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# Plasma–maser instability of the ion acoustics wave in the presence of lower hybrid wave turbulence in inhomogeneous plasma

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**Abstract.** A theoretical study is made on the generation mechanism of ion acoustics wave in the presence of lower hybrid wave turbulence field in inhomogeneous plasma on the basis of plasma–maser interaction. The lower hybrid wave turbulence field is taken as the low-frequency turbulence field. The growth rate of test high frequency ion acoustics wave is obtained with the involvement of spatial density gradient parameter. A comparative study of the role of density gradient for the generation of ion acoustics wave on the basis of plasma–maser effect is presented. It is found that the density gradient influences the growth rate of ion acoustics wave.

**Keywords.** Weak turbulence theory; wave–particle interaction; resonant and non-resonant waves; growth rate; density gradient.

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## 1. Introduction

Plasma–maser instability is a nonlinear wave–particle interaction in plasma turbulence [1]. It occurs when nonresonant as well as resonant oscillations are present in the system. The resonant waves are those for which the Cherenkov resonance condition $\omega - \vec{k}\cdot\vec{v} = 0$ is satisfied, whereas the nonresonant waves are those for which both Cherenkov and the nonlinear scattering conditions are not satisfied, i.e. $\Omega - \vec{K}\cdot\vec{v} \neq 0$ and $(\Omega-\omega) - (\vec{K}-\vec{k})\cdot\vec{v} \neq 0$. Here $\omega$ and $\Omega$ are the frequencies of the resonant and nonresonant waves, respectively, and $\vec{k}$ and $\vec{K}$ are the corresponding wave numbers. It has been established that, plasma–maser effect is found to be effective in an open system, where free energy from external sources are available [2] (in the form of continuous beam injection, external magnetic field etc.). It corresponds to vortex correction and is effective without electron population inversion which differs it markedly from other wave–particle interaction processes (e.g., inverse nonlinear Landau interaction). In a closed plasma system the plasma–maser instability vanishes because of exact cancellation with the reverse absorption process due to the quasi-linear interaction [3] and in this system of plasma [4], the contribution of time and space variation of the distribution function is essential.

In almost all the previous studies carried out on plasma–maser effect, the spatial inhomogeneity of plasma has been ignored. But the space plasma is basically inhomogeneous. In magnetosphere, there are various plasma regions of strong density gradients as well as confining magnetic field gradients. Moreover, under laboratory conditions complete homogeneity of confined plasma cannot be achieved. In any realistic situation, effect of spatial gradient cannot be ignored. Analysis of wave phenomena in inhomogeneous plasma was formulated by Mikhailovoskii [5]. In view of practical importance, effect of density gradient has been studied with great importance in the recent years. Recently, Vranjes and Poedts [6] have pointed out the modification of ion cyclotron frequency due to density gradients in the system.

In this paper, we have considered the plasma–maser interaction of ion acoustics wave with the low-frequency lower hybrid wave turbulence. Since, in the inhomogeneous plasma the drifting particles may provide free source of momentum to the system, it is important to clarify the role of free energy of these drifting particles in this particular mode–mode coupling process of turbulent inhomogeneous plasma. In this study, we have estimated the growth rate of ion acoustics wave through the plasma–maser effect in the presence of lower hybrid wave turbulence and investigated the role of free energy due to drifting particles in inhomogeneous plasma.

## 2. Formulation

We consider inhomogeneous plasma with spatial gradient in the $y$-direction. The confining magnetic field with negligibly small gradient is taken along the $\vec{z}$ direction, $\vec{B}_0 = B_0(y)$. For such an inhomogeneous system [7] the particle distribution function is considered as

$$\begin{aligned} f_0\left(v_\perp, v_\parallel, y + \frac{v_x}{\Omega_e}\right) &\simeq f(v,y) + \frac{v_x}{\Omega_e}\frac{\partial f_{0e}}{\partial x} \\ &\simeq f_{0e}(\vec{v},y)\left\{1 + \frac{v_x}{\Omega_e}\epsilon'\right\}, \end{aligned} \tag{1}$$

where

$$\epsilon' = \left[\frac{1}{f_{0e}}\frac{\partial f(v,y)}{\partial y}\right]_{y=0} \tag{2}$$

is the density gradient, $f(v,y)$ is the distribution function for guiding center, $v_\perp$ and $v_\parallel$ are velocity components along and perpendicular to the external magnetic field, $\phi$ is the phase angle of the particle in the orbit and $\Omega_e$ is the electron cyclotron frequency.

The interaction of high-frequency ion acoustics wave with low-frequency lower hybrid wave turbulence is goverened by Vlasov–Poisson equations

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial\vec{r}}-\frac{e}{m}\left(\vec{E}+\frac{\vec{v}\times\vec{B}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\right]f_{0e}(\vec{r},\vec{v},t)=0 \tag{3}$$

and

$$\vec{\nabla}\cdot\vec{E}=-4\pi en_e\int f(\vec{r},\vec{v},t)\mathrm{d}v. \tag{4}$$

According to the linear response theory of a turbulent plasma [8], the unperturbed electron distribution function and fields are

$$F_{0e}=f_{0e}+\epsilon f_{1e}+\epsilon^2 f_{2e} \tag{5}$$

and

$$\vec{E}_{0e}=\epsilon\vec{E}_l, \tag{6}$$

where $\epsilon$ is a small parameter associated with turbulence field $\vec{E}_l=(0,0,E_{l\|})$ with propagating vector $\vec{k}=(0,0,k_\|)$, $f_{0e}$ is the space and time averaged part and $f_{1e}$,$f_{2e}$ are fluctuating parts of the distribution function.

To the order of $\epsilon$ from eq. (3), we have

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial\vec{r}}-\frac{e}{m}\left(\frac{\vec{v}\times\vec{B}_0}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\right]f_{1e}=\frac{e}{m}\left(\vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}f_{0e}\right). \tag{7}$$

To find $f_{1e}$ we use Fourier transforms of the form

$$A(\vec{r},\vec{v},t)=\sum_{\vec{k},\omega}A(\vec{k},\omega)\exp[i(\vec{k}\cdot\vec{r}-\omega t)]. \tag{8}$$

From eq. (7), we have

$$f_{1e}(\vec{k},\omega)=\left(\frac{ie}{m}\right)\frac{E_{l\|}(\vec{k},w)\frac{\partial}{\partial v_\|}f_{0e}}{w-k_\|v_\|+i0}, \tag{9}$$

where $i0$ is a small imaginary part of lower hybrid wave frequency $\omega$ and $\vec{k}$ is the corresponding wave number.

We now perturb the quasi-steady state by a high-frequency test ion acoustics wave field $\mu\delta\vec{E}_h$ with wave vector $\vec{K}=(0,K_\perp,K_\|)$. The total perturbed electric field, magnetic field and the electron distribution function are

$$\begin{aligned}&\delta\vec{E}=\mu\delta\vec{E}_h+\mu\epsilon\delta\vec{E}_{lh}+\mu\epsilon^2\Delta\vec{E},\\&\delta\vec{B}=0,\\&\delta f=\mu\delta f_h+\mu\epsilon\delta f_{lh}+\mu\epsilon^2\Delta f,\end{aligned} \tag{10}$$

where $\delta\vec{E}_{lh}$, $\Delta\vec{E}$ are the modulation electric fields, $\delta f_h$ is the fluctuating part due to high-frequency ion acoustics wave, $\delta f_{lh}$ and $\Delta f$ are electron distribution functions corresponding to modulation fields. In eq. (9),we have omitted the second-order field quantities, which is justified under random phase approximations. Linearizing the Vlasov eq. (3) to the order $\mu, \mu\epsilon, \mu\epsilon^2$, we obtain

$$P\delta f_h = \frac{e}{m}\delta\vec{E}_h \cdot \frac{\partial}{\partial\vec{v}} f_{0e}, \tag{11}$$

$$P\delta f_{lh} = \frac{e}{m}\vec{E}_l \cdot \frac{\partial}{\partial\vec{v}}\delta f_h + \frac{e}{m}\delta\vec{E}_h \cdot \frac{\partial}{\partial\vec{v}} f_{1e} + \frac{e}{m}\delta\vec{E}_{lh} \cdot \frac{\partial}{\partial\vec{v}} f_{1e}, \tag{12}$$

$$P\Delta f = \frac{e}{m}\vec{E}_l \cdot \frac{\partial}{\partial\vec{v}}\delta f_{lh} + \frac{e}{m}\delta\vec{E}_{lh} \cdot \frac{\partial}{\partial\vec{v}} f_{1e}, \tag{13}$$

where the operator $P$ is given by

$$P = \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial\vec{r}} - \frac{e}{m}\left(\frac{\vec{v}\times\vec{B}_0}{c}\right) \cdot \frac{\partial}{\partial\vec{v}}.$$

Using the method of characteristics [9], we solve these equations using Fourier transform given by eq. (8) for the fluctuating parts of the distribution function $\delta f_h$, $\delta f_{lh}$ and $\Delta f$ over the electron trajectories.

In obtaining the other fluctuating parts of the perturbed distribution function, we use

$$\frac{\partial f_{0e}}{\partial\vec{v}} \simeq \left(-\vec{v}\frac{m}{T_e} + \hat{x}\frac{\epsilon'}{\Omega_e}\right) f_{0e}. \tag{14}$$

Using the Fourier transform of eq. (8) and integrating along the unperturbed orbits we evaluate the various perturbed distribution functions from eqs (11)–(13) to obtain the nonlinear dielectric function of the ion-acoustics wave frequency $\Omega$, in the presence of the lower hybrid wave turbulence

$$\begin{aligned} \delta f_h(\vec{K},\Omega) = \frac{ie}{m}\frac{\delta E_h}{K_\perp}\frac{m}{T_e}\Bigg[1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right) \\ \times \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp i(a-b)\theta'}{a\Omega_e + K_\parallel v_\parallel - \Omega}\Bigg] f_{0e}, \end{aligned} \tag{15}$$

where $\alpha = K_\perp v_\perp/\Omega_e$ and $\theta' = \frac{\pi}{2} - \theta$.

$$\begin{aligned} \delta f_{lh}(\vec{K}-\vec{k},\Omega-\omega) &= \frac{e}{m}\int_{-\infty}^{0}\left[\vec{E}_l \cdot \frac{\partial \delta f_h}{\partial\vec{v}} + \delta\vec{E}_h \cdot \frac{\partial f_{1e}}{\partial\vec{v}} + \delta\vec{E}_{lh} \cdot \frac{\partial f_{0e}}{\partial\vec{v}}\right] \mathrm{d}\vec{v} \\ &= \frac{e}{m}\int_{-\infty}^{0}\left[\vec{E}_l \cdot \frac{\partial \delta f_h}{\partial\vec{v}} + \delta\vec{E}_h \cdot \frac{\partial f_{1e}}{\partial\vec{v}} + \delta\vec{E}_{lh} \cdot \frac{\partial f_{0e}}{\partial\vec{v}}\right] \\ &\quad \times \exp(i[(\vec{K}-\vec{k})\cdot(\vec{r}\,'-\vec{r}) - (\Omega-\omega)\tau])\mathrm{d}\tau \\ &= I_1 + I_2 + I_3, \end{aligned} \tag{16}$$

where

$$\begin{aligned}
I_1 &= \frac{e}{m}\int_{-\infty}^{0}\left(\vec{E}_l\cdot\frac{\partial\delta f_h}{\partial\vec{v}}\right)\exp[i\{(\vec{K}-\vec{k})\cdot(\vec{r}'-\vec{r})-(\Omega-\omega)\tau\}]\mathrm{d}\tau \\
&= -\left(\frac{ie}{m}\right)E_{l\|}\frac{\partial}{\partial v_\|}\left[\frac{ie}{m}\frac{\delta E_h}{K_\perp}\frac{m}{T_e}\left(1+\left\{\Omega-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\right.\right. \\
&\quad \left.\left.\times\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-\Omega+K_\|v_\|}\right)f_{0e}\right] \\
&\quad \times\left(\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp i(q-p)\theta'}{p\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\right), \qquad (17)
\end{aligned}$$

$$\begin{aligned}
I_2 &= \frac{e}{m}\int_{-\infty}^{0}\delta\vec{E}_h\cdot\frac{\partial f_{1e}}{\partial\vec{v}}\exp[i\{(\vec{K}-\vec{k})\cdot(\vec{r}'-\vec{r})-(\Omega-\omega)\tau\}]\mathrm{d}\tau \\
&= -\frac{ie}{m}\frac{\delta E_h}{K_\perp}\left[-\frac{m}{T_e}\left(\frac{ie}{m}\right)\left(\frac{E_{l\|}(\vec{k},w)\frac{\partial}{\partial v_\|}f_{0e}}{w-k_\|v_\|+i0}\right)\right. \\
&\quad \times\left[1+\left\{(\Omega-\omega)-(K_\|-k_\|)v_\|-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\right] \\
&\quad \times\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|} \\
&\quad +K_\|\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|} \\
&\quad \left.\times\frac{\partial}{\partial v_\|}\left\{\left(\frac{ie}{m}\right)\left(\frac{E_{l\|}(\vec{k},w)\frac{\partial}{\partial v_\|}f_{0e}}{w-k_\|v_\|+i0}\right)\right\}\right], \qquad (18)
\end{aligned}$$

$$\begin{aligned}
I_3 &= \frac{e}{m}\int_{-\infty}^{0}\left(\vec{E}_{lh}\frac{\partial f_{0e}}{\partial\vec{v}}\right)\exp[i\{(\vec{K}-\vec{k})\cdot(\vec{r}'-\vec{r})-(\Omega-\omega)\tau\}]\mathrm{d}\tau \\
&= \left(\frac{ie}{m}\frac{\delta E_{lh}(\vec{K}-\vec{k})}{|\vec{K}_\perp-\vec{k}_\perp|}\frac{m}{T_e}\left[1+\left\{(\Omega-\omega)-\frac{\epsilon' T_e K'_\perp}{m\Omega_e}\right\}\right.\right. \\
&\quad \left.\left.\times\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(a-b)\theta']}{a\Omega_e+(K_\|-k_\|)v_\|-(\Omega-\omega)}\right]\right)f_{0e}. \qquad (19)
\end{aligned}$$

Making use of the results (17), (18) and (19), we can write the expression for $\delta f_{lh}$.

Now from eq. (12), we have

$$\begin{aligned}
\Delta f_h(\vec{K},\Omega) &= \frac{e}{m}\int_{-\infty}^{0}\left(\vec{E}_l\cdot\frac{\partial\delta f_{lh}}{\partial\vec{v}}+\delta\vec{E}_{lh}\cdot\frac{\partial f_{1e}}{\partial\vec{v}}\right) \\
&\quad \times\exp[i\{\vec{K}\cdot(\vec{r}'-\vec{r})-\Omega\tau\}]\mathrm{d}\tau \\
&= I_{\Delta_1}+I_{\Delta_2}. \qquad (20)
\end{aligned}$$

Now,

$$
\begin{aligned}
I_{\Delta_1} &= \frac{e}{m}\int_{-\infty}^{0}\left(\vec{E}_l\cdot\frac{\partial\delta f_{lh}}{\partial\vec{v}}\right)\exp[i\{\vec{K}\cdot(\vec{r}\,'-\vec{r})-\Omega\tau\}]\mathrm{d}\tau \\
&= -\frac{ie}{m}E_{l\|}(\vec{k},\omega)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(a-b)\theta']}{a\Omega_e+K_\| v_\| -\Omega}\frac{\partial}{\partial v_\|} \\
&\quad\times\left[-\frac{ie}{m}E_{l\|}\frac{\partial}{\partial v_\|}\left\{\frac{ie}{m}\frac{\delta E_h}{K_\perp}\frac{m}{T_e}\left(1+\left\{\Omega-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\right.\right.\right. \\
&\quad\left.\left.\times\sum_{s,t}\frac{J_s(\alpha)J_t(\alpha)\exp[i(t-s)\theta']}{s\Omega_e-\Omega+K_\| v_\|}\right)f_{0e}\right\} \\
&\quad\times\left(\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\right)+\left(-\frac{ie}{m}\right)\frac{\delta E_h}{K_\perp} \\
&\quad\times\left[-\frac{m}{T_e}\left(\frac{ie}{m}\frac{E_{l\|}(\vec{k},w)\frac{\partial}{\partial v_\|}f_{0e}}{w-k_\| v_\| +i0}\right)\left(1+\left\{(\Omega-\omega)-(K_\|-k_\|)v_\|\right.\right.\right. \\
&\qquad\left.\left.-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-(\Omega-\Omega)+(K_\|-k_\|)v_\|}\right) \\
&\quad+K_\|\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\frac{\partial}{\partial v_\|} \\
&\quad\left.\times\left\{\frac{ie}{m}\frac{E_{l\|}(\vec{k},\omega)\frac{\partial}{\partial v_\|}f_{0e}}{\omega-k_\| v_\| +i0}\right\}\right]\frac{ie}{m}\frac{\delta E_{lh}}{|\vec{K}-\vec{k}|}\frac{m}{T_e} \\
&\quad\left.\times\left(1+\left\{(\Omega-\omega)-\frac{\epsilon' T_e K'_\perp}{m\Omega_e}\right\}\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\right)\right], \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
I_{\Delta_2} &= \frac{e}{m}\int_{-\infty}^{0}\left(\delta\vec{E}_{lh}\cdot\frac{\partial f_{1e}}{\partial\vec{v}}\right)\exp[i\{\vec{K}\cdot(\vec{r}\,'-\vec{r})-\Omega\tau\}]\mathrm{d}\tau \\
&= -\frac{ie}{m}\frac{\delta E_{lh}}{|\vec{K}-\vec{k}|}\left[\frac{m}{T_e}\left(\frac{ie}{m}\frac{E_{l\|}(\vec{k},w)\frac{\partial}{\partial v_\|}f_{0e}}{w-k_\| v_\| +i0}\right)\right. \\
&\quad\times\left(1+\left\{\Omega-K_\| v_\| -\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e+K_\| v_\| -\Omega}\right) \\
&\quad+(K_\|-k_\|)\left(\frac{ie}{m}\right)\frac{\partial}{\partial v_\|}\left(\frac{E_{l\|}(\vec{k},w)\frac{\partial}{\partial v_\|}f_{0e}}{w-k_\| v_\| +i0}\right) \\
&\quad\left.\times\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e+K_\| v_\| -\Omega}\right]. \qquad (22)
\end{aligned}
$$

Using eqs (21) and (22), we can write the expression for $\Delta f(\vec{K},\Omega)$.

Now we calculate modulated field $\delta E_{lh}(\vec{K}-\vec{k})$ by Poisson's equation,

$$\nabla\cdot\delta\vec{E}_{lh} = -4\pi e n_e\int \delta f_{lh}\mathrm{d}\vec{v}$$

in the form

$$\begin{aligned}
\delta E_{lh}(\vec{K}-\vec{k}) &= -\frac{4\pi e n_e}{i|\vec{K}-\vec{k}|}\int \delta f_{lh}\mathrm{d}\vec{v}\\
&= -\frac{4\pi e^2 n_e}{m|\vec{K}-\vec{k}|R}\frac{\delta E_h}{K_\perp}\int\Bigg[\left(\frac{e}{m}\right)\left(\frac{m}{T_e}\right)E_{l\|}\frac{\partial}{\partial v_\|}f_{0e}\\
&\quad\times\left(1+\left\{\Omega-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-\Omega+K_\| v_\|}\right)\\
&\quad\times\left(\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\right)+\left(\frac{e}{m}\right)\left(\frac{m}{T_e}\right)\\
&\quad\times\left(\frac{E_{l\|}(\vec{k},\omega)\frac{\partial}{\partial v_\|}f_{0e}}{\omega-k_\| v_\|+i0}\right)\Bigg(1+\Bigg\{(\Omega-\omega)-(K_\|-k_\|)v_\|\\
&\quad-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigg\}\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\Bigg)\\
&\quad+K_\|\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\\
&\quad\times\frac{\partial}{\partial v_\|}\left(\frac{e}{m}\times\frac{E_{l\|}(\vec{k},\omega)\frac{\partial}{\partial v_\|}f_{0e}}{\omega-k_\| v_\|+i0}\right)\Bigg],
\end{aligned}\tag{23}$$

where

$$\begin{aligned}
R = 1&+\frac{4\pi e^2 n_e}{m|\vec{K}-\vec{k}|}\frac{m}{T_e}\int\Bigg(1+\left\{(\Omega-\omega)-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\\
&\times\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(a-b)\theta']}{a\Omega_e-(\Omega-\omega)+(K_\|-k_\|)v_\|}\Bigg)f_{0e}\mathrm{d}\vec{v}.
\end{aligned}\tag{24}$$

Now we obtain dielectric response function for high-frequency ion acoustic wave by using Poisson's equation

$$\nabla\cdot\delta\vec{E}_h = -4\pi e n_e\int\left[\delta f_h(\vec{K},\Omega)+\Delta f(\vec{K},\Omega)\right]\mathrm{d}\vec{v}. \tag{25}$$

We have

$$\delta E_h(\vec{K},\Omega)\epsilon_h(\vec{K},\Omega)=0.$$

The dispersion relation $\epsilon_h(\vec{K},\Omega)$ can be written as

$$\epsilon_h(\vec{K},\Omega)=\epsilon_0(\vec{K},\Omega)+\epsilon_d(\vec{K},\Omega)+\epsilon_p(\vec{K},\Omega), \tag{26}$$

where $\epsilon_0(\vec{K},\Omega)$, $\epsilon_d(\vec{K},\Omega)$ and $\epsilon_p(\vec{K},\Omega)$ are respectively the linear part, direct coupling part and polarization coupling part. Their expressions are given by

$$\epsilon_0(\vec{K},\Omega) = 1 + \left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{m}{T_e}\right)\int\left\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\right.$$
$$\left.\times \sum_{a,b} \frac{J_a(\alpha)J_b(\alpha)\exp\{i(b-a)\theta'\}}{a\Omega_e + K_\parallel v_\parallel - \Omega}\right\} f_{0e}\mathrm{d}\vec{v}, \tag{27}$$

$$\epsilon_d(\vec{K},\Omega) = -\left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2 |E_{l\parallel}(\vec{k})|^2 \int\left[\frac{\partial}{\partial v_\parallel}\left\{\frac{m}{T_e}\right.\right.$$
$$\times\left(\frac{\frac{\partial}{\partial v_\parallel}f_{0e}}{-\omega + k_\parallel v_\parallel + i0}\right) \times \left(1 + \left\{(\Omega-\omega) - (K_\parallel - k_\parallel)v_\parallel\right.\right.$$
$$\left.\left. - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e - (\Omega-\omega) + (K_\parallel - k_\parallel)v_\parallel}\right)$$
$$+ K_\parallel \frac{\partial}{\partial v_\parallel}\left(\frac{\frac{\partial}{\partial v_\parallel}f_{0e}}{-\omega + k_\parallel v_\parallel + i0}\right)$$
$$\left.\times \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e - (\Omega-\omega) + (K_\parallel - k_\parallel)v_\parallel}\right\}$$
$$\left.\times\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta]}{p\Omega_e - \Omega}\right]\mathrm{d}\vec{v}. \tag{28}$$

Now

$$\epsilon_p(\vec{K},\Omega) = -\left(\frac{\omega_{pe}^4}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2 \frac{|E_{l\parallel}(\vec{K}-\vec{k})|^2}{|\vec{K}-\vec{k}|^2 R}$$
$$\times[(A+B)\times(C+D)], \tag{29}$$

where $A, B, C$ and $D$ are given by

$$A = -\int\left(\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e + K_\parallel v_\parallel - \Omega}\right)\left(\frac{m}{T_e}\right)$$
$$\times\frac{\partial}{\partial v_\parallel}\left[1 + \left\{(\Omega-\omega) - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\right.$$
$$\left.\times\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e - (\Omega-\omega) + (K_\parallel - k_\parallel)v_\parallel}\right] f_{0e}\mathrm{d}\vec{v}, \tag{30}$$

$$B = \int\left[\left(\frac{m}{T_e}\right)\left(\frac{\frac{\partial}{\partial v_\parallel}f_{0e}}{\omega - k_\parallel v_\parallel + i0}\right)\left(1 + \left\{\Omega - K_\parallel v_\parallel\right.\right.\right.$$

$$-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigg\}\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e+(K_\parallel-k_\parallel)v_\parallel-(\Omega-\omega)}\Bigg)$$
$$-K_\parallel\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e+K_\parallel v_\parallel-\Omega}$$
$$\times\frac{\partial}{\partial v_\parallel}\left(\frac{\frac{\partial}{\partial v_\parallel}f_{0e}}{\omega-k_\parallel v_\parallel+i0}\right)\Bigg]f_{0e}\mathrm{d}\vec{v}, \tag{31}$$

$$C=\int\Bigg[\left(\frac{m}{T_e}\right)\frac{\partial}{\partial v_\parallel}\Bigg(1+\left\{\Omega-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}$$
$$\times\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta']}{a\Omega_e+K_\parallel v_\parallel-\Omega}\Bigg)$$
$$\times\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e-(\Omega-\omega)-(K_\parallel-k_\parallel)v_\parallel}\Bigg]f_{0e}\mathrm{d}\vec{v}, \tag{32}$$

$$D=-\int\Bigg[\left(\frac{m}{T_e}\right)\left(\frac{\frac{\partial}{\partial v_\parallel}f_{0e}}{-\omega+k_\parallel v_\parallel+i0}\right)\Bigg(1+\Bigg\{(\Omega-\omega)$$
$$-(K_\parallel-k_\parallel)v_\parallel-\frac{\epsilon' T_e K_\perp}{m\Omega_e}\Bigg\}$$
$$\times\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e+(K_\parallel-k_\parallel)v_\parallel-(\Omega-\omega)}\Bigg)$$
$$+K_\parallel\sum_{p,q}\frac{J_p(\alpha)J_q(\alpha)\exp[i(q-p)\theta']}{p\Omega_e+(K_\parallel-k_\parallel)v_\parallel-(\Omega-\omega)}$$
$$\times\frac{\partial}{\partial v_\parallel}\left(\frac{\frac{\partial}{\partial v_\parallel}f_{0e}}{-\omega+k_\parallel v_\parallel+i0}\right)\Bigg]f_{0e}\mathrm{d}\vec{v}. \tag{33}$$

## 3. Instabilities

The growth rate of ion acoustics wave is calculated by using the following formula:

$$\frac{\gamma_h}{\Omega}=-\left[\frac{\left(I_m\epsilon_h+\frac{1}{2}\frac{\partial^2\epsilon_0}{\partial\Omega\partial t}\right)}{\Omega\frac{\partial\epsilon_0}{\partial\Omega}}\right]_{\Omega_r}. \tag{34}$$

The second part of the growth rate formula (34) is due to reverse absorption effect which in our case is

$$\frac{\partial^2 \epsilon_0}{\partial\Omega\partial t} = \frac{\omega_{pe}^2}{K_\perp^2}\left(\frac{m}{T_e}\right)\int\left[\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp\{i(b-a)\theta'\}}{a\Omega_e + K_\parallel v_\parallel - \Omega}\right.$$
$$\left.\times\left\{1+\left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\frac{1}{a\Omega + K_\parallel v_\parallel - \Omega}\right\}\frac{\partial f_{0e}}{\partial t}\right]\mathrm{d}\vec{v}, \tag{35}$$

where the slow time change of electron distribution function is given by

$$\frac{\partial f_{0e}(v_\parallel)}{\partial t} = \pi\left(\frac{e}{m}\right)^2\frac{\partial}{\partial v_\parallel}\sum_{\vec{k},\omega}|E_{l\parallel}(\vec{k},\omega)|^2\delta(\omega - k_\parallel v_\parallel)\frac{\partial f_{0e}}{\partial v_\parallel}. \tag{36}$$

After partial integration this contribution due to the reverse absorption effect becomes zero.

We consider the plasma–maser interaction between ion acoustics wave and lower hybrid wave turbulence. The condition for the plasma–maser is $\omega = k_\parallel v_\parallel$ and assuming $\Omega < K v_\parallel$, we first estimate the linear part of the dielectric function of ion acoustics wave from eq. (26), and considering the fact that for ion acoustics wave, (we have $\alpha = (K_\perp v_\perp/\Omega_e) \simeq 10^{-2}$, i.e. the argument of Bessel's function is small), the most dominant contribution to Bessel's sum comes from the term $a = s = p = 0$.

$$\epsilon_0(\vec{K},\Omega) = 1 + \frac{2\Lambda_1\omega_{pe}^2}{v_e^2 K_\perp^2}\left(1 - \frac{\epsilon' T_e K_\perp}{m\Omega_e\Omega}\right)\left(-1 - \frac{K_\parallel^2 v_e^2}{2\Omega^2}\right), \tag{37}$$

where

$$\Lambda_1 = \int_0^\infty 2\pi J_0^2(\alpha) v_\perp f_{0e}(v_\perp)\mathrm{d}v_\perp. \tag{38}$$

Equation (37) is obtained by using argument series of the plasma dispersion function $Z(\Omega/K_\parallel v_\parallel)$. We obtain

$$\frac{\partial\epsilon_0(\vec{K},\Omega)}{\partial\Omega} \simeq \frac{2\Lambda_1\omega_{pe}^2}{\Omega^2}\left(\frac{K_\parallel}{K_\perp}\right)^2\frac{1}{\Omega}\left(1 - \frac{\epsilon' T_e K_\perp}{m\Omega_e\Omega}\right). \tag{39}$$

Now we calculate the imaginary part of the direct coupling term from eq. (28) as follows:

$$I_m\epsilon_d(\vec{K},\Omega) = -\left(\frac{\omega_{pe}^2}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2|E_{l\parallel}(\vec{k})|^2\left[(\Lambda_2 - \Lambda_1)\frac{m}{T_e} + \frac{\epsilon' K_\perp}{\Omega\Omega_e}\right.$$
$$\left. - \frac{K_\parallel(K_\parallel - k_\parallel)}{\Omega^2}\Lambda_2\right]\frac{2\sqrt{\pi}}{v_e^3}\frac{\omega}{k_\parallel|k_\parallel|}\exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}, \tag{40}$$

where

$$\Lambda_2 = \int_0^\infty 2\pi J_0^4(\alpha) v_\perp f_{0e}(v_\perp)\mathrm{d}v_\perp.$$

Next we calculate $\epsilon_p(\vec{K},\Omega)$ from eq. (29). We observe that $A$ and $C$ are real and $B \propto D^\star$, where $\star$ denotes complex conjugate.

So we have

$$I_m\epsilon_p(\vec{K},\Omega) = -\left(\frac{\omega_{pe}^4}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2 \frac{|E_{l\parallel}(\vec{K}-\vec{k})|^2}{|\vec{K}-\vec{k}|^2 R}\left[A \times I_m D + C \times I_m B\right]. \tag{41}$$

After partial integration the contribution of $A$ is zero, i.e.

$A = 0.$

Hence we need to calculate $I_m B$ and $C$ only.

$$I_m B = -\left[\frac{m}{T_e}\left\{(1-\Lambda_1) + \frac{\epsilon' T_e K_\perp}{m\Omega_e\Omega}\right\} + \frac{T_e K_\parallel (K_\parallel - k_\parallel)}{m\Omega_e\Omega}\Lambda_1\right] \times \frac{2\sqrt{\pi}}{v_e^3}\frac{\omega}{k_\parallel |k_\parallel|}\exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}, \tag{42}$$

$$C = \left\{\frac{K_\parallel - k_\parallel}{(\Omega-\omega)^2}\right\}\left(\frac{m}{T_e}\right)\Lambda_1 + \frac{m}{T_e}\left(1 - \frac{\epsilon' T_e K_\perp}{m\Omega_e\Omega}\right)\frac{K_\parallel}{\Omega(\Omega-\omega)}\Lambda_2. \tag{43}$$

In calculating expressions from eq. (40) to eq. (43), we have used

$$\int_{-\infty}^{\infty} f_{0e}(v_\parallel)\mathrm{d}v_\parallel = 1, \tag{44}$$

$$\int_0^{\infty} f_{0e}(v_\perp)2\pi v_\perp \mathrm{d}v_\perp = 1, \tag{45}$$

$$I_m\int_{-\infty}^{\infty}\frac{\frac{\partial}{\partial v_\parallel}f_{0e}(v_\parallel)}{-\omega + k_\parallel v_\parallel + i0}\mathrm{d}v_\parallel = -\int_{\infty}^{\infty}\pi\delta(\omega - k_\parallel v_\parallel)\frac{\partial}{\partial v_\parallel}f_{0e}(v_\parallel) = \frac{2\sqrt{\pi}}{v_e^3}\frac{\omega}{k_\parallel|k_\parallel|}\exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}. \tag{46}$$

Hence we have

$$\begin{aligned} I_m\epsilon_p(\vec{K},\Omega) &= -\left(\frac{\omega_{pe}^4}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2\frac{\sum_{\vec{k},\omega}|E_{l\parallel}(\vec{K}-\vec{k})|^2}{|\vec{K}-\vec{k}|^2R}\left[C\times I_m B\right] \\ &= -\left(\frac{\omega_{pe}^4}{K_\perp^2}\right)\left(\frac{e}{m}\right)^2\frac{\sum_{\vec{k},\omega}|E_{l\parallel}(\vec{k})|^2}{k_\parallel^2}\left[\left\{\frac{K_\parallel - k_\parallel}{(\Omega-\omega)^2}\right\}\left(\frac{m}{T_e}\right)\Lambda_1\right. \\ &\quad \left. + \frac{m}{T_e}\left(1 - \frac{\epsilon' T_e K_\perp}{m\Omega_e\Omega}\right)\frac{K_\parallel}{\Omega(\Omega-\omega)}\Lambda_2\right]\left[-\left(\frac{m}{T_e}\left\{(1-\Lambda_1)\right.\right.\right. \\ &\quad \left.\left. + \frac{\epsilon' T_e K_\perp}{m\Omega_e\Omega}\right\} + \frac{T_e K_\parallel(K_\parallel - k_\parallel)}{m(\Omega-\omega)^2}\Lambda_1\right)\frac{2\sqrt{\pi}}{v_e^3}\frac{\omega}{k_\parallel|k_\parallel|} \\ &\quad \left.\times\exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}\right]. \end{aligned} \tag{47}$$

There are two branches of the lower hybrid wave. Of these two branches, the $\omega = \omega(\vec{k})[-\omega(\vec{k})]$ one resonates with electrons only for $k_\parallel > 0$ $[k_\parallel < 0]$.

Thus the dominant contribution to the growth rate due to the polarization term is calculated using eq. (34) as

$$\frac{\gamma_p}{\Omega} = 2\sqrt{\pi}\sqrt{\frac{m}{M}}\left(\frac{\omega_{pe}}{v_e}\right)^2 \sum_{+,-} |E_{l\parallel}(\vec{k},\omega)|^2 \left(\frac{K_\parallel \Lambda_2}{\Omega - \omega}\right) \times \left(1 - \frac{\epsilon' T_e K_\perp}{m\Omega\Omega_e}\right) \exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}, \tag{48}$$

where $+$ and $-$ in $\sum$ means summation over the two branches of lower hybrid waves. Furthermore, for plasmas in the presence of a uniform external magnetic field $\vec{B}_0$, the spectral energy density $|E_l[\vec{k},\omega]|^2$ is azimuthally symmetric about $\vec{B}_0$, i.e.

$$|\vec{E}_l[\vec{k},\omega]|^2 = |\vec{E}_l|^2(0,k_\parallel). \tag{49}$$

It should be mentioned that in cylindrical coordinates for $\vec{k} = (0,0,k_\parallel)$, we have $-\vec{k} = (0,0,-k_\parallel)$.

Thus eq. (48) reduces to

$$\frac{\gamma_p}{\Omega} = 2\sqrt{\pi}\sqrt{\frac{m}{M}}\left(\frac{\omega_{pe}}{v_e}\right)^2 \sum_{\vec{k}} |E_{l\parallel}(\vec{k})|^2 \left(\frac{K_\parallel \Lambda_2}{\Omega - \omega}\right) \times \left(1 - \frac{\epsilon' T_e K_\perp}{m\Omega\Omega_e}\right) \exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}. \tag{50}$$

Accordingly, the growth of the ion acoustics wave occurs if one (or both) of the following conditions is (or are) violated:

(i) The two branches of the lower hybrid waves have equal intensity, i.e.

$$|E_l^+[\vec{k},\omega]|^2 = |E_l^-[(\vec{k},-\omega)]|^2. \tag{51}$$

(ii) The spectral energy density is symmetric with respect to the parallel wave vector $(k_\parallel)$

$$|E_l^+|^2(k_\parallel > 0) = |E_l^-|^2(k < 0). \tag{52}$$

In obtaining $R|\vec{K}-\vec{k}|^2$, we have expanded $R$, from eq. (24), about small argument $\vec{k}$ and $\omega$ in $R$ and we have used the relation $\epsilon_0(\vec{K},\Omega) = 0$. To the lowest order approximation we can estimate $R|\vec{K}-\vec{k}|^2 \simeq k_\parallel^2$.

## 4. Discussion

The nonlinear dispersion relation (eq. (26)) involves the effect of density gradient $\epsilon'$. The growth rate of ion acoustics wave in the presence of lower hybrid wave turbulence is estimated by using eq. (34).

The contributions $I_m\epsilon_d(\vec{K},\Omega)$ and $I_m\epsilon_p(\vec{K},\Omega)$, in the estimation of growth rate of ion acoustics are the plasma–maser contribution due to direct coupling term and polarization coupling term. It was pointed out in refs [3] and [4] that in unmagnetized plasma $I_m\epsilon_p(\vec{K},\Omega) = 0$ and the plasma–maser contribution for $I_m\epsilon_d(\vec{K},\Omega)$ from direct coupling term exactly cancels with quasi-linear interaction $\partial^2\epsilon_0(\vec{K},\Omega)/\partial\Omega\partial t$ because, for a closed system, both low frequency turbulence and the background electron distribution function are not fixed by external agents, but are free to evolve self-consistently to form a quasi-linear plasma. Then both the plasma–maser and the reverse process due to quasi-linear effect co-exist and there is no net growth of the nonresonant high-frequency wave.

On the other hand, for an open system with a particle supply from outside, external agents fix the electron distribution function and the reverse absorption effect vanishes, that the stationary state without quasi-linear plateau is possible. Accordingly the energy transferred from low-frequency wave by nonlinear resonant interaction must go into an unstable high-frequency test wave.

The plasma–maser process is much enhanced for the magnetized plasma [10] where magnetic field provides the external source of momentum to the system.

In our study the plasma–maser effect is considered for magnetized inhomogeneous plasma. The lower hybrid turbulence wave plays a vital role in fusion [11,12] and in space plasmas [13,14]. In laboratory plasma [15] lower hybrid wave has been shown to be extremely effective in accelerating electron parallel to the magnetic field and producing high-energy tail in the electron distribution function. Recent observations by the Viking satellite [16] have reported the sporadic auroral kilometric radiation (AKR) together with lower hybrid turbulence. So lower hybrid turbulence can be identified as the possible source of radiation phenomenon in magnetosphere plasma [17]. We have considered plasma–maser interaction between lower hybrid wave turbulence and ion acoustics wave. The dispersion relation for inhomogeneous plasma was discussed by Mikhailovskii [5] and symmetry properties were investigated by Nambu [18]. In the investigation of plasma–maser effect in inhomogeneous plasma, nonlinear dispersion relation for an inhomogeneous plasma was obtained by Deka [19]. In this problem, nonlinear dispersion relation (eq. (26)) is obtained by neglecting the confining magnetic field gradients. This dispersion relation is used to estimate the growth rate. If the gradient in confining magnetic field $\vec{\nabla}\cdot\vec{B}$ is considered, the particle trajectories will be modified and one has to adopt a different integration strategy. As a consequence, in such a situation the gradient parameter $\epsilon = \frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}y}$ will appear in the denominator of nonlinear dispersion relation (eq. (26)). The term involving $\epsilon$ allows a resonance with the phase velocity $v_{\mathrm{ph}} = \frac{\omega}{k}$ equal to the drift speed [9]. This may further enhance the plasma–maser effect. Since polarization coupling is the dominating term in the plasma–maser effect, we estimate the growth rate due to polarization coupling term (eq. (50)) only.

When there is no density gradient ($\epsilon' = 0$) the growth rate of ion acoustics wave for homogeneous plasma is obtained as

$$\frac{\gamma_p}{\Omega} = 2\sqrt{\pi}\sqrt{\frac{m}{M}}\,(\omega_{pe})^2\left(\frac{|E_{l\parallel}(\vec{k})|}{v_e}\right)^2\left(\frac{K_\parallel}{\Omega-\omega}\right)\Lambda_2. \tag{53}$$

When high density gradient is present in the system of plasma, then the growth rate of ion acoustics wave is estimated as

$$\frac{\gamma_p}{\Omega} = 2\sqrt{\pi}\sqrt{\frac{m}{M}}\,(\omega_{pe})^2\left(\frac{|E_{l\|}(\vec{k})|}{v_e}\right)^2\left(\frac{K_\perp}{\Omega-\omega}\right)\left(\frac{\epsilon'}{\Omega_e}\right)\Lambda_2. \tag{54}$$

However, for small order gradient, there is no effect in the growth rate and eq. (53) will give the estimate for such a gradient situation.

We wish to estimate the growth rate by using the observational data, according to the observations by Viking satellite in space [16]: $\omega_{pe} \sim 10$, $K_\| \sim 2\pi \times 10^{-7}$ cm$^{-1}$, $E_{l\|} \sim 140$ $\mu$vm$^{-1}$, $\Lambda_2 \sim 1$, $\Omega \sim 16\pi \times 10^3$ Hz, $v_e \sim 10^{-1}$, $\Omega_e \sim 2\pi \times 10^3$ Hz, $K_\perp \sim 2\pi \times 10^{-5}$ cm$^{-1}$ and $\sqrt{m/M} \sim 43$.

From eq. (53) we have

$$\frac{\gamma_p}{\Omega} \sim 10^{-2}, \tag{55}$$

from eq. (54)

$$\frac{\gamma_p}{\Omega} \sim 10^{-3}\epsilon', \tag{56}$$

and from eq. (56),

$$\frac{\gamma_p}{\Omega} \sim 10^{-2}, \tag{57}$$

taking $\epsilon' = 10$.

This reveals that if the density gradient is large then the growth rates in the case of homogeneous and inhomogeneous plasma remain unaltered. The growth rates as obtained in eqs (55) and (57) are sufficiently large to generate AKR. We wish to refer to the observations by Viking satellite [16], and the observations indicate that sporadic auroral kilometer radiation (AKR) occurs well above the electron gyrofrequency together with electrostatic lower hybrid turbulence [20–23] and upward going electron beams with broad spectra (∼100 eV to 1 keV). The growth of the AKR is possible up to $\Omega = 1.2\Omega_e$, because plasma–maser interaction does not require any matching conditions among wave numbers and frequencies. Moreover, the plasma–maser interaction predicts that the accelerated electrons along the parallel direction produce bursts of high-frequency radiation near the electron gyrofrequency even for the universal electron distribution functions. Similar high-frequency radiations are also observed in laboratory experiments [24].

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