

Theoretical expectations for σ^{tot} at the large hadron collider

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Abstract. In this note, we summarize and compare various model predictions for pp total cross-section σ_{tot}^{pp} , giving an estimate of the range of predictions for the total cross-section, σ_{tot}^{pp} expected at the LHC. We concentrate on the results for σ_{tot}^{pp} obtained in a particular QCD based model of the energy dependence of the total cross-section, including the effect of soft gluon radiation. We obtain the range of predictions in this model by exploring the allowed range of model parameters. We further give a handy parametrisation of these results which incidentally spans the range of various other available predictions at the LHC as well.

Keywords. Quantum chromodynamics; total cross-sections; large hadron collider.

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1. Introduction

Energy dependence of total hadronic cross-sections has been the focus of intense theoretical interest as a sensitive probe of strong interactions long before the establishment of QCD as ‘the’ theory of hadrons. Even now, notwithstanding creditable successes of perturbative and lattice QCD, a first principle description of total/elastic and inelastic hadronic cross-section is unavailable. More pragmatically, for a correct projection of the expected underlying activity at LHC, a reliable prediction of total non-diffractive cross-section is essential to ensure the extraction of new physics from the LHC data. Surely we will have to depend – at the initial stages of LHC – upon predictions based on our current understanding of these matters. Only much later it may become feasible to use the LHC data itself towards this goal. Hence, a critical evaluation of the range of theoretical predictions, is absolutely essential.

The hadronic cross-section data exhibit, and require explanation of, three basic features:

- (i) the normalization of the cross section,
- (ii) an initial decrease and
- (iii) a subsequent rise with energy.

Various theoretical models exist which are motivated by our theoretical understanding of the strong interactions. The parameters in these models, in most cases, are fitted to explain the observed low energy data and the model predictions are then extrapolated to give the σ_{tot}^{pp} at the LHC energies. There are different classes of models. The highly successful Donnachie–Landshoff (DL) parametrisation [1] of the form

$$\sigma_{\text{tot}}(s) = Xs^\epsilon + Ys^{-\eta}, \quad (1)$$

has been used for a very long time. Here the two terms are understood as arising from the Regge and the Pomeron trajectories, the ϵ being approximately close to zero and η being close to 0.5. These values seem to be consistent with a large, but not all, body of the hadronic cross-sections. In this note we will first present phenomenological arguments for the approximate values of these parameters which seem to be required to describe the data satisfactorily. As a matter of fact, there also exist in the literature discussions of the ‘hard’ Pomeron [2] motivated by the discrepancies in the rate of energy rise observed by E710 [3], E811 [4] and the CDF [5]. In addition, a variety of models exist wherein the observed energy dependence of the cross-section, along with few very general requirements of factorisation, unitarity and/or ideas of finite energy sum rules (FESR), is used to determine the values of model parameters [6–12]. The so-obtained parametrisations are then extended to make predictions at the LHC energies. There also exist QCD motivated models based on the mini-jet formalism [13–15], wherein the energy rise of the total cross-sections is driven by the increasing number of the low- x gluon–gluon collisions. These models need to be embedded in an eikonal formalism [16] to soften the violent energy rise of the mini-jet cross-sections. Even after eikonalisation the predicted energy rise is harder than the gentle one observed experimentally [15,17]. A QCD based model where the rise is further tamed by the phenomenon of increasing emission of soft gluons by the valence quarks in the colliding hadrons, with increasing energy [18,19], offers a consistent description of σ_{tot}^{pp} . Thus we have a variety of model predictions for σ_{tot}^{pp} at the LHC. In this note we compare these predictions with each other to obtain an estimate of the ‘theoretical’ uncertainty in them.

2. Phenomenological models

The two terms of eq. (1) [1] reflect the well-known duality between resonance and Regge pole exchange on the one hand and background and Pomeron exchange on the other, established in the late 60’s through FESR [20]. This correspondence meant that, while at low energy the cross-section could be written as due to a background term and a sum of resonances, at higher energy it could be written as a sum of Regge trajectory exchanges and a Pomeron exchange.

It is well to ask (i) where the ‘two-component’ structure of eq. (1) comes from and (ii) why the difference in the two powers (in s) is approximately a half. Our present

knowledge of QCD and its employment for a description of hadronic phenomena can be used to provide some insight into the ‘two-component’ structure of eq. (1). This begins with considerations about the bound state nature of hadrons which necessarily transcends perturbative QCD. For hadrons made of light quarks (q) and gluons (g), the two terms arise from $q\bar{q}$ and gg excitations. For these, the energy is given by a sum of three terms: (i) the rotational energy, (ii) the Coulomb energy and (iii) the ‘confining’ energy. If we accept the Wilson area conjecture in QCD, (iii) reduces to the linear potential [21,22]. Then the hadronic rest mass for a state of angular momentum J can be obtained by minimising the expression for the energy of two massless particles ($q\bar{q}$ or gg) separated by a distance r .

Explicitly, in the CM frame of two massless particles, either a $q\bar{q}$ or a gg pair separated by a relative distance r with relative angular momentum J , the energy is given by

$$E_i(J, r) = \frac{2J}{r} - \frac{C_i\bar{\alpha}}{r} + C_i\tau r, \quad (2)$$

where $i = 1$ refers to $q\bar{q}$, $i = 2$ refers to gg , τ is the ‘string tension’ and the Casimir’s are $C_1 = C_F = 4/3$, $C_2 = C_G = 3$. $\bar{\alpha}$ is the QCD coupling constant evaluated at some average value of r and whose precise value will disappear in the ratio to be considered. The hadronic rest mass for a state of angular momentum J is then computed through minimising the above energy

$$M_i(J) = \min_r \left[\frac{2J}{r} - \frac{C_i\bar{\alpha}}{r} + C_i\tau r \right]. \quad (3)$$

This is then given by

$$M_i(J) = 2\sqrt{(C_i\tau)[2J - C_i\bar{\alpha}]}. \quad (4)$$

This can then be used to obtain the two sets of linear Regge trajectories

$$\alpha_i(s) = \frac{C_i\bar{\alpha}}{2} + \left(\frac{1}{8C_i\tau} \right) s = \alpha_i(0) + \alpha'_i s, \quad (5)$$

Note that α_i are *not* the coupling constants.

Thus, the ratio of the intercepts is given by

$$\frac{\alpha_{gg}(0)}{\alpha_{q\bar{q}}(0)} = C_G/C_F = \frac{9}{4}. \quad (6)$$

Employing our present understanding that resonances are $q\bar{q}$ bound states while the background, dual to the Pomeron, is provided by gluon–gluon exchanges [22], the above equation can be rewritten as

$$\frac{\alpha_P(0)}{\alpha_R(0)} = C_G/C_F = \frac{9}{4}. \quad (7)$$

If we restrict our attention to the leading Regge trajectory, namely the degenerate $\rho - \omega - \phi$ trajectory, then $\alpha_R(0) = \eta \approx 0.48-0.5$, and we obtain for $\epsilon \approx 0.08-0.12$, a rather satisfactory value. The same argument for the slopes gives

$$\frac{\alpha'_{gg}}{\alpha'_{q\bar{q}}} = C_F/C_G = \frac{4}{9}. \quad (8)$$

Hence, if we take for the Regge slope $\alpha'_R \approx 0.88-0.90$, we get for $\alpha'_P \approx 0.39-0.40$, in fair agreement with lattice estimates [23].

We now have good reasons for a break up of the amplitude into two components. To proceed further, it is necessary to realize that precisely because massless hadrons do not exist, eq. (1) violates the Froissart bound and thus must be unitarized. To begin this task, let us first rewrite eq. (1) by putting in the ‘correct’ dimensions

$$\bar{\sigma}_{\text{tot}}(s) = \sigma_1(s/\bar{s})^\epsilon + \sigma_2(\bar{s}/s)^{1/2}, \quad (9)$$

where we have imposed the nominal value $\eta = 1/2$. It is possible to obtain [19] rough estimates for the size of the parameters in eq. (9). A minimum occurs in $\bar{\sigma}_{\text{tot}}(s)$ at $s = \bar{s}$, for $\sigma_2 = 2\epsilon\sigma_1$. If we make this choice, then eq. (9) has one less parameter and it reduces to

$$\bar{\sigma}_{\text{tot}}(s) = \sigma_1[(s/\bar{s})^\epsilon + 2\epsilon(\bar{s}/s)^{1/2}]. \quad (10)$$

We can isolate the rising part of the cross-section by rewriting the above as

$$\bar{\sigma}_{\text{tot}}(s) = \sigma_1[1 + 2\epsilon(\bar{s}/s)^{1/2}] + \sigma_1[(s/\bar{s})^\epsilon - 1]. \quad (11)$$

Equation (11) separates cleanly the cross-section into two parts: the first part is a ‘soft’ piece which shows a saturation to a constant value (but which contains no rise) and the second a ‘hard’ piece which has all the rise. Moreover, \bar{s} naturally provides the scale beyond which the cross-sections would begin to rise. Thus, our ‘Born’ term assumes the generic form

$$\sigma_{\text{tot}}^B(s) = \sigma_{\text{soft}}(s) + \vartheta(s - \bar{s})\sigma_{\text{hard}}(s) \quad (12)$$

with σ_{soft} containing a constant (the ‘old’ Pomeron with $\alpha_P(0) = 1$) plus a (Regge) term decreasing as $1/\sqrt{s}$ and with an estimate for their relative magnitudes ($\sigma_2/\sigma_1 \sim 2\epsilon$). In the eikonalised mini-jet model used by us [19] the rising part of the cross-section σ_{hard} is provided by jets which are calculable by perturbative QCD, obviating (at least in principle) the need of an arbitrary parameter ϵ . An estimate of σ_1 can also be obtained [19] and is ~ 40 mb.

As said earlier, the DL parametrisation [1] is a fit to the existing data of the form given by eq. (1), with $\epsilon = 0.0808, \eta = 0.4525$. This fit has been extended to include a ‘hard’ Pomeron [2] due to the discrepancy between different data sets. The BH model [6] gives a fit to the data using duality constraints. The BH fit for $\sigma^\pm = \sigma^{\bar{p}p}/\sigma^{pp}$ as a function of beam energy ν , is given as

$$\sigma^\pm = c_0 + c_1 \ln(\nu/m) + c_2 \ln^2(\nu/m) + \beta_{P'}(\nu/m)^{\mu-1} \pm \delta(\nu/m)^{\alpha-1},$$

where $\mu = 0.5, \alpha = 0.415$ and all the other parameters in mb are $c_0 = 37.32, c_1 = -1.440 \pm 0.07, c_2 = 0.2817 \pm 0.0064, \beta_{P'} = 37.10, \delta = -28.56$. The fit obtained by Igi and Ishida [8] using the finite energy sum rules (FESR) gives LHC predictions very similar to those given by the BH [6] fit. Avila *et al* [9] gave a fit using analyticity arguments whereas Cudell *et al* [10] gave predictions at the LHC energies

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by extrapolating fits obtained to the current data again using constraints from unitarity, analyticity of the S-matrix, factorisation, coupled with a requirement that the cross-section asymptotically goes to a constant as $\ln(s)$ or $\ln^2(s)$, in the framework of the COMPETE program.

In the mini-jet models the energy rise of σ_{tot}^{pp} is driven by the increase with energy of the σ_{jet}^{AB} given by

$$\sigma_{\text{jet}}^{AB}(s) = \int_{p_{t\text{min}}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^1 dx_1 \int_{4p_t^2/(x_1s)}^1 dx_2 \sum_{i,j,k,l} f_{i|A}(x_1) f_{j|B}(x_2) \times \frac{d\hat{\sigma}_{ij \rightarrow kl}(\hat{s})}{dp_t}, \quad (13)$$

where subscripts A and B denote particles (γ, p, \dots), i, j, k, l are partons and x_1, x_2 the fractions of the parent particle momentum carried by the parton. $\hat{s} = x_1 x_2 s$ and $\hat{\sigma}$ are hard partonic scattering cross-sections. This parton model used in the calculation is illustrated in figure 1. Note here that the experimentally measured parton densities in the proton and the elements of perturbative QCD is all the input needed for the calculation of σ_{jet}^{AB} . The rate of rise with energy of this cross-section is determined by $p_{t\text{min}}$ and the low- x behaviour of the parton densities. As said before, the rise with energy of this cross-section is much steeper than can be tolerated with the Froissart bound. Hence it has to be embedded in an eikonal formulation given by

$$\sigma_{\text{tot}}^{AB} = 2 \int d^2\vec{b} [1 - e^{-2\Im m \chi^{AB}(b,s)}], \quad (14)$$

where $\text{Re} \chi^{AB}(b,s) = 0$ and $2\Im m \chi^{AB}(b,s) = n^{AB}(b,s)$ is the average number of multiple collisions which are Poisson distributed. As outlined in eq. (11) this quantity too has contributions coming from soft and hard physics and can be written as

$$n^{AB}(b,s) = n_{\text{soft}}^{AB} + n_{\text{hard}}^{AB} \simeq A_{\text{soft}}^{AB}(b) \sigma_{\text{soft}}^{AB}(s) + A_{\text{jet}}^{AB}(b) \sigma_{\text{jet}}^{AB}(s). \quad (15)$$

In the second step the number $n(b,s)$ has been assumed to be factorisable into an overlap function $A(b)$ and the cross-section σ .

The sketch in figure 2 indicates the relationship between the multiple scatterings and the overlap function. The assumption of factorisation as well as the split up between the two contributions, hard and soft, for the $n(b,s)$ are only approximate. The extent to which this softens the energy rise, depends on the b dependence of $n(b,s)$, i.e., that of $A(b)$ in the factorised case. The normal assumption of using the same form of $A(b)$ for both the hard and the soft part, given by the Fourier transform of the electromagnetic form factor (FF), still gives too steep a rise even in this eikonalised mini-jet model (EMM) [16]. In our model this rise is tamed by including the effect on the transverse momentum distribution of the partons in the proton, of the soft gluon emission from the valence quarks in the proton [18]; the effect increasing with increasing energy. In this description, the transverse

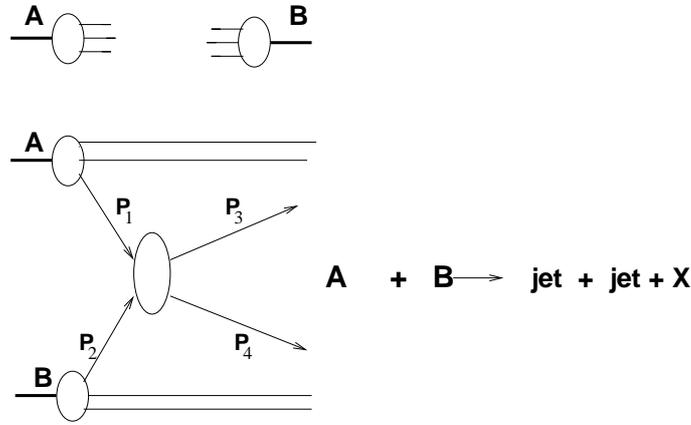


Figure 1. Parton model picture for jet production.

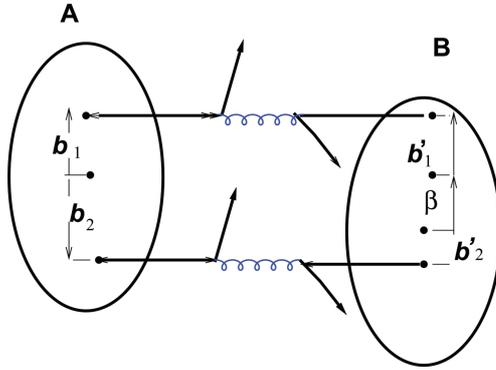


Figure 2. Multiple scattering giving rise to the transverse overlap of the hadrons.

momentum distribution of the quarks in a proton can then be calculated in a semi-classical picture as arising from the re-summation of a large number of soft gluons. This in turn allows us to calculate the transverse overlap function.

Figure 3 sketches the multiple emissions of gluons which gives rise to the transverse momentum distribution of the valence quarks and hence the overlap function. The non-perturbative soft part of the eikonal includes only limited low energy gluon emission and leads to the initial decrease in the proton-proton cross-section. On the other hand, the rapid rise in the hard, perturbative jet part of the eikonal is tamed into the experimentally observed mild increase by soft gluon radiation whose maximum energy (q_{\max}) rises slowly with energy. Thus the overlap functions $A(b)$ are no longer a function of b alone. We denote the corresponding overlap function by $A_{BN}(b, q_{\max})$. The σ_{tot}^{pp} can then be computed using eq. (14) where $2\Im m \chi^{pp}(b, s)$ is given by

$$\begin{aligned}
 2\Im m \chi(b, s) &= n(b, s; q_{\max}, p_{t \text{ min}}) = n_{\text{soft}} + n_{\text{jet}} \\
 &= A_{\text{soft}}(b, s)\sigma_{\text{soft}}(s) + A_{BN}(b, q_{\max})\sigma_{\text{jet}}(s).
 \end{aligned}
 \tag{16}$$

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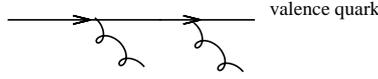


Figure 3. Multiple gluon emission giving rise to the transverse momentum distribution of the valence quarks.

The function $A_{BN}(b, q_{\text{max}})$ [18] is determined by q_{max} , which in turn depends on the energy and the kinematics of the subprocess. What we use is its average value over all the momentum fractions of the parent partons. We need to further make a model for the ‘soft’ part A_{soft} , which is determined by the non-perturbative dynamics. It is this part of the eikonal that contributes to the σ_{tot}^{pp} at high energies through its impact on the turn around from the decreasing Regge behaviour to the softly rising behaviour around $\sqrt{s} \simeq 15$ GeV, where the hard part contribution is minuscule. Note that we have taken n_{soft} to be factorised into a constant soft cross-section σ_{soft} and taken A_{soft} to be given by the function $A_{BN}(b, q_{\text{max}}^{\text{soft}})$ except for the fact that it is not possible to calculate the $q_{\text{max}}^{\text{soft}}$ for the soft processes, as in the case of hard processes. We further postulate that the q_{max} is the same for the hard and soft processes at low energy, parting company around 10 GeV where the hard processes start becoming important. A good fit to the data requires q_{max} at low energies to be a very slowly decreasing function of energy, with a value around 0.20 MeV at $\sqrt{s} = 5$ GeV rising to about 0.24 MeV, $\sqrt{s} \geq 10$ GeV, the upper value of this soft scale being completely consistent with our picture of the proton. Further, we need to fix one more parameter for non-perturbative region, the σ_{soft} . For the pp case it is a constant σ_0 which will fix the normalization of σ_{tot}^{pp} , whereas for the $p\bar{p}$ the duality arguments suggest that there is an additional \sqrt{s} dependent piece $\simeq 1/\sqrt{s}$. Thus neglecting the real part of the eikonal, $n(b, s)$ in our model is given by

$$n(b, s) = A_{BN}(b, q_{\text{max}}^{\text{soft}})\sigma_{\text{soft}}^{pp,\bar{p}} + A_{BN}(b, q_{\text{max}}^{\text{jet}})\sigma_{\text{jet}}(s; p_{t \text{ min}}), \quad (17)$$

where

$$\sigma_{\text{soft}}^{pp} = \sigma_0, \quad \sigma_{\text{soft}}^{p\bar{p}} = \sigma_0 \left(1 + \frac{2}{\sqrt{s}} \right). \quad (18)$$

Thus the parameters of the model are $p_{t \text{ min}}$ and σ_0 . In addition, the evaluation of A_{BN} involves α_s in the infrared region, for which we use a phenomenological form inspired by the Richardson potential [18]. This involves a parameter p which for the Richardson potential takes value 1. Values of $p_{t \text{ min}}, \sigma_0$ and p which give a good fit to the data with the GRV parametrisation of the proton densities [24] are 1.15 GeV, 48 mb and 3/4 respectively, as presented in ref. [19]. These values are consistent with the expectations of the general argument [19]. We expect these best fit values to change somewhat with the choice of parton density functions (PDF). Since we are ultimately interested in the predictions of the model at TeV energies, we need PDF parametrisation which cover both the small and large Q^2 range ($2 < Q^2 < 10^4$) as well as are valid up to rather small values of x ($\sim 10^{-5}$). Further, since our calculation here is only LO, for consistency we have to use LO densities. We have repeated the exercise then for a range of PDF’s [25–27] meeting these requirements. For each PDF, it is onset of the rise that fixes the $p_{t \text{ min}}$,

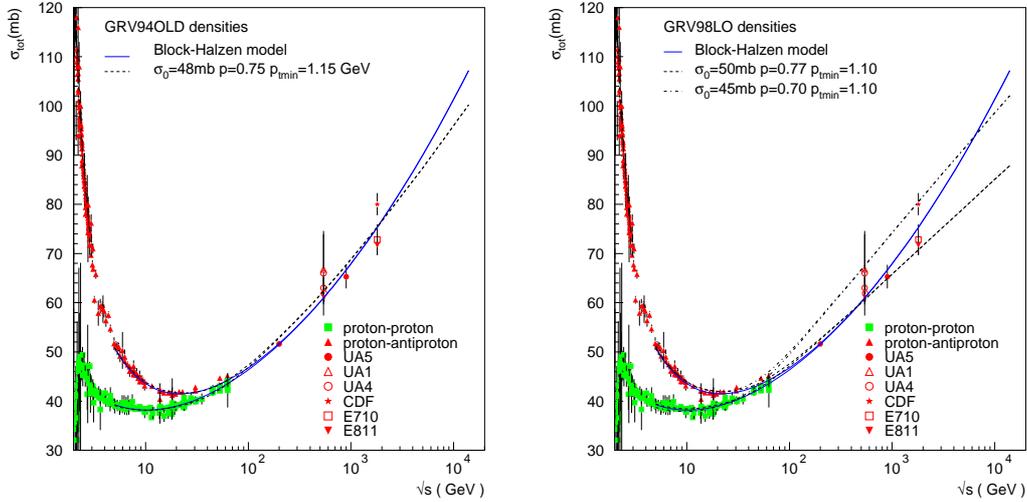


Figure 4. Comparison of the G.G.P.S. model [19] predictions for GRV [24] and GRV98lo [26] densities with the BH [6] predictions.

σ_0 controlling the normalisation and p determining the slope of the rising part of the cross-section. We find that it is possible to get a satisfactory description of all the current data, for all the choices of PDF's considered. The corresponding range of values of $p_{t\min}$, σ_0 and p are given in table 1. Figure 4 compares with the data the predictions of our QCD based model along with those obtained by the considerations of unitarity and factorisation [6]. As can be seen clearly, both are able to describe the current data on total cross-sections equally satisfactorily. As can be seen from figures 4 and 5, it is possible to get equally satisfactory description of the data in our QCD based model, for all the chosen PDF's, by tuning the soft parameters by a small amount. In the next section we compare these model results with the predictions of all the other models.

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Figure 6 summarises the predictions of the different models described in the previous section. The shaded area gives the range of predictions in the eikonalised mini-jet model with soft gluon re-summation [19] (the G.G.P.S. model), the different PDF's used giving the range as described in the earlier section. The solid line gives prediction obtained using the GRV parton densities [24] in the model. The curve d indicates predictions of the DL fit [1]. The (BH) curve c and the uppermost curve a , are the results of two analytical models incorporating constraints from unitarity and analyticity, from [6] and [9], respectively. The predictions obtained by Igi and Ishida, using FESR follows very closely to that given by the BH curve. Further, curve b is the result of a fit by the COMPETE collaboration [10]. The parametrisation for the DL curve and BH curve is already given in the last section.

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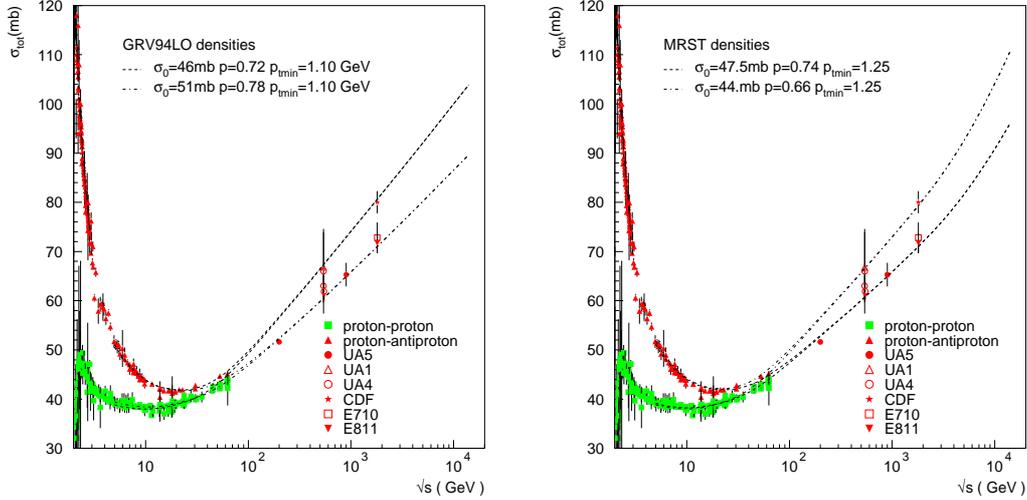


Figure 5. Predictions of the G.G.P.S model [19] for the GRV94lo [25] and the MRST [27] densities for model parameters mentioned in table 1.

It is gratifying to see that the range of results of our QCD motivated mini-jet models for the LHC span the other predictions obtained in models using unitarity, factorisation, analyticity along with fits to the current data. Thus the two sets of predictions seem consistent with each other.

We have parametrised the results of our EMM model with a $\ln^2(s)$ fit. We found that in most cases this gave a better representation of our results than a fit of the Regge–Pomeron type of the form of eq. (1). The top edge of the EMM model prediction is obtained for the MRST parametrisation whereas the lower edge for the GRV98lo. We give fits to our results for σ^{pp} of the form

Table 1. Values of $p_{t \min}$ and σ_0 corresponding to the different parton densities in the proton, for which the EMM (as described in ref. [19]) gives a satisfactory description of σ_{tot}^{pp} .

PDF	$p_{t \min}$ (GeV)	σ_0 (mb)	p
GRV [24]	1.15	48	0.75
GRV94lo [25]	1.10	46	0.72
	1.10	51	0.78
GRV98lo [26]	1.10	45	0.70
	1.10	50	0.77
MRST [27]	1.25	47.5	0.74
	1.25	44	0.66

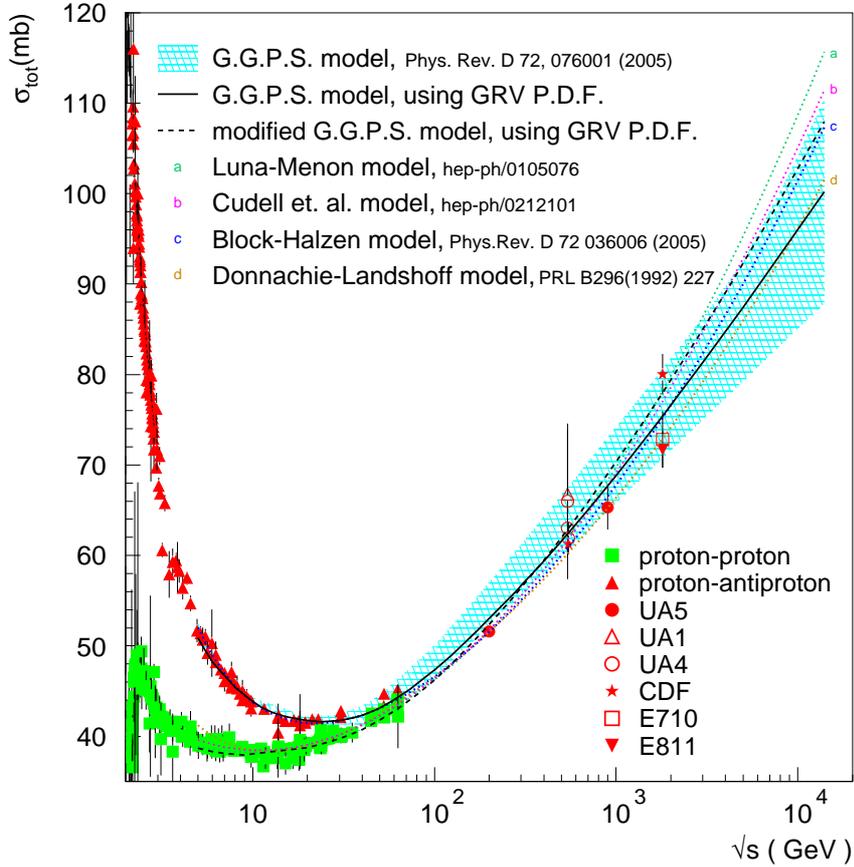


Figure 6. Predictions for σ_{tot}^{pp} in various models. The shaded area gives the range of results in the eikonalised mini-jet model with soft gluon re-summation [19] (the G.G.P.S. model) the solid line giving the prediction obtained using the GRV parton densities [24] in the model. Curve *d* indicates predictions for the DL fit [1]. Curve *c* and the uppermost curve *a*, are the results of two analytical models incorporating constraints from unitarity and analyticity, from [6] and [9], respectively. The prediction obtained by Igi and Ishida, using FESR follows very closely to that given by the BH curve. Further, curve *b* is the result of a fit by the COMPETE collaboration [10].

$$\sigma_{tot}^{pp} = a_0 + a_1 s^b + a_2 \ln(s) + a_3 \ln^2(s). \tag{19}$$

The values of various parameters for the top end lower edge as well as the central curve are given in table 2.

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Table 2. Values of a_0, a_1, a_2, a_3 and b parton densities in the proton, for which the EMM (as described in ref. [19]) gives a satisfactory description of σ_{tot}^{pp} .

	a_0 (mb)	a_1 (mb)	b	a_2 (mb)	a_3 (mb)
Top edge	23.61	54.62	-0.52	1.15	0.17
Center	-139.80	193.89	-0.11	13.98	-0.14
Lower edge	-68.73	125.80	-0.16	11.05	-0.16

4. Conclusions

We thus see that the range of the results for the σ_{tot}^{pp} from our QCD motivated EMM model [19] spans the range of predictions made using the current data and general arguments of unitarity and/or factorisation. Further, we give $\ln^2(s)$ parametrisation of the model results for σ_{tot}^{pp} which may be used in evaluating the range of predictions for the underlying event at the LHC.

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