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Pentaquarks in chiral color dielectric model

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Abstract. Recent experiments indicate that a narrow baryonic state having strangeness +1 and mass of about 1540 MeV may be existing. Such a state was predicted in chiral model by Diakonov *et al.* In this work I compute the mass and width of this state in chiral color dielectric model. I show that the computed width is about 30 MeV. I find that the mass of the state can be fitted to the experimentally observed mass by invoking a color neutral vector field and its interaction with the quarks.

Keywords. Pentaquark mass; width; chiral color dielectric model.

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1. Introduction

Hadrons consisting of exotic quark structure have been proposed since the inception of the quark model [1]. Theoretically mesons having two quarks and two antiquarks and baryons having four quarks and an antiquark can form a color neutral object and such states can, in principle, exist. Regge pole theories [2] also indicate possible existence of such hadronic states. However, such states are expected to have large widths for decay into a pair of mesons or a baryon–meson pair and therefore are difficult to observe experimentally. Recently, the search for baryons having four quarks and an antiquark got an impetus after the prediction of Diakonov *et al* [3]. Their calculation was based on the chiral model and it predicted a pentaquark state, which they called Θ pentaquark, with mass of about 1540 MeV and an exceptionally narrow width of about 15 MeV. Unlike commonly observed 3-quark baryons, which have negative or zero strangeness, the strangeness of Θ pentaquark was predicted to be +1 and therefore it was considered to be exotic. Further, the computation of Diakonov *et al* predicted the spin-parity of Θ state which had to be $\frac{1}{2}^+$ and was supposed to belong to the $\overline{10}$ representation of flavor $SU(3)$. Recently, such a state has been observed at about 1540 MeV in a number of experiments [4] with a very small width. However, results of some experiments have been negative [5]. The spin-parity of this state has not yet been determined experimentally. We may note here that some analyses of kaon–nucleon scattering have also indicated the existence of such a state [6].

Although the exotic hadrons have been investigated earlier, the study of such objects received an impetus after the experimental observation of Θ baryon and a number of theoretical calculations for the determination of the properties of Θ pentaquark have been done in the last few years. Many of these [7] assume a strong diquark correlation in color $\overline{3}$ channel. Thus, the Θ state is assumed to be a two diquark–one antiquark state. These calculations yield positive parity Θ. On the other hand, naive quark models assuming the quarks and antiquarks to be in $s_{1/2}$ orbitals predict a negative parity Θ. The mass of the pentaquark state computed in naive quark models is, however, much larger than 1540 MeV. Some lattice calculations for Θ have also been done [8] but these are still inconclusive. Some indicate the existence of a negative parity state whereas others are inconclusive. For a review of current status of Θ pentaquarks, refer to the article by Hosaka [9] and references therein.

In the present work, the mass and width of Θ has been computed in chiral color dielectric (CCD) model. The CCD model is based on the work of Nielsen and Patkos [10]. In their work, Nielsen and Patkos showed that, on averaging the gluon field over lattice plaquette, one obtains, in addition to the effective quark and gluon fields, a scalar color neutral field (as well as other effective fields). Furthermore, the interaction of the scalar field with quark and gluon fields is such that the quark and gluon fields are excluded from the region where the scalar field vanishes. Thus, it is possible to construct a dynamical model in which color neutral objects can emerge. The model has been further extended to include chiral invariance [11] and a number of calculations for the properties of low-lying baryons have been done. The model has also been used in meson–baryon scattering calculations [12], computation of quark matter equation of state [13] etc. It may be noted here that, because of the chiral invariance which is incorporated in the model, one can generate the coupling between Θ baryon and NK system in a natural way and one can therefore compute the width of Θ. Our calculation yields the Θ width of about 30 MeV. The computed mass of Θ is more than 1800 MeV. However, we show that it is possible to lower the Θ mass by introducing a color-neutral vector field which interacts with quarks. With a suitable coupling between this vector field and quark field it is possible to fit the Θ mass. The reason for the reduction of Θ mass in relation to the other nonexotic baryons is that the vector field produces a repulsive interaction for quarks but an attractive interaction for antiquarks. This is similar to what happens in the Walecka model [14] which has been extensively used in nuclear structure calculations. We may mention here that the Nielsen–Patkos procedure [10] does not rule out such an effective vector field arising from averaging procedure. Such a field was not used earlier possibly because one gets a good agreement for baryon properties without it.

The paper is organised as follows. In §2 we briefly describe the extended CCD model, construct the baryonic pentaquark states and compute their masses. In §3 we present the results of the calculations and finally in §4 we discuss the future plans.

2. The chiral color dielectric model

The Lagrangian of the CCD model with perturbative pseudoscalar meson coupling to quarks is [11]

$$\mathcal{L} = \bar{\Psi}\Big[i\gamma^\mu\partial_\mu - g_s\lambda_a\gamma^\mu A^a_\mu - g_V\gamma^\mu V_\mu - m_0 - \frac{m}{\chi}\Big(1+\frac{i}{f}\gamma_5\lambda^f_a\phi^a\Big)\Big]\Psi + \frac{\sigma_v^2(\partial_\mu\chi)^2}{2} - U(\chi) - \frac{m_V^2 V_\mu^2}{2} + \frac{(\partial_\mu V_\nu - \partial_\nu V_\mu)^2}{4} + \frac{\chi^4(F^a_{\mu\nu})^2}{4} + \frac{(\partial_\mu\phi^a)^2}{2} - \frac{m_\phi^2(\phi^a)^2}{2}, \tag{1}$$

where Ψ, A^a_μ, χ, V_μ and ϕ^a are the (effective) quark, gluon, scalar (color dielectric), vector and pseudoscalar meson fields respectively, $F^a_{\mu\nu}(x)$ is the color electromagnetic field tensor, g_s is the color coupling constant, g_V is the coupling constant for coupling between quark and color-neutral vector field, f is the pion decay constant and λ^f_a are the flavor $SU(3)$ Gell–Mann metrices. The pseudoscalar meson mass terms have been included explicitly and therefore, to that extent, the chiral invariance is broken. Further, we do not consider the full non-linear chiral Lagrangian here since we will be treating the pseudoscalar fields perturbatively. We have found that the perturbative treatment of pseudoscalar fields gives a very good agreement for meson–nucleon scattering etc. and therefore it is justified. The dielectric field self-interaction $U(\chi)$ is defined to be of the form

$$U(\chi) = \alpha B\big[\chi^2 - 2(1-2/\alpha)\chi^3 + (1-3/\alpha)\chi^4\big].$$

The form of $U(\chi)$ has been chosen to have an absolute minimum at $\chi = 0$ and a secondary minimum at $\chi = 1$ with $U(\chi = 1) = B$. The coupling of χ with the quark and gluon fields is such that these fields cannot exist in the region where χ vanishes. Thus $\chi = 0$ can be identified with the physical vacuum of the MIT bag model [15] and B with the bag pressure. The quantity $m_{GB} = \sqrt{\frac{\alpha B}{\sigma_v^2}}$ can be identified with the mass of the scalar field. The parameters of the model are α and B which enter in the definition of the self-interaction of the scalar field, g_s, the color coupling constant, σ_v (or equivalently m_{GB}), m the u and d quark mass, m_0, the mass difference between u and s quark masses and g_V, the coupling of the color-neutral vector field with the quarks. These parameters are adjusted to fit the masses of baryon octet and decouplet. For a detailed discussion of the CCD model the reader is referred to [11].

2.1 *Mean-field equations of motion*

Following perturbative chiral bag model approach [16], we solve quark and scalar and vector field equations in mean-field approximation. We further assume that the dielectric field is spherically symmetric and consider the s-state quark and antiquark wave functions [17]. The problem then reduces to a self-consistent solution of quark (and antiquark) as well as dielectric field equations of motion derived from the Lagrangian of eq. (1). These are

$$\left[i\gamma_\mu\partial^\mu - \gamma_0 g_V V_0(r) - m_0 - \frac{m}{\chi(r)}\right]\psi(\vec{r}) = 0, \tag{2}$$

$$\chi''(r) + \frac{2}{r}\chi'(r) + \frac{2\alpha B}{\sigma_v^2}\chi(r)[1 - 3(1-2/\alpha)\chi(r) + 2(1-3/\alpha)\chi^2(r)] \\ - \frac{m}{\sigma_v^2\chi^2(r)}\sum_i \bar{\psi}_i(\vec{r})\psi_i(\vec{r}) = 0 \tag{3}$$

and

$$V_0''(r) + \frac{2}{r}V_0'(r) + m_V^2 V_0(r) \\ - g_V\left[\sum_q \psi^\dagger(\vec{r})\psi(\vec{r}) - \sum_{\bar{q}} \psi^\dagger(\vec{r})\psi(\vec{r})\right] = 0. \tag{4}$$

Here the summation in $\sum_i \bar{\psi}_i(\vec{r})\psi_i(\vec{r})$ is over the valance quarks and antiquarks. Thus, in case of Θ, the Dirac wave functions of four u, d quarks and an s antiquark are included in the sum.

2.2 Θ *Wave function and bare mass*

The wave function of Θ baryon is constructed by putting two u and two d quarks and s antiquark in $1s_{1/2}$ orbital. This defines the space part of the wave function. We need to couple the spin-flavor-color wave functions of individual quarks such that the total wave function is color-singlet, flavor-antidecuplet and has spin 1/2. Construction of such wave functions is given in literature [18]. However, we construct the wave function as follows. We first construct a product wave function consisting of a nucleon cluster (three quarks) and a kaon cluster (a quark and the s antiquark), coupling the isospins of two particles to zero. This product wave function is color-neutral, has spin 1/2 and belongs to anti-decuplet of flavor. We then antisymmetrize this wave function between the quarks in the nucleon cluster and the quark in the kaon cluster. Although this procedure is not elegant, it is useful in many computations.

The bare mass of Θ is defined as the expectation value of the Hamiltonian H,

$$H = \int \mathrm{d}^3r\Big[\bar{\psi}(\vec{r})\gamma_0\partial_t\psi(\vec{r}) + \frac{\sigma_v^2}{2}(\nabla\chi(r))^2 \\ + U(\chi(r)) - \frac{1}{2}(\nabla V_0)^2 - \frac{1}{2}m_V^2 V_0^2\Big] \tag{5}$$

in Θ wave function. In mean-field approximation, the expectation value is

$$\langle H\rangle = \sum_i \epsilon_i + \int \mathrm{d}^3r\Big[\frac{\sigma_v^2}{2}(\nabla\chi(r))^2 + U(\chi(r)) \\ - \frac{1}{2}(\nabla V_0)^2 - \frac{1}{2}m_V^2 V_0^2\Big]. \tag{6}$$

We need to correct for the spurious center-of-mass motion and we use Pierls–Yoccoz method for this purpose [19]. We also include the contribution arising from color-magnetic energy. For this, we need to compute the gluon propagator in the presence of the dielectric field [11].

2.3 *Meson self-energy and Θ width*

One more important contribution, which is not included in the previous subsection is the contribution arising from meson–quark interaction. In perturbative chiral model calculations, this contribution is essentially coming from the meson loop diagrams shown in figure 1. This contribution is often referred to as the meson self-energy correction. The self-energy computation requires the matrix element of pseudoscalar meson–quark interaction Hamiltonian. In the perturbative chiral models, this interaction Hamiltonian is

$$H_{\rm int} = \frac{im}{f}\sum_a \int {\rm d}^3 r \frac{1}{\chi(r)} \bar{\psi}(\vec{r})\gamma_5 \lambda_a^f \psi(\vec{r})\phi_a(\vec{r}), \tag{7}$$

where the summation is over the flavor index a. There is also an implied sum over the color index of quarks. Note that the interaction Hamiltonian of eq. (7) allows coupling of Θ state to a nucleon state as the quark-field operators in the interaction term can annihilate the antiquark and one of the quarks leaving a three-quark state. By construction, this state is color-neutral and therefore a nucleon state. To proceed further, we quantize the pseudoscalar meson fields in eq. (7) and evaluate the matrix element between the nucleon and Θ states. The result of this computation can be expressed in the form

$$\langle N|H_{\rm int}|\Theta\rangle = \frac{1}{(2\pi)^{3/2}} \int \frac{{\rm d}^3 k}{\sqrt{2\omega(k)}} \Big[V_a^{N\Theta}(\vec{k}) a_a(\vec{k}) + V_a^{N\Theta *}(\vec{k}) a_a^{\dagger}(\vec{k}) \Big], \tag{8}$$

where a_a and $a_a^{\dagger}$ are the annihilation and creation operators for pseudoscalar mesons, $\omega(k)$ is the energy of the mesons, and $V_a^{N\Theta}(\vec{k})$ is the form factor. In terms of the quark wave functions, the form factor is

$$V_a^{N\Theta}(\vec{k}) = \frac{4\pi i m}{f} \int r^2 {\rm d}r j_0(kr) \frac{g_u(r) g_{\bar{s}}(r) + f_u(r) f_{\bar{s}}(r)}{\chi(r)} \times A_f^{N\Theta}. \tag{9}$$

Here $A_f^{N\Theta}$ is the coefficient arising from the spin-flavor-color matrix element. The value of this turns out to be $\frac{\sqrt{6}}{2\sqrt{2}}$. The denominator is essentially the coefficient of fractional parentage (in the language of nuclear physics) of finding a nucleon and a kaon in Θ and the numerator arises from the summation over the color and spin indices of quark–antiquark pair in Θ baryon.

In order to compute the self-energy $\Sigma(M_\Theta)$ in the lowest order, we need to evaluate the self-energy diagram in figure 1 and the meson–baryon vertex here is given

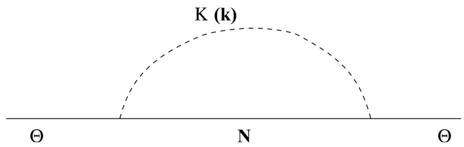

Figure 1. Θ self-energy diagram. We have nucleon and K meson in the intermediate state. The two vertices arise from meson–quark interaction $H_{\rm int}$.

in terms of the matrix elements given in eq. (8). Using 'old-fashioned' field theory, the result is

$$\Sigma(M_\Theta) = \frac{1}{(2\pi)^3} \int \frac{\mathrm{d}^3k}{\omega(k)} \frac{(V^{N\Theta}(\vec{k}))^2}{M_\Theta - \sqrt{k^2 + M_n^2} - \sqrt{k^2 + m_K^2} + i\eta}. \tag{10}$$

Note that the self-energy is computed at Θ mass. This means that one has to do a self-consistent calculation as computation of Θ mass requires the self-energy. Here we have not imposed the self-consistency and simply calculated the self-energy at the experimental value of Θ mass. It should also be noted that the Θ mass is larger than the sum of nucleon and kaon masses which means that the integrand in eq. (10) has a singularity. The integral is regularized by $i\eta$ prescription and as a result the self-energy is complex. Its imaginary part gives the width of Θ due to decay of kaon–nucleon channel. Thus,

$$\Gamma = \Im(\Sigma(M_\Theta)) = \frac{1}{2\pi^2} \frac{\sqrt{k_\Theta^2 + M_n^2}}{M_\Theta} k_\Theta (V^{N\Theta}(\vec{k}_\Theta))^2, \tag{11}$$

where $k_\Theta = \sqrt{\frac{(M_\Theta^2 - (M_n^2 + m_K^2))^2 - 4M_N^2 m_K^2}{4M_\Theta^2}}$ is the momentum of kaon in the frame in which Θ is at rest and M_Θ, M_n and m_K are Θ, nucleon and kaon masses, respectively.

The results of our calculation are discussed in the following section.

3. Results and discussions

The parameters of the modified CCDM are non-strange and strange quark masses (m and $m + m_0$ respectively), strong coupling constant $\alpha_s = g_s^2/4\pi$, bag constant B, pion decay constant f and the coupling constant of quark–vector field interaction g_V). The constant α defining the shape of the dielectric field self-energy has been chosen to be $\alpha = 12$. We find that the results are not sensitive to the value of this quantity [11]. The pion decay constant f has been set to its experimental value (93 MeV). Other parameters are adjusted to fit the masses of octet and decuplet baryons. Table 1 shows the variation of the parameters as the quark–vector field coupling is changed. We find that the glueball mass $m_{GB} = \sqrt{\frac{2\alpha B}{\sigma_v^2}}$ needs to be decreased to fit the masses of octet and decuplet baryons. This is because the quark eigenenergy (in the units of glueball mass) increases when the vector coupling constant increases. The result of this is that there is a small decrease in the quark eigenenergy of pentaquark system but the drop in the antiquark eigenenergy is much larger.

3.1 Θ *Mass*

As mentioned in the preceding section, the calculation in the present work differs from earlier calculations because of the introduction of the color-neutral vector field.

Table 1. The dielectric model parameters and non-strange quark and strange antiquark energies for different values of vector field coupling.

g_V	α_s	m_{GB}	m	m_0	ϵ_q	$\epsilon_{\bar{s}}$
0.5	0.6528	930.70	77.56	156.59	247.32	357.43
1.0	0.6505	927.02	77.25	156.00	245.40	354.46
2.0	0.6745	912.47	76.04	154.46	243.41	347.69
3.0	0.7043	890.57	74.21	152.43	240.19	337.32
4.0	0.7413	863.44	71.95	149.74	235.00	323.20
5.0	0.7869	832.99	69.42	146.84	230.44	307.80
6.0	0.8570	801.32	66.78	144.13	224.68	291.16
7.0	0.9189	769.48	64.12	140.94	220.14	274.16

We have considered the coupling constant of the vector field with the quarks as an adjustable parameter. The result of the fitting is shown in table 1. Our calculation shows that one can fit the baryon octet and decuplet masses over a wide range of the vector coupling constant and the quality of the fit seems to be independent of the coupling constant. This means that the coupling between vector field and quarks is not required for fitting octet and decuplet baryon masses.

After fitting the baryon octet and decuplet masses, we compute the Θ mass. Figure 2 displays the Θ mass as a function of the vector field-quark coupling constant. As expected, the Θ mass decreases with the increase in the coupling constant. The reason for this decrease is basically the fact that the vector field gives a repulsive contribution to the quark energy but an attractive contribution to the antiquark energy. Since we are fitting the octet and decuplet masses, the decrease in the quark eigenenergies needs to be compensated by a decrease in the scalar field mass $m_{GB} = \sqrt{\frac{2\alpha B}{\sigma_v^2}}$. But when one computes the pentaquark mass, one finds that there is a substantial decrease because of the drop in the strange antiquark mass.

First of all, the computation shows that for vanishing quark–vector field coupling, the Θ mass comes out to be 250–300 MeV larger than the experimental value. This is a general phenomenon with most of the quark models [9]. However, the Θ mass decreases with the increase in the coupling constant sufficiently so that it is possible to get the expected Θ mass of 1540 MeV when the vector coupling

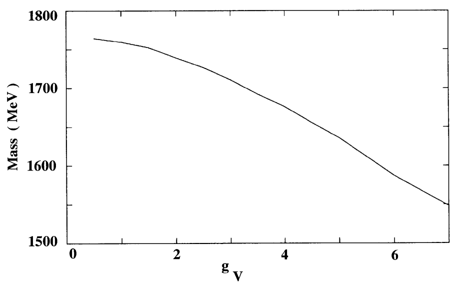

Figure 2. Θ mass (in MeV) as a function of the coupling constant for interaction between quarks and the color-neutral vector field.

constant is about 7. We therefore propose that the existence of color-neutral vector field may give an alternate mechanism for obtaining correct Θ mass. One now needs to compute the masses of other pentaquark baryons and investigate the effect of the vector field on their masses.

3.2 Θ *Width*

The formula for the width of Θ baryon is given in eq. (11). Putting the values of the masses of kaon, nucleon and Θ, we get $k_\Theta = 280$ MeV and $\frac{\sqrt{k_\Theta^2+M_n^2}}{M_\Theta=0.637}$. Thus the width of Θ becomes [20]

$$\begin{aligned}\Gamma_\Theta &= \frac{\sqrt{k_\Theta^2+M_n^2}}{2\pi^2 M_\Theta} k_\Theta |V_{N\Theta}(k_\Theta)|^2 \\ &= 9.36|V_{N\Theta}(k_\Theta)|^2. \end{aligned} \tag{12}$$

The form factor $V_{N\Theta}(k)$ is computed numerically. The result is $\Gamma_\Theta = 18.3$ MeV. The reason for this small value is essentially the small value of the integral $\int r^2 \mathrm{d}r (g_q(r)g_{\bar{s}}(r) + f_q(r)f_{\bar{s}}(r))/\chi(r)$. Although computed Γ_Θ is about twice as large as the experimental upper limit, the chiral model is able to give the result in the right ball park. This we think is satisfactory, particularly because other models [9] predict a very large width if Θ has negative parity.

4. Concluding remarks

The chiral color dielectric model has been used in this work to compute the properties of Θ pentaquark. We have considered the Θ state to be formed by putting the four quarks and a strange antiquark in $s_{1/2}$ orbital. Thus, the spin-parity of the state has been assumed to be $1/2^-$. We find that the width of the state is about twice as large as the experimental upper limit. We also find that in order to obtain an agreement with the mass, we need to introduce a color-neutral vector field and its interaction with the quark field. With this addition, it is possible to fit the Θ mass, albeit for a rather large value of the quark–vector field coupling constant. We find that this happens because the vector field generates an attractive interaction for antiquarks, thus lowering the energy of the antiquark. We are extending our calculations to compute the masses and widths of the other members of $\overline{10}$ flavor multiplet partners of Θ.

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