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Diffraction model analysis of pion-^{12}C elastic scattering at 800 MeV/c: Optical potential by inversion

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Abstract. Elastic scattering of 800 MeV/c pions by ^{12}C has been studied in the diffraction model with a view to determine pion optical potential by the method of inversion. Finding an earlier diffraction model analysis to be deficient in some respects, we propose a Glauber model based parametrization for the elastic S-matrix and show that it provides an exceedingly good fit to the pion–carbon data. The proposed elastic S-matrix gives a closed expression for the pion–^{12}C optical potential by the method of inversion in the high energy approximation.

Keywords. Diffraction model; pion optical potential; pion–nucleus scattering.

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1. Introduction

Elastic scattering of 800 MeV/c positive and negative pions from ^{12}C and ^{40}Ca has been studied by several authors. Contrary to expectations, these studies show that the microscopic models that take the elementary πN interaction and the target ground state nucleon density as the input information do not provide a satisfactory description of the experimental data (see, for example, refs [1–6]). This situation provided motivation for the application of semi-phenomenological and phenomenological models for the description of the pion–nucleus elastic scattering. An example of the first category is the Glauber model analysis by Arima *et al* [5] who varied the πN amplitude to achieve a reasonably satisfactory description of the pion–nucleus scattering data. However, even in this case notable disagreements between the calculated and experimental angular distributions are present in the region of the minima.

On the phenomenological front two different models, namely, the phenomenological optical potential model and the diffraction model have been applied for the analysis of pion–nucleus experiment at 800 MeV/c. In the former, six-parameter Saxon–Woods optical potential has been used to fit the data [7,8]. This model has the advantage of directly giving the optical potential needed for nuclear reaction studies involving pions. However, it suffers from the well-known parameter ambiguities [8]. Moreover, these phenomenological analyses show that pion–^{12}C optical potential at 800 MeV/c is attractive whereas the first-order microscopic theories strongly suggest that it should be repulsive [2,4,5].

In the diffraction model an appropriate form for the elastic S-matrix S_l is chosen and its parameters are varied to obtain an acceptable fit to the data. Choudhary and Scura [9] used this model to study the 800 MeV/c pion–nucleus scattering. Ignoring the Coulomb scattering and using a simple form for S_l they have derived a closed expression for the pion–nucleus elastic scattering amplitude under the strong absorption approximation. Their calculation shows that the derived amplitude that involves only three adjustable parameters provides a fairly good description of the data except in a small forward angle region where theory overpredicts the cross-sections. Despite some of its limitations which are to be discussed shortly, the study of Choudhary and Scura [9] suggests that the diffraction model is capable of describing the pion–^{12}C data with lesser number of parameters than the phenomenological optical model. Moreover, at the energy of our interest the high-energy approximation developed by Glauber [10] holds well that permits evaluation of the optical potential from the phase-shift by the method of inversion in a simple way. It is therefore of some interest to undertake a refined diffraction model analysis of the pion data to determine S_l and use it to determine pion–^{12}C optical potential at 800 MeV/c by the method of inversion.

It is generally known that at energies above the delta resonance the incident pion, because of weaker pion–nucleon interaction, has a longer mean free path in the nuclear medium. It is estimated to be about 3–4 fm at higher energies [6]. This implies that for pion–nucleus system at 800 MeV/c the strong absorption approximation might not work sufficiently well especially for a lighter nucleus like ^{12}C whose radial extension is about the same as the mean free path. In view of this it is desirable that the correctness of S_l as determined by Choudhary and Scura [9] be examined if it is to be used for the determination of the optical potential by inversion. Indeed, as it will be shown later, the strong absorption approximation does not work well for the pion–^{12}C system at 800 MeV/c.

The problem of determination of optical potential by 'S-matrix fitting plus inversion' has been discussed at length by McEvan, Cooper and Mackintosh (MCM) in the context of ^{12}C–^{12}C scattering at medium energies [11]. These authors, recognizing the difficulties in uniquely determining S_l from the elastic angular distribution data have suggested that the situation could be ameliorated by choosing a physically reasonable S_l. One way of achieving it is to choose a form for S_l that has some theoretical basis.

In this work we first study the working of the strong absorption approximation as developed by Choudhary and Scura [9] for pion–nucleus system at 800 MeV/c and also the effect of the Coulomb scattering that has been neglected by them. This is done by comparing the predictions of Choudhary and Scura's closed expression with that of the exact partial wave expression for the scattering amplitude. As it will be shown later the approximation does not work well in this case. Next, following

MCM's suggestion we propose a parametrization for the phase-shift function for pion–^{12}C system that has some theoretical basis. This parametrization involves four adjustable parameters and it describes the experimental data exceedingly well. Moreover, it enables us to derive a closed expression for the pion–^{12}C optical model potential by the method of inversion.

2. Theoretical considerations

The elastic scattering amplitude for the scattering of a charged nuclear particle from a target nucleus of mass number A, and charge number Z may be written as

$$F_{\rm el}(\theta) = F_{\rm c}(\theta) + \frac{i}{2k}\sum_{l=0}^{\infty}(2l+1){\rm e}^{2i\sigma_l}[1-S_l]P_l(\cos\theta), \tag{1}$$

where $F_{\rm c}(\theta)$ is the point Coulomb scattering amplitude, k is the c.m. momentum, σ_l is the point Coulomb phase shift, $P_l(\cos\theta)$ is the Legendré polynomial, and S_l is the elastic S-matrix element. We write the S- matrix element as

$$S_l = S(b){\rm e}^{i\delta_{\rm c}(b)}|_{kb=l+\frac{1}{2}}, \tag{2}$$

where b is the impact parameter, S is the nuclear part of the S-matrix, and $\delta_{\rm c}(b)$ is the difference between the phase-shift functions of the Coulomb potentials due to the extended charge distribution of the target nucleus and the corresponding point charge. In ref. [9] the Coulomb effect is completely neglected and the quantity S is parametrized as

$$S(b) = g(b) + i\frac{\mu}{k}\frac{{\rm d}g}{{\rm d}b} \tag{3}$$

with

$$g(b) = \frac{1}{1+\exp[(R-b)/a]}, \tag{4}$$

where R and a are respectively the effective radius and surface diffuseness parameters of the diffraction model. The parameter μ is directly related to the real part of the nuclear phase-shift and hence it describes the refraction. Neglecting the Coulomb effect in eq. (1) and invoking the strong absorption approximation, Choudhary and Scura [9] obtained the following expression for the elastic scattering amplitude:

$$F_{\rm el}(\theta) = R\left(\frac{\theta}{\sin\theta}\right)^{1/2}\frac{\pi\theta\Delta}{\sinh(\pi\theta\Delta)}\left[\frac{iJ_1(kR\theta)}{\theta} + \mu J_0(kR\theta)\right], \tag{5}$$

where $\Delta = ka$. Finally the elastic scattering differential cross-sections are calculated from the relation

$$\frac{{\rm d}\sigma}{{\rm d}\Omega} = |F_{\rm el}(\theta)|^2. \tag{6}$$

3. Comparison with realistic amplitude

In this section we compare the predictions of the approximate and exact scattering amplitudes given by eqs (5) and (1) respectively for π^+–^{12}C system. We will also study the effect of Coulomb scattering neglected in deriving eq. (5). As already mentioned the phase-shift function $\delta_c(b)$ is the difference of the phase-shift functions due to the Coulomb potentials of the extended target charge distribution and the corresponding point charge. Using realistic charge form factor of ^{12}C, Germond and Wilkin [12] have derived the following expression for it.

$$\delta_c(b) = \eta \left\{ E_1(b^2/4\beta^2) + \frac{1}{\beta^2 q_a^2} e^{-b^2/4\beta^2} \right\}, \tag{7}$$

where $\eta(= Ze^2/\hbar\nu)$ is the Sommerfeld parameter, E_1 is the exponential integral, $\beta^2 = 0.775$ fm^2 and $q_a = 3.37$ fm^{-2}.

The results of calculation for π^+–^{12}C scattering for the parametrization of S as given by eq. (3) are shown in figure 1. These calculations have been made with the parameter values, $R = 2.38$ fm, $a = 0.58$ fm, and $\mu = 0.60$. These parameter values have been obtained by Choudhary and Scura [9] by fitting the experimental data using the approximate amplitude given by eq. (5). Their fit is shown by the dotted curve. The solid and the dashed curves in figure 1 show the predictions of the exact expression (1) with and without the Coulomb scattering using eq. (2). It is seen that the predictions of the approximate expression (5) differ greatly from the realistic values calculated from the exact expression (1) with the same set of parameter values. Similar results are obtained for π^-–^{12}C system. This implies

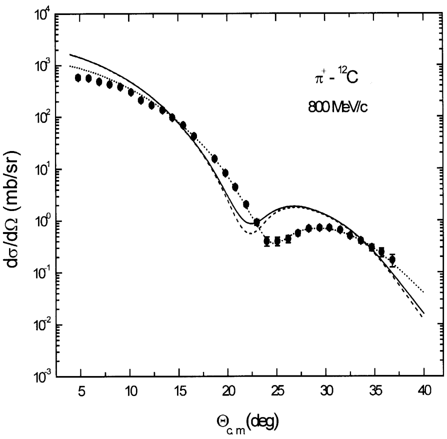

Figure 1. Elastic scattering differential cross-sections for 800 MeV/c π^+ mesons on ^{12}C. The dotted curve shows the predictions of the approximate expression (5). The solid and dashed curves show the predictions of expression (1) with and without the Coulomb scattering. In each case the parameter values are [9]: $R = 2.38$ fm, $a = 0.58$ fm and $\mu = 0.60$. The experimental data are of ref. [1].

that the strong absorption approximation that forms the basis of the derivation of the expression (5) does not work sufficiently well for π–^{12}C system at higher energies. In other words, the S-matrix determined by fitting the experimental data using the approximate amplitude (5) is unrealistic and hence unsuitable for the determination of the optical potential by inversion. From figure 1, it is also seen that the Coulomb effect is not negligible in the region of the first minimum in the angular distribution. However, the contribution of the phase-shift function $\delta_{\rm c}(b)$ given by eq. (7) is not significant even in this region.

4. Phase-shift function for ^{12}C

Having found that the approximate expression (5) does not give realistic results, we made an attempt to fit the data using the exact expression (1) for the scattering amplitude with the same parametrization for $S(b)$ as given by eq. (3). Our attempt did not give satisfactory results. We could not get any set of parameter values that fits the data satisfactorily well. It seems that perhaps the form of $S(b)$ given by eq. (3) is unsuitable for pion–^{12}C system at higher energies. This situation led us to parametrize the S-matrix in the following manner. Instead of $S(b)$ we parametrize the phase-shift function $\chi(b)$ defined by the relation

$$S(b) = {\rm e}^{i\chi(b)}, \tag{8}$$

and assume that it is of the form

$$\chi(b) = \chi_0(1 + c_1 b^2){\rm e}^{-c_2 b^2}, \tag{9}$$

where c_1 and c_2 are real parameters and $\chi_0(= c_{\rm r} + ic_{\rm i})$ is a complex parameter.

The above parametrization is based on the consideration that in the zero-range optical limit approximation of the Glauber model [10], the phase-shift function for the hadron–nucleus system depends linearly upon the thickness function: $T(b) = \int_{-\infty}^{\infty} \rho(r){\rm d}z$, where $\rho(r)$ is the target density. The form of $\chi(b)$ given by eq. (9) is the same as that of $T(b)$ for ^{12}C evaluated with the modified harmonic oscillator density [2]. Thus our parametrization for $\chi(b)$, and hence of the elastic S-matrix has a theoretical basis in accordance with the suggestion made by MCM [11]. Further, the proposed parametrization has a desirable property that it gives a closed expression for pion–^{12}C optical potential $V_{\rm op}(r)$ as given by the high-energy inversion formula [10]:

$$V_{\rm op}(r) = \frac{\hbar\nu}{\pi}\,\frac{\rm d}{r\,{\rm d}r}\int_r^{\infty}\frac{\chi(b)b\,{\rm d}b}{\sqrt{b^2 - r^2}}, \tag{10}$$

where ν denotes the velocity. Several authors have used the above formula to calculate the optical potential from the phase-shift function at intermediate and high energies (see refs [13,14]). It is readily seen that substitution of eq. (9) in eq. (10) gives the following expression for the optical potential:

$$V_{\rm op}(r) = \frac{\hbar\nu\chi_0}{\sqrt{\pi c_2}}\left(\frac{c_1 - 2c_2}{2} - c_1 c_2 r^2\right){\rm e}^{-c_2 r^2}. \tag{11}$$

It may be pointed out that the symbol ν in eqs (10) and (11) denotes the lab velocity which at non-relativistic energies is the same as the velocity in the center-of-mass system. Taking ν as the lab velocity ensures that the phase-shift function $\chi(b)$ of the Glauber high-energy approximation that forms the basis of eq. (11) gives the correct expression for the point Coulomb scattering amplitude at relativistic energies. However, in pion–nucleus optical model phenomenology one generally uses a Schrödinger-type equation that is obtained by truncating the Klein–Gordon equation obeyed by pions. In such cases the optical potential given by eq. (11) should be multiplied by an appropriate kinematical factor. For example, to apply the optical potential given by eq. (11) in the Schrödinger-type equation of ref. [8] one should multiply it by the factor $k/(\nu\mu)$, where, k is the pion c.m. momentum and $\mu = M_\pi m_{\rm T}/(M_\pi + m_{\rm T})$ with M_π as the total pion energy in the c.m. system and $m_{\rm T}$ the mass of the target nucleus.

Although the present work is mainly concerned with the application of the diffraction model phenomenology for the determination of the pion–carbon optical potential by the method of inversion, it is useful to briefly discuss the physical content of our parametrization of $\chi(b)$ given by eq. (9). It has already been mentioned that the proposed parametrization is motivated by the form of the phase-shift function in the optical limit approximation of the Glauber theory under the zero-range approximation. In this approximation the phase-shift function for pion–nucleus system is given by

$$\chi^{\rm G}(b) = \frac{A\sigma(i+\alpha)}{2} T(b), \tag{12}$$

where A is the target mass number, σ and α are respectively the πN total cross-section and the ratio of real to imaginary parts of the forward πN scattering amplitude and $T(b)$ is the same as defined before. Taking the target density to be of the modified harmonic oscillator form: $\rho(r) = \rho_0(1+\alpha_m r^2/a^2)\exp(-r^2/a^2)$ with a, and α_m as the density parameters, the thickness function can be evaluated easily. The resulting expression for the Glauber phase-shift function is

$$\chi^{\rm G}(b) = \frac{A\sigma(i+\alpha)(2+\alpha_m)}{2\pi a^2(2+3\alpha_m)} \left(1 + \frac{2\alpha_m b^2}{a^2(2+\alpha_m)}\right) \exp(-b^2/a^2). \tag{13}$$

Comparing the above expression with the expression (9) it is seen that the geometrical parameters c_1 and c_2 portray the density parameters through the relations

$$c_1 = \frac{2\alpha_m}{a^2(2+\alpha_m)}; \quad c_2 = \frac{1}{a^2} \tag{14}$$

and

$$c_{\rm r} + ic_{\rm i} = \frac{A\sigma(i+\alpha)(2+\alpha_m)}{2\pi a^2(2+3\alpha_m)}. \tag{15}$$

From eqs (14) and (15) it follows that having obtained the best-fit parameters $c_{\rm r}, c_{\rm i}, c_1$ and c_2 from the phenomenological analysis one can determine the effective values of the πN amplitude and the density parameters from the following relations:

$$\sigma = \frac{2\pi(c_1+c_2)c_\mathrm{i}}{Ac_2^2}, \quad \alpha = \frac{c_\mathrm{r}}{c_\mathrm{i}}, \quad a = \frac{1}{\sqrt{c_2}}, \quad \alpha_m = \frac{2c_1}{(2c_2-c_1)}. \tag{16}$$

Needless to say, the values of the effective πN amplitude and the density parameters determined from the above relations and the best-fit phenomenological parameters $c_\mathrm{r}, c_\mathrm{i}, c_1$ and c_2 will be somewhat different from their true values for several reasons. The most important being the modification of the πN amplitude within the nuclear medium. This follows from the fact that the microscopic studies using realistic densities and the free πN amplitude do not provide satisfactory description of the data as discussed earlier. The other reasons which are worth mentioning are the zero-range approximation that has been used to derive the relations (16) and the optical limit approximation itself that neglects the correlation terms in the expansion of the full Glauber phase-shift function [10].

5. Results and discussion

Using the phase-shift function given by eq. (9) and incorporating the Coulomb effect we have made χ^2-fit to the 800 MeV/c $\pi^\pm$–^{12}C scattering data [1] by treating c_1, c_2, and the real and imaginary parts of χ_0 as adjustable parameters. The results of the fit are shown in figures 2 and 3. The corresponding parameter values, and per point χ^2-values are given in table 1. In the last two columns of the table, we also give the calculated values of the reaction cross-section σ_R and the total cross-section σ_T. From the figures it is seen that the experimental data are very nicely fitted. This is also evident from small χ^2 values. During the fitting process we observed that acceptable fits to both negative and positive pion–carbon data could be obtained with negative as well as positive Re χ_0 which respectively correspond to repulsive and attractive real optical potential. However, for π^-–^{12}C system the best-fit χ^2-value with positive Re χ_0 was found to be about three times larger than

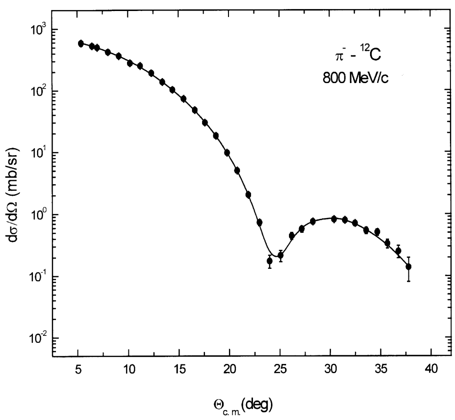

Figure 2. Elastic scattering differential cross-sections for 800 MeV/c π^- mesons on ^{12}C. The solid curve shows the results of our fit with the parameter values given in table 1.

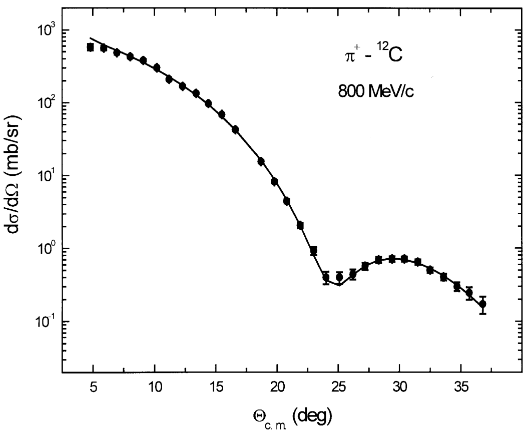

Figure 3. Elastic scattering differential cross-sections for 800 MeV/c π^+ mesons on ^{12}C. The solid curve shows the results of our fit with the parameter values given in table 1.

Table 1. Parameter values of the phase-shift function for $\pi^{\mp}$–^{12}C system at 800 MeV/c.

	Re χ_0	Im χ_0	c_1 (fm^{-2})	c_2 (fm^{-2})	χ^2/N	$\sigma_{\rm R}$ (mb)	$\sigma_{\rm R}$ (mb)
π^-–^{12}C	−0.58	1.58	0.234	0.387	0.55	213.84	294.18
π^+–^{12}C	−0.48	1.42	0.226	0.370	1.65	208.81	278.11

the value given in table 1 which is obtained with negative Re χ_0. The present phenomenology strongly suggests that the real part of the pion optical potential at 800 MeV/c is repulsive. This agrees with the predictions of the microscopic models (see, for example, [4]). In view of this we considered only negative Re χ_0 in fitting the π^+–^{12}C scattering data.

From table 1 it is seen that the parameters c_1 and c_2 that determine the geometry of the phase-shift function are almost the same for negative and positive pions. However, the real and imaginary parts of χ_0 are different in the two cases. The absolute values of the real and imaginary parts of χ_0 for positive pions are somewhat smaller than those for negative pions. Further, the calculated values of $\sigma_{\rm R}$ and $\sigma_{\rm T}$ for π^- are in good agreement with the experimental values at neighboring energy as shown in figure 2 of ref. [8].

Having determined the phenomenological parameters $c_{\rm r}, c_{\rm i}, c_1$ and c_2 (table 1), the effective πN amplitude and density parameters can be determined using eq. (16). The calculated values of the effective parameters are: (i) $\sigma = 3.43$ fm^2, $\alpha = -0.37$, $a = 1.61$ fm and $\alpha_m = 0.867$ for π^-–^{12}C scattering, and (ii) $\sigma = 3.24$ fm^2, $\alpha = -0.34$, $a = 1.64$ fm and $\alpha_m = 0.879$ for π^+–^{12}C scattering. We note that (i) the parameter values in the two cases are very close to each other as expected, (ii) the effective πN amplitude parameter values are enhanced in magnitude compared to their isospin averaged free values $\sigma = 2.62$ fm^2, $\alpha = -0.23$ as follows from the values given in ref. [2], and (iii) the density parameter values differ much from their realistic point density parameter values $a = 1.5$ fm and $\alpha_m = 2.33$ but are closer to the charge density parameter values $a = 1.649$ fm and $\alpha_m = 1.247$ as given in table 6 of ref. [15]. This behavior of the calculated density parameters is mainly due to the zero-range approximation that has been used to establish correspondence between the parameters of the phenomenological and microscopic models. At this incident pion energy the slope parameter β^2 of the Gaussian parametrization of the πN amplitude $\approx r_{\mathrm{p}}^2/3$, where r_{p} is the proton charge rms radius. Hence working with the point nucleon density without invoking the zero-range approximation is equivalent to working with the charge density with the zero-range approximation.

One of the objectives of analyzing elastic scattering data is to obtain a reliable optical potential for generating distorted waves for DWBA or DWIA calculations. In this respect the presently proposed parametrization has advantage over the generally employed parametrization of the S-matrix, since, as shown in the previous section, it gives an analytical expression for the optical potential. In figure 4 we show the optical potential for $\pi^{\pm}$–^{12}C system at 800 MeV/c that has been calculated from eq. (11) using the parameter values given in table 1. The real and imaginary parts of the potentials for π^+ and π^- are shown by the dashed and solid curves respectively. It is seen that the forms of the potentials for π^- and π^+ are

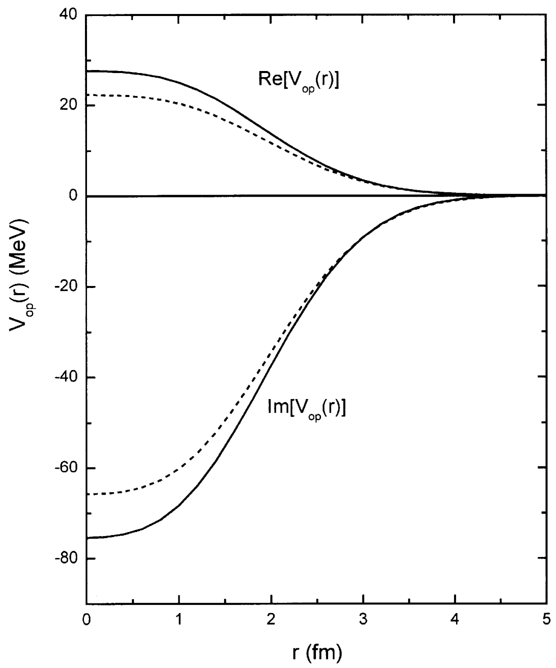

Figure 4. Real and imaginary parts of the $\pi^{\mp}$–^{12}C optical potential at 800 MeV/c. The solid and dashed curves show the potentials for π^- and π^+ mesons respectively.

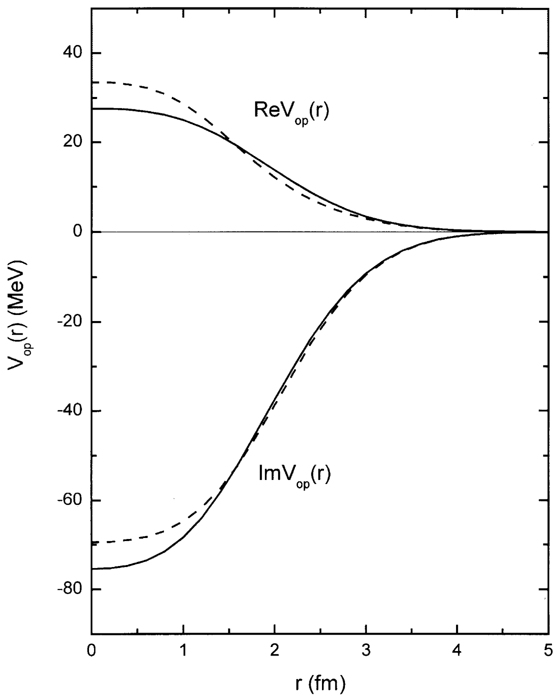

Figure 5. Effect of non-eikonal correction on π^-–^{12}C inversion optical potential at 800 MeV/c. The solid and the dashed curves show the uncorrected and corrected potentials respectively.

the same as expected. It is further seen that the two potentials have almost the same values in the surface region. However, both the real and imaginary parts of the optical potential for π^+ meson are weaker than those of the π^- meson.

The inversion optical potential given by eq. (10) is based on the eikonal approximation. The effect of non-eikonal correction on the calculated inversion optical potential has been discussed by Ahmad *et al* [16]. They have also given an approximate prescription to account for the non-eikonal correction that amounts to multiplying the inversion potential by the factor

$$\left(1+\frac{1}{k\nu}\left(V_{\rm op}+r\frac{{\rm d}V_{\rm op}}{{\rm d}r}\right)\right)^{-1}.$$

Using this prescription we have studied the effect of the non-eikonal correction on the inversion potential for π^-–^{12}C system at 800 MeV/c. The results of the study are shown in figure 5. The solid and the dashed curves show the corrected and uncorrected potentials respectively. It is seen that except for small values of the radial distance r the non-eikonal corrections are generally small. The relatively large non-eikonal effect for small r is not unexpected. From eq. (10) it follows that the optical potential at large r is determined by the phase-shift function $\chi(b)$ for $b \geq r$. Since the conditions for the applicability of the high-energy approximation are better satisfied for large b [10], it follows that the non-eikonal correction to the inversion potential should be small for large r. Further, the somewhat larger non-eikonal effect for small r values is not of much concern because the scattering cross-section calculation does not depend sensitively upon the optical potential in the interior region as the scattering at high energies is dominated by the partial waves corresponding to the large impact parameter values. We also studied the non-eikonal effect on inversion potential for π^+–^{12}C system the results (not shown) of which are found to be similar to that for π^-–^{12}C system. On the basis of the above discussion, it may be said that the expression (11), which is based on four-parameter phenomenology, gives a reasonably reliable optical potential.

To summarize, in this work we have demonstrated that the results of the previously applied strong absorption approximation are unrealistic for pion–nucleus scattering at 800 MeV/c. Next, we have proposed a new parametrization for the phase-shift function that has four adjustable parameters, and shown that it provides an excellent fit to 800 MeV/c $\pi^\pm$–^{12}C elastic scattering data. Using the proposed phase-shift function, we have also calculated the pion optical potential by the method of inversion. Our study provides some phenomenological evidence that the real part of the pion potential at this energy is repulsive.

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