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Weak pion production from nuclei

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Abstract. The charged current pion production induced by neutrinos in ^{12}C, ^{16}O and ^{56}Fe nuclei has been studied. The calculations have been done for the coherent as well as the incoherent processes assuming Δ dominance and takes into account the effect of Pauli blocking, Fermi motion and the renormalization of Δ in the nuclear medium. The pion absorption effects have also been taken into account.

Keywords. Neutrino nucleus reactions; resonance production; nuclear effects; local density approximation; pion absorption.

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1. Introduction

The pion production processes from nucleons and nuclei at intermediate energies are important tools to study the hadronic structure. The dynamic models of the hadronic structure are used to calculate the various nucleon and transition form factors which are tested by using the experimental data on photo, electro and weak pion production processes on nucleons. The weak pion production along with the electropion production from nucleon in the Δ-resonance region is used to determine the various electroweak $N-\Delta$ transition form factors. The neutrino-induced pion production experiments performed at CERN [1], ANL [2] and BNL [3] laboratories have been analyzed to obtain informations on these form factors. The early neutrino experiments performed at CERN [1] have low statistics and are done using heavy nuclear targets and their analysis have uncertainties related to the nuclear corrections. The later experiments performed at ANL [2] and BNL [3] are done using hydrogen and deuterium and are free from nuclear medium corrections. These experiments also have high statistics and provide reasonable estimates of the dominant form factors in $N-\Delta$ transitions. However, with the availability of the new neutrino beams in intermediate energies at K2K [4] and MiniBooNE [5], it is desirable that various $N^\star - N$ weak transition form factors are determined for low-lying nuclear resonances like $\Delta(1232)$, $N^\star(1440)$, $N^\star(1535)$, etc. There is a considerable activity in this field, specially, in the determination of electromagnetic transition form factors using the photo and electroproduction data from MAINZ,

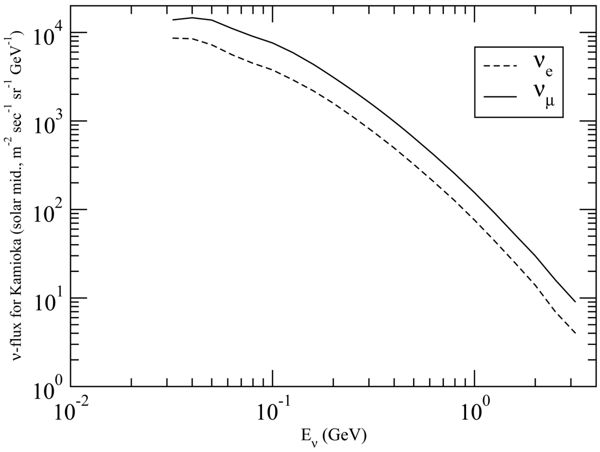

Figure 1. The atmospheric ν-flux for Kamioka site [7].

BONN and TJNAF laboratories [6]. It is desirable that such attempts be extended to the determination of weak transition form factors also.

Recently, the weak pion production processes have become very important in the analysis of the neutrino oscillation experiments with atmospheric neutrinos. The energy spectrum of atmospheric neutrino at Kamioka site [7], shown in figure 1 is such that the weak pion production contributes about 20% of the quasielastic lepton production and is a major source of uncertainty in the identification of electron and muon events. In particular, the neutral current π^0 production contributes to the background of $e^{\pm}$ production while $\pi^{\pm}$ production contributes to the background of $\mu^{\pm}$ production. This is because both the particles, i.e. π^0 and $e^{\pm}$ are identified through the detection of photons and $\pi^{\pm}$ and $\mu^{\pm}$ are identified through single track events in the detection of neutrino oscillation experiments. Moreover, the neutral current π^0 production plays a very important role in distinguishing between the two oscillation mechanisms $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_s$ [8].

The neutrino oscillation experiments are generally performed with detectors which use materials with nuclei like ^{12}C, ^{16}O, ^{56}Fe, etc. as targets. It is therefore desirable that nuclear medium effects be studied in the production of leptons and pions induced by the atmospheric as well as accelerator neutrinos used in these neutrino oscillation experiments.

There have been many attempts in the past to calculate the weak pion production [9], but very few attempts have been made to estimate the nuclear effects and their influence on the weak pion production in nuclei [10]. Recent experiments on neutrino oscillation experiments with atmospheric and accelerator neutrinos in the intermediate energy region have started a fresh interest in the study of weak pion production of pions from nuclei [11–13]. We report in this paper the results of a calculation of the weak pion production in nucleus assuming Δ dominance. The effect of Pauli blocking, Fermi motion and renormalization of weak Δ properties in the nuclear medium are taken into account in a local density approximation following the methods of ref. [13].

In §2, we describe the matrix elements for the production of Δ resonance from free nucleons using the information about the $N-\Delta$ transition form factors as determined from experiments. These matrix elements are then used to calculate the weak pion production in nuclei. We have described and discussed the incoherent weak production of pions in §3 and the coherent weak pion production in §4 with conclusion given in §5.

2. Weak pion production from nucleons

The weak pion production processes like

$$\nu_l(\bar{\nu}_l) + N \rightarrow l^-(l^+) + N + \pi^\alpha \tag{1}$$

and

$$\nu_l(\bar{\nu}_l) + N \rightarrow \nu_l(\bar{\nu}_l) + N + \pi^\beta, \tag{2}$$

where N can be a proton or a neutron and $\pi^{\alpha}(^{\beta})$ are the different possible charged states (π^+, π^- and π^0) of the pions produced and are determined by the lepton number and charge conservation in the charged current and neutral current reactions in eqs (1) and (2) respectively.

The pion production induced by charged current neutrino interaction is calculated in the standard model of electroweak interactions using the Lagrangian given by

$$L = \frac{G_{\mathrm{F}}}{\sqrt{2}} l_\mu(x) J^\mu(x)^W, \tag{3}$$

where $l_\mu(x) = \bar{\psi}(k')\gamma_\mu(1-\gamma_5)\psi(k)$ and $J^\mu(x)^W$ is the hadronic current given by $J^\mu(x)^W = \cos\theta_c(V^\mu(x) + A^\mu(x) + \text{h.c.})$ for the charged current reactions.

The matrix element of the hadronic current $J^{\mu W}$ is generally calculated using the nucleon and meson exchanges and the resonance excitation diagrams. However, it has been shown that in the intermediate energy region of about 1 GeV, the dominant contribution to the pion production from nucleons as well as from nuclei comes from the Δ resonance due to very strong P-wave pion nucleon coupling leading to Δ resonance. Furthermore, the angular distribution and the energy distribution of the pions is dominated by the Δ contribution while the other diagrams contribute to the tail region due to the interference of the nucleon and meson exchange diagrams with the Δ resonance diagram [12,14].

In this model the matrix element for the neutrino excitation of the Δ resonance through charged current reaction, i.e.

$$\nu_l(k) + p(p) \rightarrow l^-(k') + \Delta^{++}(p') \tag{4}$$

and

$$\nu_l(k) + n(p) \rightarrow l^-(k') + \Delta^{+}(p') \tag{5}$$

is written as

$$T = \frac{G_{\rm F}}{\sqrt{2}} \cos\theta_c l_\mu (V^\mu + A^\mu), \tag{6}$$

where l_μ is the leptonic current and V^μ and A^μ define the vector and axial vector part of the transition hadronic current between N and Δ states for the charged current interaction. The most general form of the matrix elements of hadronic currents between the p and Δ^{++} states, in eq. (4) are given by [2,9,13]:

$$\begin{aligned}\langle \Delta^{++}|V^\mu|p\rangle = \sqrt{3}\bar{\psi}_\alpha(p') \Bigg[\Bigg(& \frac{C_3^{\rm V}(q^2)}{M}(g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) \\ & + \frac{C_4^{\rm V}(q^2)}{M^2}(g^{\alpha\mu} q\cdot p' - q^\alpha p'^\mu) \\ & + \frac{C_5^{\rm V}(q^2)}{M^2}(g^{\alpha\mu} q\cdot p - q^\alpha p^\mu) + \frac{C_6^{\rm V}(q^2)}{M^2} q^\alpha q^\mu \Bigg) \gamma_5 \Bigg] u(p),\end{aligned} \tag{7}$$

$$\begin{aligned}\langle \Delta^{++}|A^\mu|p\rangle = \sqrt{3}\bar{\psi}_\alpha(p') \Bigg[\Bigg(& \frac{C_3^{\rm A}(q^2)}{M}(g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) \\ & + \frac{C_4^{\rm A}(q^2)}{M^2}(g^{\alpha\mu} q\cdot p' - q^\alpha p'^\mu) \\ & + C_5^{\rm A}(q^2) g^{\alpha\mu} + \frac{C_6^{\rm A}(q^2)}{M^2} q^\alpha q^\mu \Bigg) \Bigg] u(p).\end{aligned} \tag{8}$$

Here $\psi_\alpha(p')$ and $u(p)$ are the Rarita Schwinger and Dirac spinors for Δ and nucleon of momenta p' and p respectively, $q(= p' - p = k - k')$ is the momentum transfer and $C_i^{\rm V}$ $(i = 3$–$6)$ are vector and $C_i^{\rm A}$ $(i = 3$–$6)$ are axial vector transition form factors. The vector form factors $C_i^{\rm V}$ $(i = 3$–$6)$ are determined by using the conserved vector current (CVC) hypothesis which gives $C_6^{\rm V}(q^2) = 0$ and relates $C_i^{\rm V}$ $(i = 3,4,5)$ to the electromagnetic form factors which are determined from photoproduction and electroproduction of Δ's. Using the analysis of these experiments [12,15] we take for the vector form factors

$$C_5^{\rm V} = 0, \quad C_4^{\rm V} = -\frac{M}{M_\Delta} C_3^{\rm V}$$

and

$$C_3^{\rm V}(q^2) = \frac{2.05}{(1 - \frac{q^2}{M_{\rm V}^2})^2}, \quad M_{\rm V}^2 = 0.54\ {\rm GeV}^2. \tag{9}$$

The axial vector form factor $C_6^{\rm A}(q^2)$ is related to $C_5^{\rm A}(q^2)$ using PCAC and is given by

$$C_6^{\rm A}(q^2) = C_5^{\rm A}(q^2) \frac{M^2}{m_\pi^2 - q^2}. \tag{10}$$

The remaining axial vector form factor $C_{i=3,4,5}^{\rm A}(q^2)$ are taken from the experimental analysis of the neutrino experiments producing Δ's in proton and deuteron targets [2,3]. These form factors are not uniquely determined but the following parametrizations give a satisfactory fit to the data

$$C^{\mathrm{A}}_{i=3,4,5}(q^2) = C^{\mathrm{A}}_i(0)\left[1+\frac{a_i q^2}{b_i - q^2}\right]\left(1-\frac{q^2}{M_{\mathrm{A}}^2}\right)^{-2} \tag{11}$$

with $C^{\mathrm{A}}_3(0) = 0, C^{\mathrm{A}}_4(0) = -0.3, C^{\mathrm{A}}_5(0) = 1.2, a_4 = a_5 = -1.21,\ \ b_4 = b_5 = 2\ \mathrm{GeV}^2$, $M_{\mathrm{A}} = 1.28$ GeV. Using the hadronic current given in eqs (7) and (8), the energy spectrum of the outgoing leptons is given by

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E_{k'}\mathrm{d}\Omega_{k'}} = \frac{1}{8\pi^3}\frac{1}{MM'}\frac{k'}{E_\nu}\frac{\frac{\Gamma(W)}{2}}{(W-M')^2+\frac{\Gamma^2(W)}{4.}}L_{\mu\nu}J^{\mu\nu}, \tag{12}$$

where $W = \sqrt{(p+q)^2}$ and M' is the mass of Δ,

$$\begin{aligned} L_{\mu\nu} &= \bar{\Sigma}\Sigma {l_\mu}^\dagger l_\nu = L^S_{\mu\nu} + iL^{\mathrm{A}}_{\mu\nu} \\ &= k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k' + i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta, \\ J^{\mu\nu} &= \bar{\Sigma}\Sigma J^{\mu\dagger}J^\nu \end{aligned} \tag{13}$$

and is calculated with the use of spin $\frac{3}{2}$ projection operator $P^{\mu\nu}$ defined as

$$P^{\mu\nu} = \sum_{\mathrm{spins}} \psi^\mu \bar{\psi}^\nu$$

and is given by

$$P^{\mu\nu} = -\frac{\not{p}' + M'}{2M'}\left(g^{\mu\nu} - \frac{2}{3}\frac{p'^\mu p'^\nu}{M'^2} + \frac{1}{3}\frac{p'^\mu\gamma^\nu - p'^\nu\gamma_\mu}{M'} - \frac{1}{3}\gamma^\mu\gamma^\nu\right). \tag{14}$$

In eq. (12), the decay width Γ is taken to be an energy-dependent P-wave decay width given by

$$\Gamma(W) = \frac{1}{6\pi}\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2\frac{M}{W}|\mathbf{q}_{cm}|^3\Theta(W-M-m_\pi) \tag{15}$$

where

$$|\mathbf{q}_{cm}| = \frac{\sqrt{(W^2-m_\pi^2-M^2)^2-4m_\pi^2M^2}}{2W}$$

and M is the mass of nucleon. The step function Θ denotes the fact that the width is zero for the invariant masses below the $N\pi$ threshold. $|\mathbf{q}_{cm}|$ is the pion momentum in the rest frame of the resonance.

3. Weak pion production in nuclei

The reaction (given in eqs (4) and (5)) taking place in the nucleus produces Δ^{++} or Δ^+ which give rise to pions as decay product through $\Delta \to N\pi$ channel. In the incoherent pion production process only these pions are observed and no observation is made on other hadrons. However, in the nucleus there is a possibility, that the Δ produced in the reaction decays producing a pion such that the nucleus stays in the ground state. This leads to the process of coherent production of pions which are characterized by the forward production of pions in the direction of the lepton momentum transfer as the recoil of the nucleus is very small and negligible.

In the following we discuss separately the incoherent and coherent production of pions in the nuclei.

3.1 *Incoherent weak production of pions*

When the reaction given by eqs (4) or (5) takes place in the nucleus, the neutrino interacts with the nucleon moving inside the nucleus of density $\rho(r)$ with its corresponding momentum $\vec{p}$ constrained to be below its Fermi momentum $k_{\mathrm{F}_{n,p}}(r) = \left[3\pi^2\rho_{n,p}(r)\right]^{1/3}$, $\rho_n(r)$ and $\rho_p(r)$ are the neutron and proton nuclear densities. The produced Δ's have no such constraints on their momentum. These Δ's decay through various decay channels in the medium. The production of Δ in the nucleus, leads to pion as it decays into nucleon and pion through $\Delta \to N\pi$ channel. However, these nucleons have to be above the Fermi momentum p_{F} of the nucleon in the nucleus thus inhibiting the decay as compared to the free decay of the Δ described by Γ in eq. (12). Therefore, in the nuclear medium the decay width Γ in eq. (15) is to be modified in the nuclear medium.

The modification of Γ due to Pauli blocking of nucleus has been studied in detail in electromagnetic and strong interactions [16]. The modified Δ decay width, i.e. $\tilde{\Gamma}$ is written as [16]:

$$\tilde{\Gamma} = \frac{1}{6\pi}\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2 \frac{M|\mathbf{q}_{cm}|^3}{W} F(k_{\mathrm{F}}, E_\Delta, k_\Delta)\Theta(W - M - m_\pi), \tag{16}$$

where $F(k_{\mathrm{F}}, E_\Delta, k_\Delta)$ is the Pauli correction factor given by

$$F(k_{\mathrm{F}}, E_\Delta, k_\Delta) = \frac{k_\Delta|\mathbf{q}_{cm}| + E_\Delta E'_{p_{cm}} - E_{\mathrm{F}} W}{2k_\Delta|\mathbf{q'}_{cm}|} \tag{17}$$

where k_{F} is the Fermi momentum, $E_{\mathrm{F}} = \sqrt{M^2 + k_{\mathrm{F}}^2}$, k_Δ is the Δ momentum and $E_\Delta = \sqrt{W + k_\Delta^2}$.

Moreover, in the nuclear medium there are additional decay channels now open due to two-body and three-body absorption processes like $\Delta N \to NN$ and $\Delta NN \to NNN$ through which Δ's disappear in the nuclear medium without producing a pion while a two-body Δ absorption process like $\Delta N \to \pi NN$ gives rise to some more pions. These nuclear medium effects on Δ propagation are included by using a Δ propagator in which the Δ propagator is written in terms of Δ self-energy Σ_Δ. This is done by using a modified mass and width of Δ in the nuclear medium, i.e. $M_\Delta \to M_\Delta + \mathrm{Re}\,\Sigma_\Delta$ and $\tilde{\Gamma} \to \tilde{\Gamma} - \mathrm{Im}\,\Sigma_\Delta$. There are many calculations of Δ self-energy Σ_Δ in the nuclear medium [16–19] and we use the results of [16], where the density dependence of real and imaginary parts of Σ_Δ are parametrized in the following form:

Table 1. Coefficients of eqs (18) and (19) for an analytical interpolation of $\mathrm{Im}\,\Sigma_\Delta$.

	C_Q (MeV)	C_{A2} (MeV)	C_{A3} (MeV)	α	β
a	−5.19	1.06	−13.46	0.382	−0.038
b	15.35	−6.64	46.17	−1.322	0.204
c	2.06	22.66	−20.34	1.466	0.613

$$\mathrm{Re}\,\Sigma_\Delta = 40\frac{\rho}{\rho_0}\ \mathrm{MeV}$$

and

$$-\mathrm{Im}\,\Sigma_\Delta = C_Q\left(\frac{\rho}{\rho_0}\right)^{\alpha} + C_{A2}\left(\frac{\rho}{\rho_0}\right)^{\beta} + C_{A3}\left(\frac{\rho}{\rho_0}\right)^{\gamma}. \tag{18}$$

In eq. (18), the term with C_Q accounts for the $\Delta N \to \pi NN$ process, the term with C_{A2} for two-body absorption process $\Delta N \to NN$ and the term with C_{A3} for three-body absorption process $\Delta NN \to NNN$. The coefficients C_Q, C_{A2}, C_{A3}, α, β and γ ($\gamma = 2\beta$) are parametrized in the range $80 < T_\pi < 320$ MeV (where T_π is the pion kinetic energy) as [16]

$$C_i(T_\pi) = ax^2 + bx + c, \quad \text{for} \quad x = \frac{T_\pi}{m_\pi}. \tag{19}$$

The values of the coefficients a, b and c are given in table 1 taken from ref. [16].

With these modifications, which incorporate the various nuclear medium effects on Δ propagation, the cross-section for pion production on proton target is

$$\sigma = \int\int \frac{\mathrm{d}\mathbf{r}}{8\pi^3}\frac{\mathrm{d}\mathbf{k}'}{E_\nu E_l}\frac{1}{MM'}\frac{\frac{\tilde{\Gamma}}{2} - \mathrm{Im}\,\Sigma_\Delta}{(W - M' - \mathrm{Re}\,\Sigma_\Delta)^2 + (\frac{\tilde{\Gamma}}{2.} - \mathrm{Im}\,\Sigma_\Delta)^2} \times \rho_p(\mathbf{r}) L_{\mu\nu} J^{\mu\nu}. \tag{20}$$

For pion production on neutron target, $\rho_p(\mathbf{r})$ in the above expression is replaced by $\frac{1}{3}\rho_n(\mathbf{r})$, where the factor $\frac{1}{3}$ with ρ_n comes due to suppression of π^+ production from neutron target as compared to the π^+ production from the proton target.

Therefore, the cross-section for π^+ production in the neutrino interaction with the nucleus is given by

$$\sigma = \int\int \frac{\mathrm{d}\mathbf{r}}{8\pi^3}\frac{\mathrm{d}\mathbf{k}'}{E_\nu E_l}\frac{1}{MM'}\frac{\frac{\tilde{\Gamma}}{2} - \mathrm{Im}\,\Sigma_\Delta}{(W - M' - \mathrm{Re}\,\Sigma_\Delta)^2 + (\frac{\tilde{\Gamma}}{2.} - \mathrm{Im}\,\Sigma_\Delta)^2} \times \left[\rho_p(\mathbf{r}) + \frac{1}{3}\rho_n(\mathbf{r})\right] L_{\mu\nu} J^{\mu\nu}. \tag{21}$$

In case of antineutrino reactions the role of $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ are interchanged and $\rho_p + \frac{1}{3}\rho_n$ in the above expression is replaced by $\rho_n + \frac{1}{3}\rho_p$ and $L^{\mathrm{A}}_{\mu\nu}$ is replaced by $-L^{\mathrm{A}}_{\mu\nu}$ in eq. (13).

3.2 *Results*

For our numerical calculation, to evaluate lepton momentum and angular distribution as well as the total scattering cross-section we use $\rho_n(r) = \frac{A-Z}{A}\rho(r)$ and $\rho_p(r) = \frac{Z}{A}\rho(r)$ in eq. (21), where $\rho(r)$ is the nuclear density which we take to be a three-parameter Fermi (3pF) for ^{12}C and ^{16}O nuclei given by

$$\rho(r) = \frac{\rho_0(1 + w\frac{r^2}{\alpha^2})}{(1 + \exp((r-\alpha)/a))}, \tag{22}$$

where $\alpha = 2.355$ fm, $a = 0.5224$ fm and $w = -0.149$ for ^{12}C nucleus and $\alpha = 2.608$ fm, $a = 0.513$ fm and $w = -0.051$ for ^{16}O nucleus and a three-parameter Gaussian (3pG) density for ^{56}Fe nucleus given by

$$\rho(r) = \frac{\rho_0(1 + w\frac{r^2}{\alpha^2})}{(1 + \exp((r^2-\alpha^2)/a^2))}, \tag{23}$$

where $\alpha = 3.475$ fm, $a = 2.33$ fm and $w = 0.401$ for ^{56}Fe nucleus. The parameters are taken from de Jager *et al* [20].

In figure 2, we present the results for the lepton momentum distribution $\mathrm{d}\sigma/\mathrm{d}p_l$ in ^{16}O as a function of lepton momentum at a fixed neutrino energy $E_\nu = 1$ GeV. The results for the momentum distribution without the nuclear medium effects have been shown by solid lines and with the nuclear medium effects have been shown by dashed lines. We find a reduction of about 15–30% around the peak region of the momentum distribution. Further, we find that around 80–85% of these Δ's produce pions and the rest of them produce particle hole excitations. This is calculated by using $\frac{\tilde{\Gamma}}{2}$ and C_3 term in $\mathrm{Im}\,\Sigma_\Delta$ for the production of pion and for medium absorption of Δ, C_1 and C_2 terms in $\mathrm{Im}\,\Sigma_\Delta$ in eq. (21). Therefore, the total effect of the medium modification is a reduction of ≈30–35% in the peak region. However, pions once produced inside the nucleus, rescatter and some of them may be absorbed while coming out of the nucleus. We have estimated this

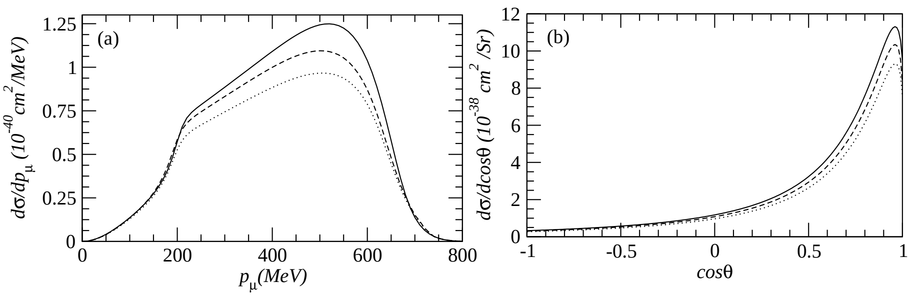

Figure 2. (**a**) $\mathrm{d}\sigma/\mathrm{d}p_\mu$ vs. p_μ at $E_\nu = 1.0$ GeV for the charged current lepton production in oxygen with (dashed line) and without (solid line) nuclear medium effects. The result with nuclear medium and pion absorption effects is shown by dotted line. (**b**) $\mathrm{d}\sigma/\mathrm{d}\cos\theta$ vs. $\cos\theta$ for various cases as depicted in figure 2a.

absorption effect in an eikonal approximation using the energy-dependent mean free path of pions taken from ref. [16]. We find a further reduction in the momentum distribution of the muon spectrum to be around 15–20% in the peak region. In figure 2b, the results for the angular distribution $(\mathrm{d}\sigma/\mathrm{d}\cos\theta)$, where θ is the angle between the outgoing lepton and the neutrino, has been shown for neutrino energy $E_\nu = 1$ GeV. We find a reduction of around 10% in the forward direction when nuclear medium modification effects are incorporated. When the pion absorption effects are taken into account there is a further reduction of about 10% in the forward direction.

In figure 3a, we have shown the effects of medium modification and pion absorption for the total scattering cross-section σ in ^{16}O. The solid line is the result for the cross-section without nuclear medium effects, the dashed line shows the effect of medium modification and dotted line shows the effect when pion absorption is also taken into account. We find that the effect of medium modification results in a reduction of the cross-section of around 5% at $E_\nu = 1$ GeV which further decreases with the increase in the neutrino energy. When pion absorption effects are taken into account there is a further reduction of around 10% at $E_\nu = 1$ GeV. In figure 3b, we have shown the results with the nuclear medium modification and final state pion interaction effects for the total scattering cross-section σ in ^{12}C (dotted line), ^{16}O (dashed line) and ^{56}Fe (solid line).

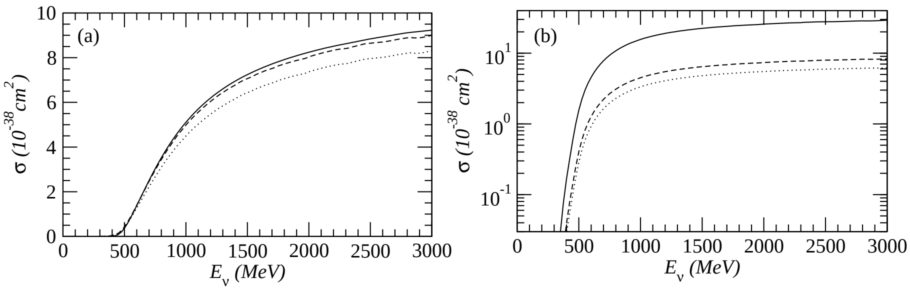

Figure 3. (**a**) Total scattering cross-section $\sigma(E_\nu)$ for the charged current lepton production in ^{16}O nucleus without (solid line) and with (dashed line) the nuclear medium effects. The result for nuclear medium and pion absorption effects is shown by dotted line. (**b**) Total scattering cross-section $\sigma(E_\nu)$ for the charged current lepton production in ^{12}C (dotted line), ^{16}O (dashed line) and ^{56}Fe (solid line) with nuclear medium and pion absorption effects.

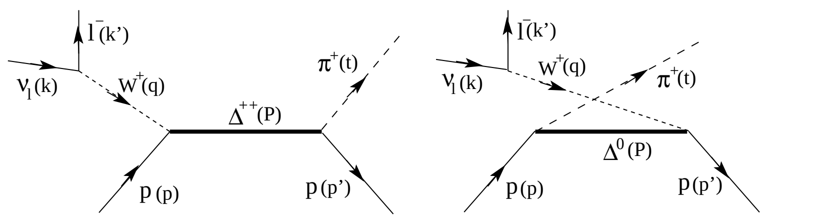

Figure 4. Direct and exchange diagrams for the reaction $\nu_l(k) + p(p) \rightarrow l^-(k') + p(p') + \pi^+(t)$.

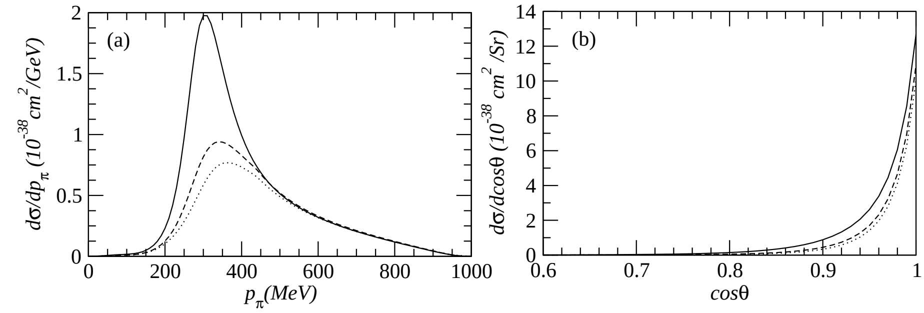

Figure 5. (**a**) $\mathrm{d}\sigma/\mathrm{d}p_\pi$ vs. p_π at $E_\nu = 1$ GeV for the charged current pion production in oxygen with (dashed line) and without (solid line) nuclear medium effects. The result with nuclear medium and pion absorption effects is shown by the dotted line. (**b**) $\mathrm{d}\sigma/\mathrm{d}\cos\theta$ vs. $\cos\theta$ at $E_\nu = 1$ GeV for various cases as depicted in figure 5a.

4. Coherent weak pion production

The coherent pion production is the process in which the nucleus remains in the ground state. The study of such processes in electromagnetic interactions has shown that these are also dominated by Δ-excitation in the intermediate energy region. Therefore, the weak pion production is also calculated using Δ dominance model. In this energy region the coherent pion production induced by neutrinos have been studied by Kelkar *et al* [21] in a non-relativistic approach and by Kim *et al* [11] in a relativistic mean-field theory of the nucleus. At very high energies, the coherent pion production has been studied using PCAC and the Adler's theorem for forward production and extrapolating the results to non-zero Q^2 [22]. Recently, these calculations have been updated by Paschos and Kartavtsev [23] using a generalized PCAC.

We calculate the coherent pion production induced by charged current interaction, i.e. $\nu(\bar{\nu}) + {}^A_Z X \rightarrow l^-(l^+) + \pi^+ + {}^A_Z X$. The calculations are done in a local density approximation using Δ dominance in which the diagrams shown in figure 4, contribute.

The matrix elements corresponding to the Feynman diagrams shown in figure 4 is given by

$$T = \frac{G_\mathrm{F}}{\sqrt{2}} \bar{u}(k')\gamma^\mu(1-\gamma_5)u(k)(J^\mu_\mathrm{direct} + J^\mu_\mathrm{exchange})F(\mathbf{q} - \mathbf{p}_\pi), \tag{24}$$

where

$$J^\mu_\mathrm{direct} = \sqrt{3}\frac{G_\mathrm{F}}{\sqrt{2}}\cos\theta_c \frac{f_{\pi N\Delta}}{m_\pi} t^\pi_\sigma \sum_s \bar{\Psi}^s(p')\Delta^{\sigma\lambda}O_{\lambda\mu}\Psi^s(p), \tag{25}$$

$$J^\mu_\mathrm{exchange} = \sqrt{3}\frac{G_\mathrm{F}}{\sqrt{2}}\cos\theta_c \frac{f_{\pi N\Delta}}{m_\pi} \sum_s \bar{\Psi}^s(p') t^\pi_\sigma O^{\sigma\lambda}\Delta_{\lambda\mu}\Psi^s(p), \tag{26}$$

$$F(\mathbf{q}-\mathbf{p}_\pi) = \int \mathrm{d}^3 r \left(\rho_p(\mathbf{r}) + \frac{1}{3}\rho_n(\mathbf{r})\right) \mathrm{e}^{i(\mathbf{q}-\mathbf{p}_\pi)\cdot\mathbf{r}}. \tag{27}$$

$\Delta^{\sigma\lambda}$ or $\Delta_{\lambda\mu}$ is the relativistic Δ propagator given by eq. (14) and $O_{\lambda\mu}$ or $O^{\sigma\lambda}$ is the weak $N-\Delta$ transition vertex given as the sum of $V^\mu + A^\mu$ using eqs (7) and (8).

Using these expressions the following form of the double differential cross-section for pion production is obtained:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_\pi \mathrm{d}E_\pi} = \frac{1}{8}\frac{1}{(2\pi)^5}\frac{(E_\nu - E_\pi)}{E_\nu}|\mathbf{p}_\pi|\Sigma|T|^2, \tag{28}$$

where $|T|^2$ is obtained by squaring the terms given in eq. (24). Similar expressions are derived for the lepton differential spectrum for the process induced by the charged currents.

The numerical evaluation of the cross-section given in eq. (28) has been done for ^{12}C, ^{16}O and ^{56}Fe using nuclear densities from eqs (22) and (23) and the results are presented in figures 5 and 6. It is found that for coherent production the axial vector current gives dominant contribution and the contribution from the vector current is almost the same. Therefore, the results for the neutrino and antineutrino reactions are almost negligible. This is in agreement with the results obtained by Kim *et al* [11] and Kelkar *et al* [21].

In figure 5a, we present the results for the momentum distribution $\mathrm{d}\sigma/\mathrm{d}p_\pi$ in ^{16}O as a function of pion momentum at a fixed neutrino energy $E_\nu = 1$ GeV. The results for the momentum distribution without the nuclear medium effects have been shown by solid lines and with the nuclear medium effects have been shown by dashed lines. We find a reduction of about 60–80% around the peak region of the momentum distribution. We find a further reduction in the pion momentum distribution to be around 15–30% in the peak region due to pion absorption effects as shown by dotted lines in this figure. In figure 5b, the results for the angular distribution $\mathrm{d}\sigma/\mathrm{d}\cos\theta$ has been shown for neutrino energy $E_\nu = 1$ GeV. The angular distribution has been found to be very sharply peaked in the forward direction. The results without(with) medium modification effects have been shown by solid line (dashed line). We find a reduction of around 15–20% in the forward direction when nuclear medium modification effects are incorporated. When the pion absorption effects are taken into account, which has been shown here by dotted line, there is a further reduction of about 8–10% in the forward direction.

In figure 6a, we have presented the results of the total scattering cross-section $\sigma(E_\nu)$ for the charged current coherent process in ^{16}O. The result of $\sigma(E_\nu)$ without the nuclear medium effects has been shown by solid line and with the nuclear medium effects has been shown by dashed line. We find the reduction in the cross-section because of the medium modification on Δ to be quite substantial at low energies. For example, at $E_\nu = 0.7$ GeV, it is $\approx$40% which decreases with the increase in energy and this reduction becomes 30% at $E_\nu = 1$ GeV and 20% at $E_\nu = 1.5$ GeV. When pion absorption effects are taken into account, there is a further reduction of 25–30% at these energies. In figure 6b we show the results for the total cross-section $\sigma(E_\nu)$ for coherent pion production in nuclei like ^{12}C (dotted line), ^{16}O (dashed line) and ^{56}Fe (solid line).

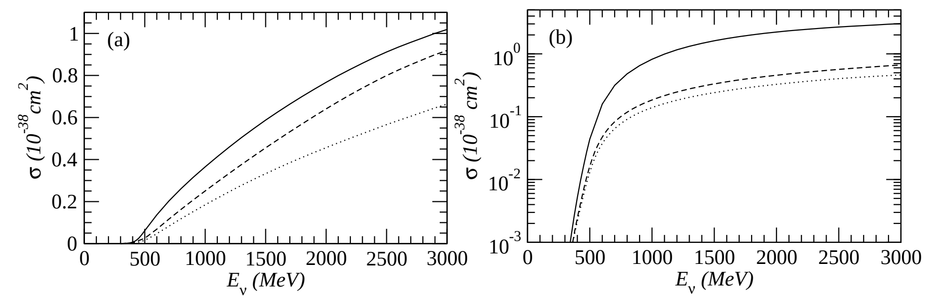

Figure 6. (**a**) Total cross-section $\sigma(E_\nu)$ for the charged current coherent pion production in ^{16}O nucleus without (solid line) and with (dashed line) nuclear medium effects. The result with nuclear medium and pion absorption effects is shown by the dotted line. (**b**) Total scattering cross-section $\sigma(E_\nu)$ for the coherent charged current pion production in ^{12}C (dotted line), ^{16}O (dashed line) and ^{56}Fe (solid line) with nuclear medium and pion absorption effects.

5. Conclusions

We have calculated the weak pion production induced by charged currents in neutrino resctions at intermediate neutrino energies of $E_\nu \approx 1$ GeV. This is relevant for the neutrino oscillation experiments being done with atmospheric neutrinos and the accelerator neutrinos at K2K and MiniBooNE.

The incoherent and coherent pion production have been calculated for ^{12}C, ^{16}O and ^{56}Fe and the numerical results have been presented. The nuclear medium effects have been calculated using the renormalization of Δ properties in the nuclear medium and the pion distortion effects using the energy-dependent mean free path of pions as determined from the electromagnetic and strong interaction processes in this energy region. The numerical results for the momentum distribution, angular distribution for lepton and/or pions and the total cross-section have been presented and discussed.

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