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Structure and reactions of pentaquark baryons

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Abstract. We review the current status of the exotic pentaquark baryons. After a brief look at experiments of both positive and negative results, we discuss theoretical methods to study the structure and reactions for the pentaquarks. First we introduce the quark model and the chiral soliton model, where we discuss the relation of mass spectrum and parity with some emphasis on the role of chiral symmetry. It is always useful to picture the structure of the pentaquarks in terms of quarks. As for other methods, we discuss a model-independent method, and briefly mention the results from the lattice and QCD sum rule. Decay properties are then studied in some detail, which is one of the important properties of Θ^+. We investigate the relation between the decay width and the quark structure having certain spin-parity quantum numbers. Through these analyses, we consider as plausible quantum numbers of Θ^+, $J^P = 3/2^-$. In the last part of this note, we discuss production reactions of Θ^+ which provide links between the theoretical models and experimental information. We discuss photoproductions and hadron-induced reactions which are useful to explore the nature of Θ^+.

Keywords. Pentaquark; quark model; chiral soliton; decay; production.

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1. Introduction

History of exotic hadrons is as old as that of the quark model [1], and the subject has been studied for a long time [2]. Yet the recent observation of the evidence of the pentaquark particle Θ^+ has triggered enormous amount of research activities both in experimental and theoretical hadron physics [3–5]. Baryons containing five valence quarks are totally new form of hadrons. The importance of knowing the nature of multi-quark states lies, for instance, in understanding the origin of matter. It is believed that in the early stage of the Universe, the matter was highly dense and was in the form of quark matter rather than ordinary hadron matter. A natural question would then be what is the mechanism of the transition from that to the present hadronic world consisting of ordinary mesons and baryons.

At this moment, the existence of the pentaquarks is still the most important issue. The whole discussions below are, therefore, based on this assumption.

From the hadron physics point of view, the understanding of five-quark systems, if they exist as (quasi-)stable states, will give us more information on the dynamics of non-perturbative QCD, such as confinement of colors and chiral symmetry breaking. Many ideas have been proposed attempting to explain the unique features of Θ^+. As it has turned out and will be discussed in this note, however, the current theoretical situation is not yet settled at all, having revealed that our understanding of hadron physics would be much poorer than we have thought [5]. We definitely need more solid ideas and methods to answer the related questions.

Turning to the specific interest in Θ^+, its would-be light mass and narrow width are the issues to be understood, together with the determination of its spin and parity. In particular, the information of parity is important, since it reflects the internal motion of the constituents.

In this lecture the following materials are discussed, with emphasis on theoretical methods.

- A quick overview on experimental situation (§2).
- Some basics of theoretical models; the quark model, chiral soliton model [3], and somewhat general treatment based on the flavor $SU(3)$ symmetry as well as a brief view over lattice and QCD sum rule studies (§3).
- Decay of Θ^+ (§4).
- Production reactions including photo- and hadronic productions (§5).

Through these discussions, we consider the possibility of Θ^+ with $J^P = 3/2^-$ as one of the likely candidates for a pentaquark state.

There are many interesting topics which cannot be discussed in this note. For readers who are interested in more details, please refer to the proceedings of the workshop PENTAQUARK04 and references therein [5].

2. Experiments

The first observation was made by the LEPS group at SPring-8 led by Nakano *et al* [4]. The backward Compton-scattered photon of energy 2.4 GeV produced at SPring-8 was used to hit a neutron target inside a carbon nucleus to produce a strangeness and antistrangeness pair (K^+ and K^-). The Fermi motion corrections were carefully analyzed, and then a missing mass analysis was performed for the K^+n final state. They have seen an excess in the K^+n invariant mass spectrum over the background at 4.6σ level in the energy region 1.54 GeV. The width of the peak was as narrow as or less than the experimental resolution (∼25 MeV). The peak was then identified with the exotic pentaquark state of strangeness $S = +1$. The absence of similar peak structure in the K^+p system suggests that the isospin of the state is likely to be $I = 0$.

The spectrum of the LEPS experiment is shown in figure 1, where the peak around 1.54 GeV is the first signal of the exotic particle [4]. Immediately after the announcement of these results, many positive signals follow [6–10]. Among them, the existence of another exotic baryon of strangeness −1, $\Xi^{0,-,--}$, was also reported [11]. Major results of experiments so far are summarized in table 1. From there, one can recognize that there is a fluctuation in absolute values of the mass of Θ^+, from 1520 to 1550 MeV. It is often said that the fluctuation of order 30 MeV is large; it is about 30% level if measured from the KN threshold.

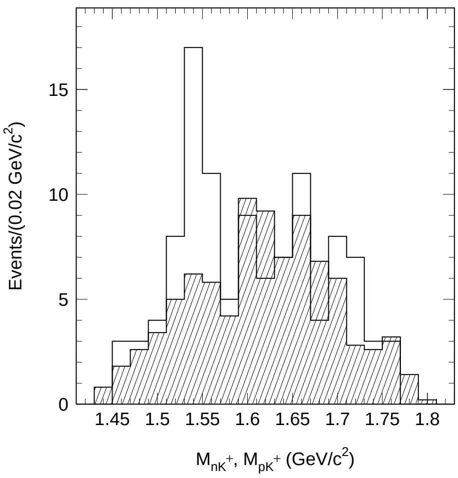

Figure 1. Invariant mass spectrum of the nK^+ extracted from the missing mass analysis of $\gamma n \to K^- K^+ n$ (unhatched histogram) and that of pK^+ extracted from the analysis of $\gamma p \to K^- K^+ p$ (hatched histogram) [4].

After many positive signals were reported, negative results also followed, mostly from the analysis of high-energy experiments [12–14]. These are also summarized in table 1. At this moment there is no theory which consistently explains these data. The high-energy experiments have much higher statistics than the low-energy experiments and should be taken seriously. If Θ^+ exists and can be seen only in the low-energy (mostly in photoproductions) experiments, one needs to understand the production mechanism [15]. A model for the suppression at high energies was proposed by Titov *et al* [16]. If it does not, we also need to understand what the signals in the low-energy experiments are for.

Very recently, CLAS (g11) reported the null result in the reaction $\gamma p \to \bar{K}^0 K^+ n$ [17]. This has much larger statistics than the previous experiment performed at SAPHIA by a factor of about twenty [9]. They extracted an upper limit of the Θ^+ production cross-section, $\sigma \lesssim$ 1–4 [nb]. The results, however, do not immediately lead to the absence of Θ^+, since there could be a large asymmetry between the reactions from the proton and neutron [18,19]. In general, photoproductions are large for charge-exchange reactions, but the reaction $\gamma p \to \bar{K}^0 K^n$ is not the one of charge exchange. Experimental studies from the neutron with higher statistics is therefore very important.

3. Theoretical methods

In this section, we discuss the structure of the pentaquarks, especially of Θ^+. Although the pioneering work of Diakonov *et al* was performed in the chiral soliton model [3], it is always instructive and intuitively understandable to work in the quark model [20,21]. After a brief look at the basics of pentaquark structure in the quark model, we discuss some essences of the chiral soliton model. Results of the

Table 1. Brief summary of the previous experiments.

Experiment	Reaction	Energy (GeV)	Mass (MeV)	Width (MeV)
LEPS	γn	$E_\gamma \sim 2.4$	$\Theta^+ \sim 1540$	$\Gamma \lesssim 25$
	γd		Being analyzed	
GRAAL	$\gamma p \to \eta p$	$E_\gamma \lesssim 1.5$	$N_5^* \sim 1715$	
			$\Theta^+ \sim 1531$	
CLAS	γp	$1.6 \lesssim E_\gamma \lesssim 2.3$	$\Theta^+ \sim 1555 \pm 10$	$\Gamma \lesssim 20$
	γd	$1.5 \lesssim E_\gamma \lesssim 3.1$	$\Theta^+ \sim 1542 \pm 5$	
CLAS(g11)	γp	$1.6 \lesssim E_\gamma \lesssim 3.8$ GeV	–	
	γd		Being analyzed	
Hall A	ep	$E_\gamma \lesssim 5$	Being analyzed	
HERMES	ed	$\sqrt{s} \sim 10$	$\Theta^+ \sim 1527 \pm 2.3$	$\Gamma = 17 \pm 9 \pm 3$
ZEUS	ep	$300 \lesssim \sqrt{s} \lesssim 319$	$\Theta^+ \sim 1521.5 \pm 1.5$	$\Gamma = 6.1 \pm 1.6^{+2}_{-1.4}$
H1	ep	$300 \lesssim \sqrt{s} \lesssim 319$	$\Theta_C^+ \sim 3100$	
NA49	pp	$\sqrt{s} \sim 17.2$	$\Xi^{--} \sim 1680$	
			$\Theta^+ = 1526 \pm 2$	
COSY-TOF	pp	$p \sim 2.95$	$\Theta^+ = 1530 \pm 5$	$\Gamma \leq 18 \pm 4$
E522(KEK)	$\pi^- p$	$p \sim 1.95$	$\Theta^+ = 1530 \pm 5$	
E690(Fermilab)	pp	$\sqrt{s} = 800$	–	
CDF(Fermilab)	$\bar{p}p$	$\sqrt{s} = 1960$	–	
BaBar	e^+e^-		–	
Belle	e^+e^-	$e^+(3.5)e^-(8)$	–	

chiral solitons are then interpreted in terms of a quark model with chiral symmetry (the chiral bag model) [22].

After the introduction of the two models, we discuss a model-independent method based on flavor $SU(3)$ symmetry, where possible spin and parity of Θ^+ are investigated [23,24]. In the last two subsections, we briefly look at the lattice QCD and QCD sum rule.

3.1 *Constituent quark model*

This model has been successfully applied to the description of the conventional mesons and baryons for their masses and various transition amplitudes [25]. In this model, a confining potential for quarks is introduced, which is usually taken to be a harmonic oscillator, to prepare basis states as single-particle states which valence quarks occupy. Then quark–quark interactions such as the spin-color interaction of one gluon exchange and the spin-flavor one of one-meson (the Nambu–Goldstone boson) exchange are introduced as residual interactions. The interaction Hamiltonian is then treated either perturbatively or diagonalized within a given model space. The role of various interactions for Θ^+ has been investigated in [21,26–28]. Here, to make the discussions simple, we consider what the structure of the five-quark states are like in a confining potential. The single-particle states of the harmonic oscillator potential are denoted by the principal and angular momentum quantum

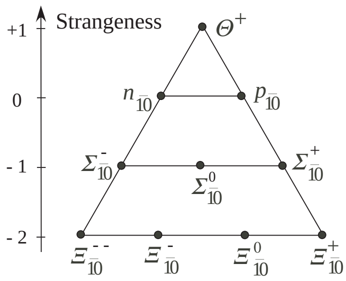

Figure 2. The weight diagram for $\overline{10}$. Particles belonging to this multiplet are also indicated.

numbers (n, l). Using the spectroscopic notation, we express them as $0s, 0p, 1s$ and so on.

Due to the many degrees of freedom of color (3), flavor (3) and spin (2), five quarks including one anti-quark ($\bar{s}$) can occupy the lowest ground state simultaneously. Thus, we denote the ground state of the five quarks as $(0s)^5$, if one quark is excited to a p-orbit, $(0s)^4 0p$, and so on. The parity of the ground state is negative, since the antiquark carries negative parity, while the parity of the first excited state is positive.

Now, let us consider the flavor structure. Under the assumption of $SU(3)$ symmetry, we need to perform irreducible decomposition of the five-quark states, the direct product of four fundamental and one conjugate representations,

$$3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3} = 1 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27 \oplus 35\,, \tag{1}$$

where multiplicities are ignored on the right-hand side. Among these representations on the right-hand side, the isosinglet state of $S = +1$ appears only in the antidecuplet representation $\overline{10}$, which is a candidate for the $SU(3)$ multiplet for the pentaquarks. The weight diagram of the $\overline{10}$ representation is shown in figure 2, where locations of various states are also indicated.

Flavor wave functions of the antidecuplet states are easily constructed, if one notices that one of the five quark is $\bar{3}$. From two $\bar{3}$'s out of two quark pairs (diquarks) in an antisymmetric combination,

$$\bar{Q}_i = \epsilon_{ijk} q_j q_k\,; \quad \bar{U} \sim [ds]\,,\ \bar{D} \sim [su]\,,\ \bar{S} \sim [ud]\,. \tag{2}$$

Then we can make symmetric products in terms of two diquarks and one antiquark, which generate antidecuplet members:

$$\begin{aligned} &\Theta^+ = \bar{S}\bar{S}\bar{s}\,, \quad N_{\overline{10}} = \frac{1}{\sqrt{3}}(\bar{S}\bar{S}\bar{u} + \bar{S}\bar{U}\bar{s} + \bar{U}\bar{S}\bar{s})\,, \\ &\Sigma^-_{\overline{10}} = \frac{1}{\sqrt{3}}(\bar{S}\bar{U}\bar{u} + \bar{U}\bar{S}\bar{u} + \bar{U}\bar{U}\bar{s})\,, \quad \Xi^{--}_{\overline{10}} = \bar{U}\bar{U}\bar{u}\,. \end{aligned} \tag{3}$$

These are analogous to the decuplet wave functions for $(\Delta, \Sigma^*, \Xi^*, \Omega)$.

What is interesting is the average number of strange (and antistrange) quarks in the wave functions. One can easily verify that it is 1 for Θ^+, 4/3 for $N_{\overline{10}}$, 5/3 for $\Sigma_{\overline{10}}$ and 2 for $\Xi_{\overline{10}}$. Namely, the strange quark content increases by equal amount 1/3 as the hypercharge decreases. This is a general consequence valid to the symmetric representation of $SU(3)$. The simple counting implies that if λ_8 is the only source of the $SU(3)$ breaking as $m_u = m_d \ll m_s$, where m_i are the constituent quark masses, the equal mass splitting of the antidecuplet baryons is expected to be $\sim(1/3)(m_s - m_u) \equiv (1/3)\Delta$. Hence we also expect

$$M(\Xi_{\overline{10}}) - M(\Theta^+) \sim m_s - m_u \sim 200\ \text{MeV}\,. \tag{4}$$

This pattern of mass splitting is shown in the left column of figure 3. The amount of total mass difference $M(\Xi_{\overline{10}}) - M(\Theta^+)$ as shown there is significantly smaller than the one originally estimated by Diakonov *et al* [3], the spectrum of which is shown on the right side of figure 3.

As pointed out by Jaffe and Wilczek [29], the antidecuplet nucleon and sigma states mix with the corresponding octet members. We will consider the mixing effects in more detail in §3.4. Here, to illustrate this mixing effect in a simple case, we also show in figure 3 the mass pattern of the octet and the ideally mixed members. Since the ideally mixed states are classified by the strange quarks, the mass splitting between neighbors is Δ. The original prediction of the chiral soliton model is close to this in values. As seen from the figure, there is significant difference in the mass patterns in the pentaquark baryons depending on the realization of the flavor $SU(3)$ symmetry (breaking).

In general, the constituent quark model cannot predict absolute values of masses. Nevertheless, if we estimate them by using typical values of constituent masses, $m_u, m_d \sim 300$ MeV and $m_s \sim 500$ MeV, we find $M_{\Theta^+} \sim 1.7$ GeV, and other masses in accordance with the equidistant rule. The mass of Θ^+ is larger than the observed values. In figure 3, however, the mass of Θ^+ is normalized.

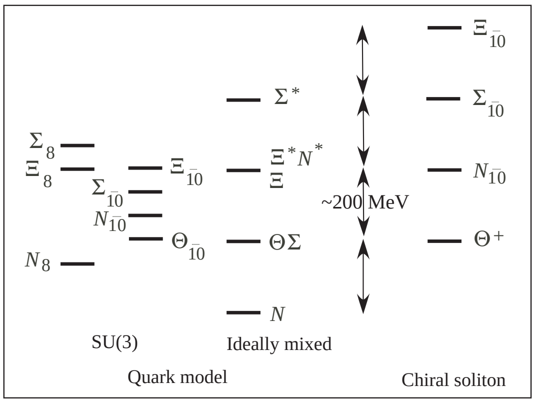

Figure 3. Expected level structure of the pentaquark baryons. The right most pattern is one from the original chiral soliton model. Absolute values of the quark model values are set such that Θ^+ appears at the same place as in the chiral soliton model.

3.2 *Chiral solitons*

This model is based on the idea of Skyrme that baryons are made from weakly interacting mesons, the solitons [30]. A microscopic basis of this model is the $1/N_c$-expansion of QCD [31–33]. The fact that there are two light flavors is also important; $SU(2)$ isospin symmetry leads to a strongly correlated pseudoscalar pion fields under rotation in the coordinate space and isospin space.

The pion field is conveniently parametrized by an $SU(2)$ matrix as

$$U(\vec{x}) = \exp(i\vec{\tau} \cdot \vec{\pi}/f_\pi)\,, \tag{5}$$

where $f_\pi = 93$ MeV is the pion decay constant. Under the strong correlation, the hedgehog configuration is realized as a static solution where the pion field points to the radial direction, $\vec{\pi}/f_\pi = \hat{r}F(r)$, where the spherical profile function $F(r)$ is determined by solving the classical field equation of motion. Nontrivial solutions for $F(r)$ define the ground states of the system. Due to nontrivial topology in the theory, different solutions $F(r)$ exist as classified by the winding number which is physically identified with the baryon number. The system of one baryon number describes the single nucleon sector.

The hedgehog solution is a classical configuration and does not correspond to a physical nucleon state. To make a link between them, we introduce the collective variables for isospin rotations $A(t) \in SU(2)$,

$$U(t, \vec{x}) = A(t)U_H(\vec{x})A(t)^\dagger\,, \quad U_H(\vec{x}) = \exp(i\vec{\tau} \cdot \hat{r}F(r))\,. \tag{6}$$

To be more precise, one needs to introduce another rotation in coordinate space, $x_i \to R_{ij}x_j$. The spatial rotation R, however, is equivalent to the isospin rotation A due to the symmetry of the hedgehog configuration; A and R cannot be independent degrees of freedom upon quantization. Consequently, the quantization of the A variable leads to the wave functions which are the $SU(2)$ D-functions for free motion in the $SU(2)$ manifold, $D^I_{t,-m}(A)$. Constraints then follow in the quantized states; spin and isospin must take the same values; $J = I$ and $t = I_z, m = J_z$ [21,34].

When this method is applied to flavor $SU(3)$, one finds several interesting consequences. We will state some of them without proof. The $SU(3)$ baryonic states are written in terms of the $SU(3)$ D-functions $D^{(p,q)}_{YII_3;Y^RI^RI^R_3}(\alpha_1, \ldots, \alpha_8)$, where the upper and lower indices label the $SU(3)$ states, and $\alpha_1, \ldots, \alpha_8$ are the Euler angles for $SU(3)$ rotations. A crucial observation here is that under the hedgehog ansatz, the right quantum numbers $Y^RI^RI^R_3$ are related to the spin and hypercharge quantum numbers. Furthermore, the Wess–Zumino term puts further constraints on the right hypercharge and the baryon number

$$(I^R, I^R_3) = (J, J_3)\,, \quad Y^R = B\,, \tag{7}$$

where the second equation holds when $N_c = 3$. From these, it follows that the number of states of $Y = 1$ is $2J + 1$. For $\overline{10}$, the state of $Y = 1$ is the nucleon and so $2J+1 = 2$, or $J = 1/2$. The parity of this state is the same as that of the nucleon, and hence the spin and parity of Θ^+ and its partner are $J^P = 1/2^+$. The mass splitting among the multiplet $\overline{10}$ is once again equidistant. In the original paper by Diakonov *et al* [3], they determined parameters in the mass formula (see eq. (9) below) from information of non-exotic sectors, then making prediction for the exotic baryons. The relatively low mass of Θ^+ was predicted this way prior to the observation. In their original work, the nucleon resonance $N(1710)$ was identified with a member of $\overline{10}$, to determine the equidistant parameter $\Delta \sim 180$ MeV. This pattern of the mass splitting is shown on the right side of figure 3.

3.3 *Role of chiral symmetry*

It is instructive to make an interpretation of the results of the chiral soliton model, especially the fact that the $1/2^+$ state appears as the lowest state of Θ^+. As it turns out, the role of chiral symmetry is important. As we have remarked in the previous subsection, a positive parity Θ^+ requires an orbital excitation of a quark to an odd parity orbit, say p-orbit ($l = 1$). This costs at least another $\hbar\omega = 500$ MeV for the mass of Θ^+. The question is then whether there is a mechanism to lower the higher state than the negative parity state of $(0s)^5$. As shown in ref. [35] the flavor dependent force due to the Nambu–Goldstone boson exchanges between quarks has a large attraction in the pentaquark state, which compensates the excess of the p-state energy.

Here we illustrate it in the chiral bag model by considering the quark single-particle states in a bag as functions of the chiral angle at the bag surface $F(R)$ (see figure 4) [23,36]. In the presence of the pion field which interact with the quarks at the bag surface, the equation of motion for the quark field is written as

$$\left(i\partial\!\!\!/ - \frac{1}{2}\exp(i\vec{\tau}\cdot\vec{\pi}(x)/f_\pi\,\gamma_5)\delta(r-R)\right)\psi = 0\,, \tag{8}$$

where the surface δ-function $\delta(r-R)$ indicates that the interaction occurs at the bag surface. In the hedgehog configuration, $\vec{\pi}(x)/f_\pi = \hat{r}F(r)$, the quark eigenstates are specified by the parity P and the grand spin which is the sum of the orbital angular momentum, spin and isospin, $\vec{K} = \vec{L} + \vec{S} + \vec{I}$. Then, the pion–quark interaction reduces to a spin–isospin interaction of the type $\vec{\sigma}\cdot\vec{\tau}$.

For a given $J = L + S = L \pm 1/2$, two K values are possible, $K = J \pm 1/2$. They are degenerate when the pion–quark interaction is zero in the large R limit as in the MIT bag model. As the bag radius is reduced and the pion–quark interaction is increased, the degeneracy is resolved. This phenomenon is similar to the spin–orbit splitting. In the present case, the state of smaller K is lowered, while the other pushed up. The change in the eigenenergies causes level crossing or a rearrangement of the pentaquark state at a certain strength of the pion–quark interaction. A crucial point is that the rearrangement occurs by the crossing of two states of opposite parities, which is followed by a flip of the parity of the pentaquark state. As shown in figure 4, for small pion–quark interaction (small $F(R)$), the five-quark configuration (denoted by the hedgehog quantum numbers K^P) is $(0^+)^4 1^+ \sim (0s)^5$, while it is replaced by the $(0^+)^4 1^- \sim (0s)^4 1p$ for $F(R) \gtrsim 0.3\pi$. In the latter, the positive parity state becomes the lowest for the pentaquark state. The parity flip occurs when the pion field is sufficiently strong. As for the nucleon, if the bag

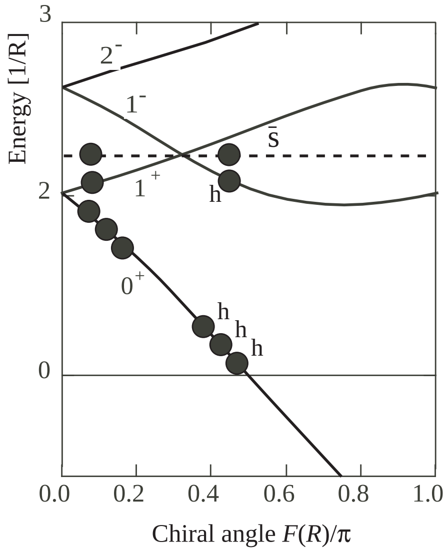

Figure 4. Quark energy levels of the chiral bag model as functions of the chiral angle F. For u, d hedgehog quarks, energy levels vary as functions of $F(R)$ as denoted by h, while the s quark is not affected by the interaction. The blobs indicate how five quarks occupy the levels.

radius of Θ^+ takes a value around $R \sim 0.6$ where the chiral angle $F(R) \sim \pi/2$ [36], the positive parity Θ^+ can be realized.

3.4 *Model independent analysis of SU*(3)

So far, we have discussed models of QCD. Instead, we can work out to a great extent by using only flavor $SU(3)$ symmetry, and derive various relations among masses and coupling constants [23,24]. The only assumption is that particles of definite spin and parity belong to certain multiplets of $SU(3)$. Symmetry puts constraints on mass and interaction Hamiltonians with several parameters, which are determined from experimental data. Since there are more physical quantities than parameters, we can make predictions. If the symmetry is only approximate, we can estimate the breaking effect either by perturbation, or by preparing a wider model space and performing diagonalization.

Our interest here is to clarify the nature of Θ^+ and its partners. If $SU(3)$ symmetry is good, they belong to pure $\overline{10}$, while if the breaking occurs (from the mass of strange quark), the nucleon and sigma states of the $\overline{10}$ start to mix with those of octet members. Presumably, we can imagine that they are also pentaquarks, which however is not a necessary condition [37]. The Ξ states do not mix because of the isospin symmetry. Therefore, we have the following set of particles to consider: $\Theta^+, (N_8, N_{\overline{10}}),\ (\Sigma_8, \Sigma_{\overline{10}}),\ \Xi\,.$

Table 2. Decay width of Θ^+ determined from the nucleon decays and the mixing angle obtained from experimental masses. Phase 1 corresponds to the same signs of g_{N_8} and $g_{\overline{10}}$, while phase 2 the opposite signs. All values are listed in MeV.

J^P	θ_N	Phase 1	Phase 2
$1/2^+$	29° (Mass)	29.1	103.3
	35.2° (Ideal)	49.3	131.8
$3/2^-$	33° (Mass)	3.1	20.0
	35.2° (Ideal)	3.9	21.3

We start with writing down the mass matrix for the antidecuplet $\overline{10}$ and octet 8,

$$H = \begin{pmatrix} M_{\overline{10}} - aY & \delta \\ \delta & M_8 - bY + c[I(I+1) - Y^2/4] \end{pmatrix}, \tag{9}$$

where the parameters are $M_8, M_{\overline{10}}, a, b, c$ and δ, while Y and I denote hypercharge and isospin. For Θ^+ and Ξ states, only the 11 element is relevant, but for N and Σ states, the full 2×2 matrix must be considered. The six parameters are determined by the six inputs from the data.

Since spin and parity are independent of flavor, we can play with different J^P's, and see how the fitting works. We have performed such fittings for $J^P = 1/2^-, 1/2^+$ and $3/2^-$, where there are sufficient number of data. For masses, the three choices of J^P work well to a similar extent.

The situation, however, changes if the method is applied to decay properties. Since the final state meson and baryon belong to the octet, there are two couplings from $\overline{10}$ and 8. Fortunately these two couplings are determined from the decay properties of two known nucleon resonances for the above three cases of J^P. Using the antidecuplet piece of the coupling constants, we can predict the decay width of Θ^+. The results are summarized in table 2, where the case of $1/2^-$ is excluded, since it gives too wide widths. Due to ambiguity in the phase of the coupling constants, we have two solutions as listed in the table. From this, we can see that the narrow decay width can be obtained in one solution of $J^P = 3/2^-$.

This result is natural, since for $3/2^-$, the final KN state is d-wave, where the centrifugal barrier suppresses the decay amplitude. The narrow width of Θ^+ could be due to the higher partial wave nature of the decaying channel.

3.5 *Lattice QCD*

The investigation of the lattice QCD was started from the early stage of the development [38,39]. Employing a baryon interpolating field with a suitable five-quark configuration, the two-point correlation function is studied. The projection into a definite parity state must also be carried out. As inspired by ref. [40], this was first performed by Sasaki [39], who found a resonance-like signal slightly above the KN threshold in the $1/2^-$ state. Recently, the significance of the signal has been somewhat weakened, being said that there is no sufficient evidence to deny resonances in the negative parity sector.

By now there are several groups who performed simulations, but their results do not always agree with each other and the issue is still controversial [41–43]. In drawing conclusions, one must know the limitation due to the approximations such as quenched approximation or finite quark mass. Instability of the results depending on the calculation scheme may indicate that the present lattice studies would not be accurate enough for the study of the pentaquark system, or that the pentaquarks might not exist.

One of the sources of different results is the use of different types of interpolating fields such as:

$$J_\Theta(x) = \epsilon_{abc}[u_a^T C\gamma_5 d_b]\{u_e(\bar{s}_e\gamma_5 d_c) \mp (u \leftrightarrow d)\}, \tag{10}$$

$$J_\Theta(x) = \epsilon_{abc}[u_a^T C\gamma_5 d_b]\{u_c(\bar{s}_e\gamma_5 d_e) \mp (u \leftrightarrow d)\}, \tag{11}$$

$$J_\Theta(x) = \epsilon_{abc}\epsilon_{aef}\epsilon_{bgh}[u_e^T C d_f][u_g^T C\gamma_5 d_h]C\bar{s}_c^T. \tag{12}$$

Ideally, if computing performance is sufficiently high, the result should not depend on the choice of the interpolating fields. In practice, results depend substantially on the choice. A possibly optimized way is to perform diagonalization of the results of different interpolating fields. Another problem which is physically important is the contamination due to the coupling to the non-resonant scattering state.

Recently, an extensive analysis was performed by Takahashi *et al*, where they considered a 2×2 matrix form of the correlation function generated by the two interpolating fields, (10) and (11), and the matrix was diagonalized to obtain states with an optimal coupling strength. Also, they investigated carefully the volume dependence [44]. They have found a resonance-like state which is rather stable against the change of the volume size in the $1/2^-$ sector. They have also studied the spectral weight factors which also support the resonance-like nature of the $1/2^-$ state. However, the resonance signal of the $1/2^-$ channel has been once again questioned in ref. [45]. Recently, higher spin states have also been investigated [46]. Definitely, further study will be needed to achieve better understanding.

3.6 *QCD sum rule*

The QCD sum rule was first applied to the pentaquarks by Zhu [47], and later by Sugiyama *et al* [40] with the proper treatment of the parity projection. In this method the two-point correlation function for the relevant baryonic state is computed in the asymptotic region in the operator product expansion (OPE). The correlation function in the asymptotic region is then analytically continued to the low-energy region to match the phenomenological spectral function. The method works reasonably well for the ground state baryons and for some resonance states, if the threshold parameter in the phenomenological side is suitably chosen. The validity for the excited states is, however, not well-tested.

The sum rule studies have been performed in many cases for the spin-1/2 sector, and signals of negative parity pentaquarks were seen around 1.5 GeV. Recently, spin-3/2 pentaquarks were also investigated [48]. The observed fact is that the OPE spectral function of the $1/2^+$ sector becomes negative or unstable [40,49], which indicates that either there is no physical state of $1/2^+$ or the present truncation of the OPE is not good enough. Also, there is a problem of contamination from the KN state. In fact, Kondo *et al*, claimed the importance of the exclusion of the KN scattering state, which may change the result of parity [50]. They have performed the separation by Fierz rearranging the operator of the type of (12) into that of KN type. Lee *et al* and Kwon *et al* also estimated a KN component in the two-point function by applying the soft kaon theorem [49,51]. Their estimation showed only a small contribution of the KN scattering state to the correlation function and therefore, the result of Sugiyama *et al* is not changed.

A fundamental question of the QCD sum rule is its applicability to the pentaquark sector, where the five-quark currents carry higher dimension than the ordinary baryon currents. In such a case, one should include higher orders of OPE in its asymptotic expansion. In this case, however, there emerge more operators of higher dimensions, the vacuum expectation values of which are not known well.

4. Decay of Θ^+

Naively, one would expect that the decay of the pentaquark Θ^+ occurs through the fall-apart process as shown in figure 5 (left). This should be so in a simple potential model for confinement. However, if, for instance, strings bind quarks, the fall-apart may not necessarily be relevant; if a string breaks by pulling two quarks apart, creation of a quark and antiquark pair must follow. Also, as discussed in ref. [52], the fall-apart decay was classified as a forbidden process; it would be interesting if there is such a selection rule which governs some particular decay processes. Here, we adopt the naive picture of the constituent quark model and test the fall-apart process [53].

For mesons and baryons of finite size, antisymmetrization among the four quarks is needed for both the initial Θ^+ and final KN scattering states. In the limit of small kaon, however, the exchange term between quarks in the kaon and nucleon can be ignored. In this case, the calculation reduces to the evaluation of the $\Theta^+ \to N$ transition matrix element of the kaon source term, or equivalently of the axial-vector current. In this section we discuss the computation of this process somewhat in detail, since such a method has not been explored much before in hadron physics.

4.1 *General remark*

Before going to actual calculations, we briefly look at the general aspect for the width of baryons. Consider a decay of Θ^+ going to the nucleon and kaon. Assuming the spin of Θ^+, $J = 1/2$, the interaction Lagrangian takes the form

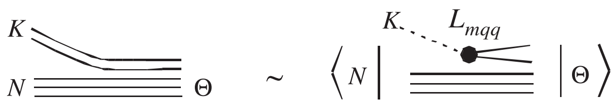

Figure 5. Decay of the pentaquark state.

$$L_{\pm} = g_{KN\Theta}\bar{\psi}_N \gamma_{\pm} \psi_\Theta K\,, \tag{13}$$

where $\gamma_+ = i\gamma_5$ if the parity of Θ^+ is positive, while $\gamma_- = 1$ if the parity of Θ^+ is negative. The decay width is then given by

$$\Gamma_+ = \frac{g^2_{KN\Theta}}{2\pi}\frac{M_N q^3}{E_N(E_N+M_N)M_\Theta}\,, \quad \Gamma_- = \frac{(E_N+M_N)^2}{q^2}\Gamma_+\,, \tag{14}$$

for the positive (+) and negative (−) parities, where M_N and M_Θ are the masses of the nucleon and Θ^+, and $E_N = \sqrt{q^2+M_N^2}$ with $\vec{q}$ being the momentum of the final-state kaon in the kaon–nucleon center of mass system, or equivalently in the rest frame of Θ^+. The difference between $\Gamma_\pm$ arises due to the different coupling nature: p-wave coupling for positive parity Θ^+ and s-wave coupling for negative parity Θ^+, representing the effect of the centrifugal repulsion in the p-wave. In the kinematical point of the Θ^+ decay, $M_\Theta = 1540$ MeV, $M_N = 940$ MeV and $m_K = 490$ MeV, the factor on the right-hand side of (14) becomes about 50, which brings a significant difference in the widths of the positive and negative parity Θ^+. If we take $g_{KN\Theta} \sim 10$ as a typical strength for strong interaction coupling constants, we obtain $\Gamma_+ \sim 100$ MeV, while $\Gamma_- \sim 5$ GeV. Both numbers are too large when compared with experimentally observed width. Therefore, the relevant question is whether some particular structure of Θ^+ will suppress the above naive values, or not.

4.2 *Calculation of decay amplitudes*

The matrix element of a fall-apart decay is written as a product of the spectroscopic factor and an interaction matrix element,

$$\mathcal{M}_{\Theta^+\to KN} = S_{KN \text{ in } \Theta^+} \cdot h_{\text{int}}. \tag{15}$$

The factor $S_{KN \text{ in } \Theta^+} \equiv \langle (\bar{s}q)_K (qqq)_N | \Theta^+ \rangle$ is a probability amplitude of finding in the pentaquark state three-quark and quark–antiquark clusters having the quantum numbers of the nucleon and kaon, respectively.

Calculations of this factor was performed in refs [53,54]. It strongly depends on the internal structure of $|\Theta^+\rangle$. Here we have investigated the four cases; one for the state of $(0s)^5$ of $J^P = 1/2^-$, and the other three for the $(0s)^4 0p$ of $J^P = 1/2^+$. The $(0s)^5$ configuration is unique, while there are four independent states for the $(0s)^4 0p$ configurations. Among them, we study the one minimizing the spin-flavor interaction of the type $\sum_{i>j}(\sigma_i\sigma_j)(\lambda_i^f\lambda_j^f)$ (SF), the one minimizing the spin-color interaction $\sum_{i>j}(\sigma_i\sigma_j)(\lambda_i^c\lambda_j^c)$ (SC), and the one of the strong diquark correlations as proposed by Jaffe and Wilczek (JW) [29]. The resulting spectroscopic factors are summarized in table 3.

Now the interaction matrix element can be computed by the meson–quark interaction of Yukawa type:

$$L_{mqq} = ig\bar{q}\gamma_5\lambda_a\phi^a q\,, \tag{16}$$

Table 3. Spectroscopic factors and decay widths (in MeV) of Θ^+ for $J^P = 1/2^\pm$. For notations SF, SC and JW, see text.

		$J^P = 1/2^+$		
	$J^P = 1/2^-$	SF	SC	JW
S-factor	$1/2\sqrt{2}$	$\sqrt{5/96}$	$\sqrt{5/192}$	$\sqrt{5/576}$
Γ (MeV)	890	63	32	11

where λ_a are $SU(3)$ flavor matrices and ϕ^a are the octet meson fields. The coupling constant g may be determined from the pion–nucleon coupling constant $g_{\pi NN} = 5g$. Therefore, using $g_{\pi NN} \sim 13$, we find $g \sim 2.6$. We have calculated the $\Theta^+ \to N$ matrix elements of (16) in the non-relativistic quark model.

Further details of calculation can be found in ref. [53], and here several results are summarized as follows. For the negative parity state of $(0s)^5$, the decay width turns out to be of the order of several hundreds MeV or more, typically 0.5∼1 GeV. In the calculation it has been assumed that the spatial wave function for the initial and final state hadrons are described by a common harmonic oscillator states. Also the masses of the particles are taken as experimental values, e.g., $M_{\Theta^+} = 1540$ MeV. For the result of the negative parity state of $(0s)^5$, the unique prediction can be made, since there is only one quark model state, meaning that the state can be expressed in terms of the totally antisymmetrized KN state. Since there is not a centrifugal barrier, that state is hardly identified with a resonant state with a narrow width.

For the positive parity state, we obtain $\Gamma = 63$ MeV, 32 MeV and 11 MeV, for the SF, SC and JW configurations, respectively. The diquark correlation of JW develops a spin-flavor-color wave function having a small overlap with the decaying channel of the nucleon and kaon. In the evaluation of these values, we did not consider spatial correlations. However, if, for instance, small diquarks are developed, spatial overlap becomes less than unity which further suppresses the decay width. In ref. [55], such suppression was shown to be significant.

The small values of the decay width for $J^P = 1/2^+$ when compared with the large values for $J^P = 1/2^-$ can be explained by the difference in the coupling structure; one is the pseudoscalar type of $\vec{\sigma} \cdot \vec{q}$ and the other the scalar type of 1. The former of the p-wave coupling includes a factor $q/(2M)$ which suppresses the decay width significantly when compared with the latter at the present kinematics, $q \sim 250$ MeV and $M \sim 1$ GeV, when the same coupling constant $g_{NK\Theta}$ is employed.

The present analyses can be extended straightforwardly to the Θ^+ of spin 3/2. For the negative parity state, the spin-1 state of the four quarks in the Θ^+ may be combined with the spin of $\bar{s}$ for the total spin 3/2. In this case the final KN state must be in a d-wave state, and therefore, the spectroscopic factor of finding a d-wave KN state in the $(0s)^5$ is simply zero. If a tensor interaction induces an admixture of a d-wave configuration, it can decay into a d-wave KN state. However, the mixture of the d-wave state is expected to be small just as for the deuteron. There could be a possible decay channel of the nucleon and the vector K^* of $J^P = 1^-$ [56]. This decay, however, does not occur since the total mass of the decay channel is larger

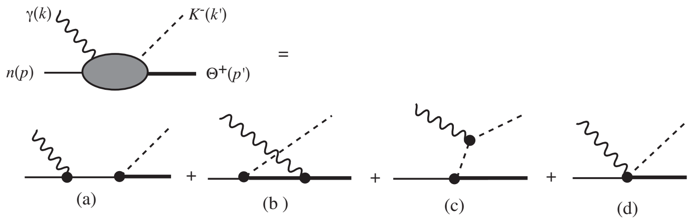

Figure 6. Born diagrams for the Θ^+ photoproduction.

than the mass of Θ^+. Hence the $J^P = 3/2^-$ state could be another candidate for the observed narrow state. This state does not have a spin–orbit partner and forms a single resonance peak around its energy. For the positive parity case, the p-state orbital excitation may be combined with the spin of $\bar{s}$ for the total spin 3/2. In this case, the calculation of the decay width is precisely the same as before (see ref. [53] for more details). After taking the average over the angle $\vec{q}$, however, the coupling yields the same factor as for the case $J = 1/2$. Hence the decay rate of spin-3/2 Θ^+ is the same as that of Θ^+ of spin 1/2 in the present treatment, if the mass of the $3/2^+$ state is the same as the $1/2^+$ state.

5. Production of Θ^+

The Θ^+ production from the nonstrange initial hadrons is furnished by the creation of $s\bar{s}$ pair, which requires energy deposit of around 1 GeV. In general, the reaction mechanism of such energy region is not well-understood. However, as one of the practical methods, we adopt an effective Lagrangian approach and perform computations of Born (tree) diagrams. Input parameters in the Lagrangians reflect the properties of Θ^+ and therefore, the comparison of calculations and experiments will help one to study the structure of Θ^+. In particular, extraction of spin and parity is an important purpose in the study of reactions. Here we briefly discuss (1) photoproduction as originally performed in the experiment by the LEPS group [57], and (2) $\Theta^+\Sigma^+$ production induced by the polarized $\vec{p}\vec{p}$ for the determination of the parity [58].

5.1 *Photoproduction*

As described in detail in ref. [57], in the effective Lagrangian method we calculate the Born (tree) diagrams as depicted in figure 6. The actual form of the interaction Lagrangian depends on the interaction schemes, i.e., either pseudoscalar (PS) or pseudovector (PV). In the PS, the three Born diagrams (a)–(c) are computed with the gauge symmetry maintained. In the PV, on the contrary, the contact Kroll–Ruderman term (d) is also necessary. In the PS scheme, the contact term may be included in the antinucleon contribution of the nucleon Z-diagram. If chiral symmetry is respected, the low-energy theorem guarantees that the two schemes provide the same result in the low-energy limit. In reality, due to the large energy deposit of order 1 GeV, the equivalence is violated. It is shown that the difference in the two schemes is proportional to the photon momentum in the first power (which therefore vanishes in the low-energy limit) and to the anomalous magnetic moment of Θ^+ [57].

In order to express the finite size effect of the nucleon, we need to consider the form factor. Here we adopt a gauge invariant one with a four-momentum cut-off [59]. This form factor suppresses the nucleon pole contributions in the PS scheme (and hence the contact term also in the PV scheme), as reflecting the fact that the nucleon intermediate state is far off-shell. Consequently, the dominant contribution is given by the t-channel process of the kaon exchange and/or K^* meson exchange. The ambiguity of the anomalous magnetic moment of Θ^+ is also not important. Therefore, the difference between the PS and PV schemes is significantly suppressed when kaon exchange term is present as for the case of the neutron target. This allows one to make rather unambiguous theoretical predictions.

We have computed the photoproduction of Θ^+ from the neutron and proton, and first for $J^P = 1/2^{\pm}$. Here are several remarks:

1. When the decay width $\Gamma_{\Theta^+ \to KN} = 15$ MeV is used, the typical total cross-section values are about 100 [nb] for the positive parity and about 10 [nb] for the negative parity. Since the total cross-section is proportional to the decay width $\Gamma_{\Theta^+ \to KN}$, experimental information on the decay width is important to determine the size of cross-sections, or *vice versa.* For instance, for a decay width of about 1 MeV or less, the total cross-sections will be of the order of 10 [nb] or less for the positive parity and 1 [nb] or less for the negative parity. In general the cross-sections are about ten times larger for the positive parity Θ^+ than for the negative parity. The p-wave coupling $\vec{\sigma}\cdot\vec{q}$ effectively enhances the coupling strength by factor of 3–4 as compared with the s-wave coupling for the negative parity, when the momentum transfer amounts to 1 GeV.
2. For the neutron target, the kaon exchange term is dominant. In this case, the K^* contributions are not important even with a large $K^*N\Theta$ coupling $|g_{K^*N\Theta}| = \sqrt{3}|g_{KN\Theta}|$ [60]. Hence the theoretical prediction for the neutron target is relatively stable. The angular dependence has a peak at $\theta \sim 60^\circ$ in the center-of-mass system, a consequence of the vertex structure of the γKK vertex in the kaon exchange term. Since this feature is common to both parities, the difference in the parity of Θ^+ may not be observed in the angular distribution.
3. The kaon exchange term vanishes for the case of the proton target. Therefore, the amplitude is a coherent sum of various Born terms, where the role of the K^* exchange is also important. The theoretical prediction for the proton target is therefore rather difficult.

The recent CLAS collaboration has reported no significant evidence of Θ^+ in the reaction $\gamma p \to \bar{K}^0\Theta^+$ [17]. This result should be taken seriously, because they have achieved significantly higher statistics when compared to the previous experiments performed close to the threshold. What could then be the fate of Θ^+? Yet their result does not lead to the absence of Θ^+ immediately, because the previous positive evidences were seen mostly in the reactions from the neutron. Due to the violation of isospin symmetry in the electromagnetic interaction, there could be asymmetry in the reactions from the proton and neutron. A well-known example is the Kroll–Ruderman term in the pion photoproduction, which survives only in the charge exchange channels such as $\gamma p \to \pi^+ n$.

We have performed a calculation once again using an effective Lagrangian, but with $J^P = 3/2^\pm$ Θ^+ [19]. There is a significant difference between 1/2 and 3/2 cases; only PV formalism is possible for the latter case. Without the equivalence between the PS and PV, the role of the contact term in the PV scheme is very much different for the two-spin cases. In the 3/2 case, the contact term dominates and the production rate from the neutron is large but that from the proton is strongly suppressed. Using the decay width $\Gamma \sim 1$ MeV, the cross-section of $\gamma p \to \bar{K}^0 \Theta^+$ was estimated to be a few [nb] which does not contradict the CLAS data. Further information from the neutron target is crucially important to settle the problem of the pentaquarks.

5.2 *Polarized proton beam and target*

This reaction was considered in order to determine the parity of Θ^+ unambiguously, independent of any reaction mechanisms [58,61]. In the photoproduction case, the determination of parity is also possible if we are able to control the polarization of both the initial and final states [62], which is however very difficult in the present experimental set-up.

The system of two protons provides a selection rule due to Fermi statistics. Since the isospin is $I = 1$, the spin and angular momentum of the initial state must be either $(S, L) = (0, \text{even})$ or $(S, L) = (1, \text{odd})$. Now consider the reaction

$$\vec{p} + \vec{p} \to \Theta^+ + \Sigma^+ \tag{17}$$

at the threshold region, where the relative motion in the final state is in s-wave. It is shown that if the initial spin state has $S = 0$, then the parity of the final state is positive and hence the parity of Θ^+ *must be* positive. Likewise, if $S = 1$ the parity of Θ^+ *must be* negative. This idea is similar to the one used to determine the parity of the pion [63].

One can compute production cross-sections by employing an effective Lagrangian of the kaon and K^* exchange model. The strength of the $KN\Theta^+$ vertex is determined once again for $\Gamma = 15$ MeV. The $K^*N\Theta^+$ vertex is expressed as the sum of the vector and tensor terms. The vector and tensor couplings are unknown, but here we take their strengths to be $|g^V_{K^*N\Theta}| = (1/2)|g^T_{K^*N\Theta}| = |g_{KN\Theta}|$. The signs are then tested for all four possible cases to see the effect of the couplings. For the $KN\Sigma$ and $K^*N\Sigma$ coupling, we take the phenomenological one from the Nijmegen potential [64]. The monopole form factor is then introduced at each vertex, with the same cut-off parameter $\Lambda = 1$ GeV for simplicity. The choice of the interaction parameter is important for such high-momentum transfer reactions. It should reflect the structure of the nucleon which has the size of order 0.5 fm or larger. Hence $\Lambda \sim 1$ GeV is crudely the upper limit which is compatible with nucleon size $\lesssim 0.5$ fm. In fact, the parameters of the Nijmegen soft core potential are chosen by such a consideration.

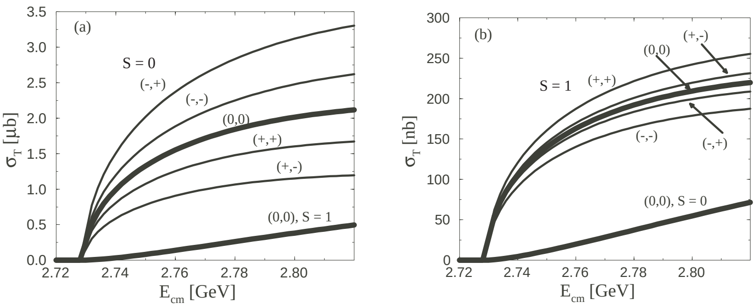

Figure 7. Total cross-sections for $pp \to \Theta^+\Sigma^+$, (**a**) for the positive parity and (**b**) for the negative parity, as functions of the center-of-mass energy. Different curves correspond to different unknown coupling $K^*N\Theta$ [57].

The results are shown in figure 7 for both positive and negative parity Θ^+. The selection rule for the positive and negative parity Θ^+ is shown clearly by the energy dependence at the threshold region. For the allowed channel the final state is in an s-wave with the energy dependence from the threshold $(E - E_{\rm th})^{1/2}$, whereas for the forbidden channel the partial wave of the final state is p-wave with the energy dependence $(s - s_{\rm th})^{3/2}$.

Recently COSY-TOF reported the result for the $\Theta^+\Sigma^+$ production in the unpolarized pp scattering at $p_p = 2.95$ GeV/c [65]. They quote the total cross-section $\sigma \sim 0.4 \pm 0.2$ [μb] at 30 MeV above the threshold in the center-of-mass energy. In comparison with theory, if we adopt a narrower width of about 5 MeV, the cross-section will be about 0.5 [μb] for the positive parity and 0.05 [μb] for the negative parity. This comparison seems to favor the positive parity Θ^+. Although there remain some ambiguities in theoretical calculations, such a comparison of the total cross-section will be useful to distinguish the parity of Θ^+.

An alternative quantity which is powerful for the determination of the parity is the spin-polarized quantity as defined by

$$A_{xx} = ({}^3\sigma_0 + {}^3\sigma_1)/(2\sigma_0) - 1\,, \tag{18}$$

where σ_0 and σ_1 are the cross-sections for the spin singlet and triplet states of the two protons. By taking the ratio of the two cross-sections, ambiguities of various coupling constants and form factors are nearly cancelled. The observation of A_{xx} as well as the energy dependence of the polarized cross-sections will provide the best opportunity to determine the parity of Θ^+ [66,67].

6. Summary

In this note we have discussed several aspects of the pentaquark baryon Θ^+ including its structure and production reactions. Among various properties of Θ^+, the importance of the spin and parity in relation with the decay width has been emphasized. The relevant points are as follows:

1. The chiral force may change the quark energy levels; with a sufficient strength, an $l = 1$ orbit may be lower than the $l = 0$ orbit. Consequently, a positive parity state can be the lowest pentaquark state. Better understanding of the role of the Nambu–Goldstone bosons is very important.
2. The decay of the pentaquark state through the fall-apart process is sensitive to the internal quark structure, especially to the spin and parity. It was shown that for $J^P = 1/2^-$, the naive ground state of $(0s)^5$ can no longer survive as a narrow resonance as the decay width is unphysically large. In fact, the uniqueness of the $(0s)^5$ configuration implies that the configuration can be written in terms of the KN state which can no longer be bound in the naive quark model where the confining force vanishes between the two-color singlet states. On the contrary, the decay widths of $1/2^+$ states were obtained to be of the order of 10 MeV, but with once again strong dependence on the configuration. We have also commented on the possibility of higher spin states, especially $J = 3/2$. The $J^P = 3/2^-$ state could be an interesting alternative possibility.
3. As recently pointed by Hiyama *et al* [68], the coupling to the KN decay channel is extremely important when considering the five-body system seriously; it could change the nature of a confined pentaquark configuration completely. Since the five-quark configuration must have components of two-color singlet hadrons, one (or some) of them can be a decay channel(s), unless there are particular selection rules. The $J^P = 1/2^\pm$ states are not subject to any such selection rules and must be accompanied by such a coupling to the two-hadron (KN) scattering state. In such a case, a coupled channel treatment is mandatory. In contrast, in the $J^P = 3/2^-$ state, unless there is d-wave mixing with the $(0s)^5$ configuration, the angular momentum conservation forbids the coupling of the state with the decay channel.
4. So far, our understanding of the reaction mechanism for the pentaquark production is rather limited. Perhaps, the best we can do is to use an effective Lagrangian approach with a reasonable choice of model parameters.
5. In the photoproduction of Θ^+, a large asymmetry between the reactions from the proton and neutron targets was found, especially when $J = 3/2$ [19]. In relation with the understanding of the narrow decay width, we once again mention that higher spin state would be an interesting possibility.
6. In order to determine the parity of the pentaquark, the polarized proton scattering $\vec{p}\vec{p} \to \Theta^+\Sigma$ provides a model-independent method. Measurement of such a reaction is extremely important to further explore the physics of pentaquarks.

The current situation for the pentaquarks is not settled. Of course, the experimental confirmation is the most important issue. However, we have also seen that different theoretical approaches make different predictions. These facts imply that there could be more aspects that we do not know yet well about the low-energy QCD. Perhaps the pentaquark has provided us with an ideal opportunity to explore further challenges to the problems.

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