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Resonances in η-light nucleus systems

K P KHEMCHANDANI[1], N G KELKAR[2], M NOWAKOWSKI[2] and B K JAIN[1]
[1]Department of Physics, University of Mumbai, Mumbai 400 098, India
[2]Departamento de Fisica, Universidad de los Andes, Bogota, Colombia
E-mail: kanchanp@magnum.barc.ernet.in

Abstract. We locate resonances in η-light nucleus elastic scattering using the time delay method. We solve few-body equations within the finite rank approximation in order to calculate the t-matrices and hence the time delay for the η-^{3}He and η-^{4}He systems. We find a resonance very close to the threshold in η-^{3}He elastic scattering, at about 0.5 MeV above threshold with a width of ∼2 MeV. The calculations also hint at the presence of sub-threshold states in both the cases.

Keywords. η-mesic nuclei; few-body equations; time delay.

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1. Introduction

The motivation attached to the η-physics and non-availability of the η-beams has given rise to a continuously growing interest in the η-mesic physics. The understanding of some of the fundamental problems like the charge symmetry breaking, the strangeness content of a nucleon etc. has been linked to the η-physics. There are also objectives relevant to the intermediate energy physics; for example, study of the N-N^* interaction, the in-medium properties of the N^* etc. In spite of the efforts that have been put into the exploration of the η-mesic physics, the strength of the η-N interaction and consequently the choice of the system (nucleus) for which the η-mesic nucleus [1] could exist, remain as unsettled issues.

We planned to study the resonances in η-light nuclei systems since the few-body systems can be treated rigorously and a good knowledge of such systems can be used to treat the η-heavier nuclei systems. There also exist strong speculations, made on the basis of theoretical studies, for the existence of η-light nuclei systems [2,3]. Recently, experimental efforts have also been carried out to look for such systems [4].

We solve few-body equations to calculate the t-matrices and then use them to calculate the time delay for the η-light nuclei elastic scattering. We find a resonance in the η-^{3}He system in addition to a very strong rise in time delay right at the threshold, hinting at the possibility of the presence of sub-threshold states. We do not find any resonance in the η-^{4}He scattering but a possibility of the existence of a sub-threshold state.

2. The η-nucleus T-matrix

We have solved the few-body equations within the finite rank approximation (FRA) to calculate the t-matrices for η-nucleus elastic scattering near threshold. The calculations made within the FRA neglect the excited states of a nucleus and allow the nucleus to remain only in its ground state during the scattering, which is justified since the calculations have been made near threshold and hence the energies involved are very low. The transition matrix within the FRA can be written as [5]

$$T(\vec{k}'\,\vec{k}\,;z) = \langle \vec{k}'\,;\,\psi_0|T^0(z)|\vec{k}\,;\,\psi_0\,\rangle + \varepsilon\int\frac{\mathrm{d}\vec{k}''}{(2\pi)^3}\frac{\langle \vec{k}'\,;\,\psi_0|T^0(z)|\vec{k}''\,;\,\psi_0\,\rangle}{(z-\frac{k''^2}{2\mu})(z-\varepsilon-\frac{k''^2}{2\mu})}T(\vec{k}'',\,\vec{k}\,;\,z), \tag{1}$$

where $z = E - |\varepsilon| + i0$. E is the energy associated with η-nucleus relative motion, ε is the binding energy of the nucleus and μ is the reduced mass of the η-nucleus system. The operator T^0 describes the scattering of an η-meson from nucleons fixed in their space positions within the nucleus. The matrix elements for T^0 are given as

$$\langle \vec{k}'\,;\,\psi_0|T^0(z)|\vec{k}\,;\,\psi_0\,\rangle = \int \mathrm{d}\vec{r}\,|\psi_0(\vec{r}\,)|^2\,T^0\,(\vec{k}',\,\vec{k}\,;\vec{r}\,;z), \tag{2}$$

where

$$T^0\,(\vec{k}',\,\vec{k}\,;\vec{r}\,;z) = \sum_{i=1}^{A} T_i^0(\vec{k}',\,\vec{k}\,;\vec{r_i}\,;z). \tag{3}$$

T_i^0 is the t-matrix for the scattering of the η-meson from the ith nucleon in the nucleus, with the re-scattering from the other (A-1) nucleons included. It is given as

$$T_i^0\,(\vec{k}',\,\vec{k}\,;\vec{r_i}\,;z) = t_i(\vec{k}',\,\vec{k}\,;\vec{r_i}\,;z) + \int\frac{\mathrm{d}\vec{k}''}{(2\pi)^3}\frac{t_i(\vec{k}',\,\vec{k}''\,;\vec{r_i}\,;z)}{z-\frac{k''^2}{2\mu}}\sum_{j\neq i}T_j^0(\vec{k}'',\,\vec{k}\,;\vec{r_j}\,;z). \tag{4}$$

The t-matrix for elementary η-nucleon scattering, t_i, is written in terms of the two-body ηN matrix $t_{\eta N}$ as

$$t_i(\vec{k}',\,\vec{k}\,;\vec{r_i}\,;z) = t_{\eta N}(\vec{k}',\,\vec{k}\,;z)\,\exp[\,i(\,\vec{k}-\vec{k}'\,)\cdot\,\vec{r_i}\,]. \tag{5}$$

The wave function ψ_0, required in the calculation of eq. (2) is taken to be of the Gaussian form for the ^{3}He and ^{4}He nuclei.

Since there exists a lot of uncertainty in the knowledge of the η-nucleon interaction, we use two different prescriptions [6,7] of the η-N t-matrix, $t_{\eta N\,\rightarrow\,\eta N}$, leading to different values of the η-N scattering length, viz. (0.88, 0.41) [6] and (0.28, 0.19) [7]. These coupled channel, off-shell t-matrices reproduce the $\pi N\,\rightarrow\,\eta N$ data well.

3. Time delay

Having obtained the t-matrices for the η-nucleus elastic scattering, we calculate the 'time delay' for each case. The concept of time delay has an interesting, intuitive physical picture of a delay caused in a scattering process due to an attractive potential, causing formation of an intermediate metastable state. The presence of a resonance shows up as a positive bump in the time delay plots with its center at the mass of the resonance. The full-width at half-maximum of the peak corresponds to the width of the resonance directly (see for example [8,9]) giving a clear picture of a resonance. The time delay originally quantified by Wigner *et al* [10] in terms of the energy derivative of the phase shift,

$$\Delta t = 2\hbar \frac{\mathrm{d}\delta}{\mathrm{d}E}, \tag{6}$$

obviously means that any significant change in phase shift, as a function of energy, will be seen clearly in the time delay plots. It should be noted that a decrease in the phase shift, with respect to energy, would give rise to a negative time delay. A detailed interpretation and discussion of the circumstances giving rise to such an observation can be found in [11].

Though the above definition of time delay as a derivative of the phase shift was derived in Wigner's work within a simple wave packet picture, it nicely emerges from the definition of the delay time matrix of Eisenbud [10] too. An element of this matrix, Δt_{ij}, which is the time delay in the emergence of a particle in the jth channel after being injected in the ith channel is given by

$$\Delta t_{ij} = \Re e\left[-i\hbar (S_{ij})^{-1} \frac{\mathrm{d}S_{ij}}{\mathrm{d}E}\right], \tag{7}$$

where S_{ij} is an element of the corresponding S-matrix. In an eigenphase formulation of the S-matrix, $S = \eta \mathrm{e}^{2i\delta}$ (with η being the inelasticity factor), one can easily see that the time delay as defined above is given by the energy derivative of the phase shift. The time delay matrix was discussed further in terms of a lifetime matrix by Smith [12]. Using the definition of the t-matrix in terms of the S-matrix:

$$S = 1 + 2iT, \tag{8}$$

where the complex t-matrix, $T\ (= T^R + iT^I)$, contains the resonant as well as the non-resonant components, the average time delay can be written in terms of the t-matrix as

$$\langle \Delta t_{ij} \rangle = 2\hbar \left[\frac{\mathrm{d}T^R_{ij}}{\mathrm{d}E} + 2T^R_{ij}\frac{\mathrm{d}T^I_{ij}}{\mathrm{d}E} - 2T^I_{ij}\frac{\mathrm{d}T^R_{ij}}{\mathrm{d}E}\right]. \tag{9}$$

Hence, the time delay in an elastic scattering can be found from

$$S^*_{ii} S_{ii} \Delta t_{ii} = 2\hbar \left[\frac{\mathrm{d}T^R_{ii}}{\mathrm{d}E} + 2T^R_{ii}\frac{\mathrm{d}T^I_{ii}}{\mathrm{d}E} - 2T^I_{ii}\frac{\mathrm{d}T^R_{ii}}{\mathrm{d}E}\right]. \tag{10}$$

In the present work, we shall use the above definition to evaluate the time delay in η-nucleus elastic scattering.

Before discussing the results of our work, it is worth mentioning here that the concept of time delay, which has been discussed in most of the standard textbooks [13] on quantum mechanics and scattering theory as a necessary condition for the existence of a resonance, had not been practically used in the past for the characterization of hadron resonances. It is only recently that we used this concept to study resonances in hadron–hadron scattering and we could successfully reproduce all the N and Δ resonances [8], the meson resonances like the ρ, the σ, the K^* [9] and, the much talked about exotic baryon resonances also [14]. This and the experience from the study of the η-^{3}He final state interaction in the $p + d \to \eta + {}^3$He reaction [15] encouraged us to locate resonances in the η-light nuclei systems.

4. Results and discussion

We present here the results of the time delay calculations made using eq. (10), where we have used the t-matrices obtained by solving the few-body equations [1–5]. Figure 1 shows the time delay plot for η-^{3}He elastic scattering as a function of excess energy. We first discuss the calculation made using the η-N t-matrix from [6], i.e., with the one giving rise to a stronger η-N scattering length. It can be seen that there is a broad bump around 0.5 MeV excitation energy with a width of about 2 MeV. There is also a sharp rise in time delay at the threshold hinting towards the presence of a sub-threshold state. The calculation made with the t-matrix obtained from [7] also shows a sharp rise in time delay near threshold. However, there is no resonance found in this case.

The time delay plots for the η-^{4}He elastic scattering have been shown in figure 2. We do not find the possibility of a resonance in this case. This calculation, in contrast to the η-^{3}He calculation, shows a large sensitivity to the different inputs in terms of the different prescriptions of the η-N t-matrices. The η-N t-matrix leading to a weaker η-N scattering length [7] does show the possibility of the presence of a sub-threshold state, whereas the one resulting into a stronger scattering length [6] shows a large negative time delay near threshold.

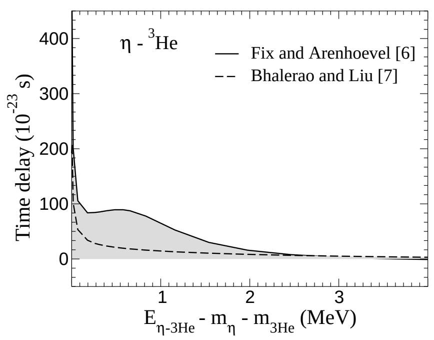

Figure 1. The time delay in $\eta\ {}^3$He $\to \eta\ {}^3$He elastic scattering.

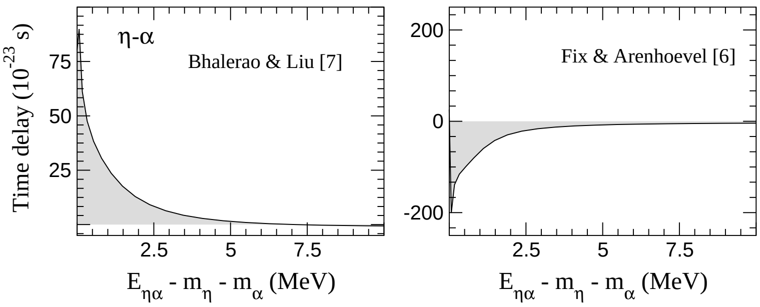

Figure 2. The time delay in $\eta\ ^4$He $\to \eta\ ^4$He elastic scattering.

The cause of the negative time delay could be a repulsive interaction. In a scattering process, a repulsive potential can cause the particle to accelerate through the interaction region, causing a 'time advancement' in contrast to the time delay caused due to an attractive potential.

To summarize this discussion, we can say that a search for possible resonances in the η-^{3}He and η-^{4}He systems has been made, on the basis of the time delay calculated from t-matrices for these systems, obtained by solving the few-body equations within the finite rank approximation. We find a resonance in the η-^{3}He scattering. These calculations also hint at the possibility of the presence of sub-threshold states in the η-light nuclei systems.

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References

[1] Q Haider and L Liu, *Phys. Lett.* **B172**, 257 (1986); **174**, 465(E) (1986); *Phys. Rev.* **C66**, 045208 (2002)

[2] T Ueda, *Phys. Rev. Lett.* **66**, 297 (1991)
A Fix and H Arenhövel, *Euro. Phys. J.* **A9**, 119 (2000)

[3] S A Rakityansky *et al*, *Phys. Rev.* **C53**, 2043 (1996)
N V Shevchenko *et al*, *Euro. Phys. J.* **A9**, 143 (2000) and the references therein

[4] M Pfeiffer *et al*, *Phys. Rev. Lett.* **92**, 252001 (2004)

[5] V B Belyaev, Lectures on the theory of few body systems, in *Springer series in nuclear and particle physics*, 1990

[6] A Fix and H Arenhövel, *Nucl. Phys.* **A697**, 277 (2002)

[7] R S Bhalerao and L C Liu, *Phys. Rev. Lett.* **54**, 865 (1985)

[8] N G Kelkar, M Nowakowski, K P Khemchandani and S R Jain, *Nucl. Phys.* **A730**, 121 (2004), hep-ph/0208197

[9] N G Kelkar, M Nowakowski and K P Khemchandani, *Nucl. Phys.* **A724**, 357 (2003), hep-ph/0307184

[10] E P Wigner, *Phys. Rev.* **98**, 145 (1955)
D Bohm, *Quantum theory* (1951)
L Eisenbud, Dissertation, Princeton, June 1948 (unpublished)
[11] N G Kelkar, *J. Phys.* **G29**, L1 (2003), hep-ph/0205188
[12] F T Smith, *Phys. Rev.* **118**, 349 (1960)
[13] C J Joachain, *Quantum collision theory* (North Holland, 1975)
J R Taylor, *Scattering theory: The quantum theory on non-relativistic collisions* (John Wiley and Sons, Inc., 1972) and many more as mentioned in refs [8,9,14]
[14] N G Kelkar, M Nowakowski and K P Khemchandani, *J. Phys.* **G29**, 1001 (2003), hep-ph/0307134; *Mod. Phys. Lett.* **A19**, 2001 (2004), nucl-th/0405008
[15] K P Khemchandani, N G Kelkar and B K Jain, *Nucl. Phys.* **A708**, 312 (2002)