{(\Lambda)}(n_0) = 2U_{\Lambda}^{(\Lambda)}(n_0) = -40 \text{ MeV}. \quad (37)$$

The scalar meson-kaon coupling constant is estimated from the real part of K^- potential depth in normal nuclear matter density

$$U_{\bar{K}}(n_0) = -g_{\sigma K}\sigma - g_{\omega K}\omega_0, \quad (38)$$

and the vector coupling constants from the quark model and isospin counting rule,

$$g_{\omega K} = \frac{1}{3}g_{\omega N} \quad \text{and} \quad g_{\rho K} = g_{\rho N}. \quad (39)$$

A strongly attractive antikaon potential depth of $U_{\bar{K}} = -160$ MeV has been used for this calculation. The strange meson σ^* and ϕ couplings with (anti)kaons are determined from the decay of $f_0(975)$ and $SU(6)$ symmetry relation respectively [4].

The abundances of various species in β -equilibrated matter containing baryons, electrons, muons and K^- and \bar{K}^0 mesons are shown in figure 1. We discuss the role of K^- and \bar{K}^0 condensation on the composition of β -equilibrated and charge neutral matter. In the pure hadronic phase where local charge neutrality is imposed,

abundances of nucleons, electrons and muons increase with density. Here, charge neutrality is maintained among protons, electrons and muons. With the onset of K^- condensation, the mixed phase begins at $2.23n_0$. We find that Λ hyperon is the first strange baryon to appear in the mixed phase at $2.51n_0$. The total baryon density in the mixed phase is the sum of two contributions from hadronic and antikaon condensed phases weighted with appropriated volume fractions. As soon as K^- condensate is formed, it rapidly grows with density and replaces electrons and muons. Being bosons, K^- mesons in the lowest energy state are energetically more favorable to maintain charge neutrality than any other negatively charged particles. Consequently, the proton density becomes equal to the density of K^- condensate. Also, the density of Λ hyperon increases with baryon density in the mixed phase. On the other hand, the neutron density decreases in the mixed phase. The reason behind it may be the creation of more protons in the presence of K^- condensate and also the growth of Λ hyperons at the expense of neutrons. The mixed phase terminates at $4.0n_0$.

Immediately after the termination of the mixed phase, a second-order \bar{K}^0 condensation sets in $\sim 4.1n_0$. With the appearance of \bar{K}^0 condensate, neutron and proton abundances become equal. The density of \bar{K}^0 condensate increases with baryon density uninterruptedly and even becomes larger than the density of K^- condensate. As soon as negatively charged hyperons Ξ^- and Σ^- appear at higher densities, the density of K^- condensate is observed to fall drastically. This is quite expected because it is energetically favorable for particles carrying conserved baryon numbers to achieve charge neutrality in the system. Leptons or mesons are no longer required for this sole purpose. Moreover, lepton number or meson number is not conserved in the star. The system is dominated by \bar{K}^0 condensate in the high density regime.

The equation of state, pressure (P) vs. energy density (ε) for β -equilibrated and charge neutral matter with and without antikaon condensates are exhibited in figure 2. The lower solid line indicates the overall EoS with hyperons and antikaon condensates whereas the EoS with hyperons and no condensate is exhibited by the upper solid line. Two kinks in the lower solid curve mark the beginning and end of the mixed phase where pure hadronic and K^- condensed phase are in thermodynamic equilibrium as dictated by Gibbs phase rules and global conservation laws. These kinks lead to discontinuity in the velocity of sound. The overall EoS with \bar{K} condensates is softer compared with the EoS without condensates. Consequently, the softer EoS gives rise to smaller maximum mass stars. The maximum mass of the static neutron star sequence calculated with the EoS including \bar{K} condensates is $1.57M_\odot$ whereas that of hyperon EoS without \bar{K} condensates is $1.79M_\odot$.

4. Summary

In this article, we discuss the formation of hyperons and Bose–Einstein condensates of \bar{K} mesons in cold and dense matter relevant for neutron stars within relativistic models. Here we consider a first-order K^- condensation followed by a second-order \bar{K}^0 condensation. The populations of neutrons, protons and hyperons are strongly modified in the presence of both K^- and \bar{K}^0 condensation. Also, antikaon

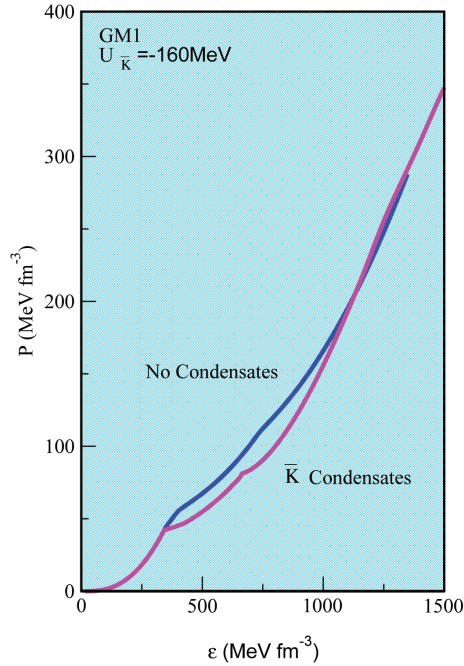


Figure 2. The equation of state, pressure (P) vs. energy density (ε), for β -equilibrated hyperon matter with and without \bar{K} condensates.

condensates make the EoS softer compared to the EoS without condensates and give rise to a smaller maximum mass neutron star.

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