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On diquark clustering in quark–gluon plasma

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Abstract. The possibility that pairs of quarks will form diquark clusters in the regime above deconfinement transition for hadron matter at finite density is revisited. Here we present the results on the diquark–diquark (dq–dq) interaction in the framework of constituent quark model taking account of spin, isospin and color degrees of freedom in the spirit of generalized Pauli principle. By constructing the appropriate spin and color states of the dq–dq clusters we compute the expectation values of the interaction Hamiltonian involving pairwise quark–quark interaction. We find that the effective interaction between two diquark clusters is quite sensitive to different configurations characterized by color and spin states, obtained after the coupling of two diquark states. The value of the coupling parameter for a particular color–spin state, i.e., $\{\bar{3}, 1\}$ is compared to the one obtained earlier by Donoghue and Sateesh, *Phys. Rev.* **D38**, 360 (1988) based on the effective ϕ^4-theory. This new value of λ derived for different color–spin dq–dq states, may lead to several important implications in the studies of diquark star and diquark gas.

Keywords. Multiquark states; diquark clustering; diquak–diquark interaction; quark–gluon plasma; diquark stars.

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1. Introduction

It is well-known that at high temperature and high density the hadronic matter will undergo a phase transition to a deconfined phase of quarks and gluons. Further, the possibility of an intermediate phase, consisting of diquarks and gluons or of quarks, diquarks and gluons, has also been the subject of study in the recent past by several authors [1–6]. In this intermediate phase, quarks remain pairwise correlated mainly due to their spin–spin interaction. A preliminary investigation on whether there is a range of densities in which the quark matter could exist primarily in the configuration of diquark clusters was carried out by Donoghue and Sateesh (DS) [2] and later pursued by Kastor and Traschen [3] and then by several authors [4–6].

The composite quark systems, like diquarks, were first introduced by Gell-Mann [7] and later by several others [8]. Out of several possible states of diquarks, the scalar diquark, i.e., color antitriplet, spin zero quark–quark (q–q) subsystems are found [9] energetically favorable. A possible relevance of diquarks in quark–gluon plasma (QGP) was first suggested by Ekelin and Fredriksson [10] by way of proposing a thermodynamical approach to quark–diquark–gluon plasma. Among other recent works, the role of diquark degrees of freedom has been exploited by Lugones and Horvath [11] to explain the high compactness of some neutron stars as revealed by the data and by Blaschke *et al* [12] to understand the phase transition to a superconducting state of a diquark condensate. In the latter case one considers the chiral quark model within the limits of field approximation. The model of DS, while has bearing on the method of Jaffe and Low [13], however deals essentially with an effective ϕ^4-theory for diquarks. As an application of this model, Kastor and Traschen [3] studied the neutron star with a core consisting of quarks and diquarks, and Sateesh [3] looked for new signals of diquarks in the QGP. Within this framework Karn *et al* [5,6] (KKM) have made an attempt to account for the extended character of the diquark. As a result, a considerable improvement in the results is obtained compared to those of DS for the case of diquark gas or for quark gas. An interesting outcome of these calculations is, following the prescription of Kastor and Traschen [3], that somewhat large value of the maximum mass ($M = 8.92$ solar mass) and radius ($R = 50.7$ km) are obtained for the diquark stars. With regard to the stability of these stars, an explanation for these abnormal figures for M and R has in fact recently been sought [14] in terms of nonlinear features [15] of the underlying field theory.

Although the results, obtained for diquark star using the crude model of DS [6] and their interpretation sought in [14], are consistent with those discussed in [15] for soliton and boson stars, fine tuning of the ingredients of the model of ref. [2] is still desirable. In computing the coupling parameter λ of the ϕ^4-term in the effective Lagrangian,

$$L_{\text{eff}} = (1/2)[(\partial_\mu \phi^{\alpha\dagger})(\partial_\mu \phi^{\alpha}) - M^2 \phi^{\alpha}\phi^{\alpha\dagger}] - \lambda(\phi^{\alpha}\phi^{\alpha\dagger})^2, \quad (1)$$

diquarks are represented [2] as spin zero boson fields ϕ^{α} (α being a color index) thereby ignoring the effective interaction between the quark constituents of the two-diquark composites.

The purpose of this paper is to compute the diquark–diquark (dq–dq) interaction in the spirit of the constituent quark model starting from the q–q interaction between all the possible pairs and demonstrate its effect on the coupling parameter λ. Here, we present a general formulation by constructing all the possible color–spin states of the diquarks such as $\{\bar{3}, 0\}, \{\bar{3}, 1\}, \{6, 0\}, \{6, 1\}$. This can then enable us to deduce the values of the coupling parameter λ, as defined in the Lagrangian (1), between two diquark clusters in different color and spin configurations. In view of the fact that these color and spin interactions between the diquarks are important, in the present work, we arrive at an improved value of the coupling parameter λ.

In the next section, we recapitulate the results on the diquark states and interaction energies of the diquark cluster in different color and spin states. In §3, we construct the states of the diquark composites by coupling two diquarks of definite color and spin symmetric states in a group theoretical framework. The interaction energies between the dq–dq clusters for different color and spin states are then computed that are related to the coupling parameter λ as defined in eq. (1). Finally concluding remarks are made in §4.

2. Diquarks in different color and spin states with their interaction energies

Quarks, being color triplet and spin doublet objects, are coupled to form pairs of quarks as color 6 and $\bar{3}$ states each with spin one and zero. Thus, there can be four possible states of a pair of quarks with color and spin quantum numbers $\{\underline{C}, \underline{S}\} = \{\bar{3}, 0\}, \{\bar{3}, 1\}, \{6, 0\}$, and $\{6, 1\}$. Restricting to the (u, d) flavor of quarks, with isospin $I = 1/2$ and assuming spatially symmetric configuration, we rewrite the interaction Hamiltonian of ref. [2], in the spirit of constituent quark model, as

$$H_{\rm int} = -A \sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j), \tag{2}$$

where the indices i, j are used to label the interacting quarks and $\vec{\sigma}$ and $\vec{\lambda}$ are the usual Pauli spin and Gell-Mann color $SU(3)$ matrices. We recall the standard results.

$$\vec{\sigma}_i \cdot \vec{\sigma}_j \left.\begin{matrix} = +1, \text{ for spin symmetric state,} \\ = -3, \text{ for spin antisymmetric state,} \end{matrix}\right. \tag{3}$$

and similar results also hold good for the isospin operators τ's. Further,

$$\vec{\lambda}_i \cdot \vec{\lambda}_j \left.\begin{matrix} = +4/3, \text{ for color symmetric state,} \\ = -8/3, \text{ for color antisymmetric state.} \end{matrix}\right. \tag{4}$$

This enables us to write the permutation operators for color, spin and isospin spaces as: $P^c_{ij} = \frac{1}{2}\vec{\lambda}_i \cdot \vec{\lambda}_j + 1/3$, $P^\sigma_{ij} = \frac{1}{2}(1 + \vec{\sigma}_i \cdot \vec{\sigma}_j)$ and $P^\tau_{ij} = \frac{1}{2}(1 + \vec{\tau}_i \cdot \vec{\tau}_j)$. For the quarks, being fermions, we can then use the generalized Pauli principle, i.e.

$$P^c_{ij} \cdot P^\sigma_{ij} \cdot P^\tau_{ij} \cdot P^x_{ij} = -1, \tag{5}$$

to write the completely antisymmetric state in the product space of color, spin, isospin and configuration space. As we restrict to the even parity interaction, we have $P^x_{ij} = +1$ all through this work. To take account of isospin explicitly for the (ud) quark pair, here we consider the diquark state, $|ud\rangle$ as

$$|ud\rangle = (1/\sqrt{2}) \left[\left(\frac{|ud\rangle + |du\rangle}{\sqrt{2}} \right) + \left(\frac{|ud\rangle - |du\rangle}{\sqrt{2}} \right) \right], \tag{6}$$

where first term on the right-hand side of this equation represents $I = 1$ (symmetric) state and the second term $I = 0$ (antisymmetric) state. With this proviso we then apply the generalized Pauli's principle (eq. (5)) for the diquark pair. Based on these considerations and employing the interaction Hamiltonian of eq. (2) we calculate the interaction energies for different color and spin diquark states, viz.,

$$\langle \bar{3},\, 0 | H_{\rm int} | \bar{3}, 0 \rangle = -8A, \tag{7}$$
$$\langle 6,\, 1 | H_{\rm int} | 6, 1 \rangle = -(4/3)A, \tag{8}$$
$$\langle \bar{3},\, 1 | H_{\rm int} | \bar{3}, 1 \rangle = +(8/3)A, \tag{9}$$
$$\langle 6,\, 0 | H_{\rm int} | 6, 0 \rangle = +4A. \tag{10}$$

Note that the above results for the matrix elements, obtained within the framework of constituent quark model, are just half of the corresponding results (cf. eqs (4a)–(4d) of ref. [2]). This is mainly because we have avoided the double counting of quarks in the pair interaction given by eq. (2). In fact, given the three-quark state with color, spin and isospin quantum numbers, we can extend the calculations to compute the interaction energies for N and Δ states (see Appendix A), i.e.,

$$\langle N|H_{\text{int}}|N\rangle = -8A \quad \text{and} \quad \langle \Delta|H_{\text{int}}|\Delta\rangle = 8A. \tag{11}$$

These results can be used to generate $N-\Delta$ mass difference, viz.,

$$M_\Delta - M_N = 16A. \tag{12}$$

The fact is that experimentally $M_\Delta - M_N = 300$ MeV enables us to determine the q–q interaction strength parameter $A = (75/4)$ MeV, appearing in eq. (2).

It is instructive to note that the diquark energy given by eq. (9) when multiplied by a factor of 3 (taking account of three pairs of quarks in a Δ-state) just reproduces the interaction energy in the Δ-state. Here it should be pointed out that DS focus exclusively on the $\{\bar{3}, 0\}$ state, ignoring thereby the role of $\{\bar{3}, 1\}$ which is not only solely responsible for the expectation value in the Δ-state but also contributes to the interaction energy in the N-state as well. It is important to realize that the pair of (u, d) quarks has both the components belonging to isospin $I = 0$ (antisysmmetric) and $I = 1$ (symmetric) states. Therefore, if the quarks are in a spatially symmetric configuration, then not only the $\{\bar{3}, 0\}$ and $\{6, 1\}$ configurations but also the $\{\bar{3}, 1\}$ and $\{6, 0\}$ can be realized in accordance with the Fermi statistics.

3. Interaction energies of two diquark composites in different color and spin states

We make use of considerations, mentioned in the pervious section, to obtain the expectation values of H_{int} in different dq–dq states. As the model of DS seems to have some limitations, their results cannot be retrieved in the present framework, particularly after accounting for the quark exchange interactions between diquarks.

Before proceeding further, we elaborate here on the symbols used for various color and spin states of a diquark. In compact form we represent the color antitriplet $(\bar{3})$ and sextet (6) states of the two quarks as

$$|\bar{3}\rangle \equiv |\mu\rangle = \tfrac{1}{\sqrt{2}}\varepsilon_{\mu\beta\gamma}u^\beta d^\gamma;\ (\mu = 1, 2, 3),$$

and

$$|6\rangle = |\mu\rangle = h_{\mu\beta\gamma}u^\beta d^\gamma,$$

where $\varepsilon_{\mu\beta\gamma}$ is the standard symbol and $h_{\mu\beta\gamma}$ is defined as follows:

$$h_{\mu\beta\gamma} = 1, \quad \text{for} \quad \mu = 1, 2, 3 \quad \text{and} \quad \beta = \gamma = (1, 2, 3);$$

and

$$h_{\mu,\beta,\gamma} = (1/\sqrt{2}) \quad \text{for} \quad \mu = 4, 5, 6 \quad \text{and} \quad \beta \neq \gamma;$$
$$(\beta, \gamma) = \{(1, 2), (2, 3), (3.1)\}.$$

Here the indices $\alpha(\beta \text{ or } \gamma) = 1, 2, 3$ refer to the color states. Similarly, for the spin singlet and triplet states, we write

$$|\bar{1}\rangle \equiv |i\rangle = \frac{1}{\sqrt{2}} \varepsilon_{imm'} X^m(u) X^{m'}(d); \quad i = 1, m \neq m' = \pm 1/2,$$

and

$$|3\rangle \equiv |i\rangle = g_{imm'} X^m(u) X^{m'}(d),$$

where

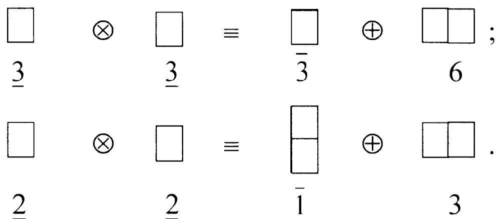
$$g_{imm'} = 1 \quad \text{for} \quad i = 1, 2 \quad \text{and} \quad m = m' = \pm 1/2;$$

$$g_{imm'} = g_{im'm} = 1/\sqrt{2} \quad \text{for} \quad i = 3 \quad \text{and} \quad m \neq m' = \pm 1/2.$$

After introducing these compact notations, it becomes easier to express two diquark states. In this way the combined color–spin states denoted by $\{\underline{C}, \underline{S}\}$ of the diquark clusters are:

$$\text{(a)} \quad |\bar{3}, 1\rangle \equiv |\mu, i\rangle = \frac{1}{\sqrt{2}} \varepsilon_{\mu\beta\gamma} u^\beta d^\gamma \cdot \frac{1}{\sqrt{2}} \varepsilon_{imm'} X^m(u) X^{m'}(d),$$
$$\text{(b)} \quad |\bar{3}, 0\rangle \equiv |\mu, i\rangle = \frac{1}{\sqrt{2}} \varepsilon_{\mu\beta\gamma} u^\beta d^\gamma \cdot g_{imm'} X^m(u) X^{m'}(d),$$
$$\text{(c)} \quad |6, 1\rangle \equiv |\mu, i\rangle = h_{\mu\beta\gamma} u^\beta d^\gamma \cdot \frac{1}{\sqrt{2}} \varepsilon_{imm'} X^m(u) X^{m'}(d),$$
$$\text{(d)} \quad |6, 1\rangle \equiv |\mu, i\rangle = h_{\mu\beta\gamma} u^\beta d^\gamma g_{imm'} X^m(u) X^{m'}(d).$$

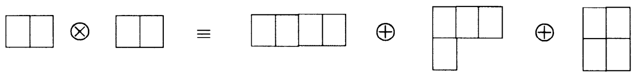
For the purpose of computing the interaction energies between two diquark composites, next we construct the irreducible representations in both the color and spin states of the dq–dq system. In fact, these representations must be the manifestation of proper eigenstates of the total Hamiltonian. Thus the diquark states arising from the coupling of the color-triplets of two quarks and spin doublets of two quarks can be constructed respectively as follows:



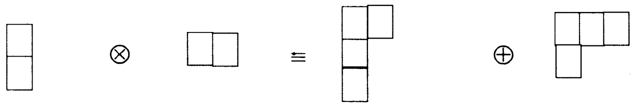
For the case of two-diquark composite, the possible color states can be constructed from the above representations as follows:

(i) 

$$|\bar{3}\rangle_{\mathrm{dq}_1} \otimes |\bar{3}\rangle_{\mathrm{dq}_2} \equiv |3_a\rangle_{\mathrm{dq}_1\mathrm{dq}_2} \oplus |6_{as}\rangle_{\mathrm{dq}_1\mathrm{dq}_2};$$

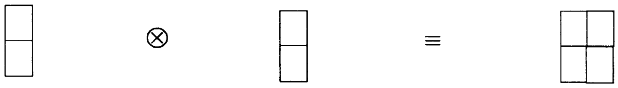
(ii) 

$$|6\rangle_{\mathrm{dq}_1} \otimes |6\rangle_{\mathrm{dq}_2} \equiv |15_s\rangle_{\mathrm{dq}_1\mathrm{dq}_2} \oplus |15_{sa}\rangle_{\mathrm{dq}_1\mathrm{dq}_2} \oplus |6_s\rangle_{\mathrm{dq}_1\mathrm{dq}_2};$$

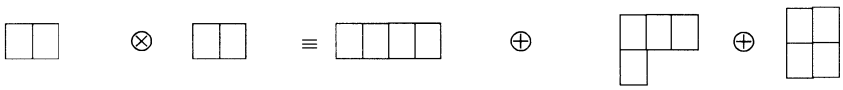
(iii) 

$$|\bar{3}\rangle_{\mathrm{dq}_1} \otimes |6\rangle_{\mathrm{dq}_2} \equiv |3_s\rangle_{\mathrm{dq}_1\mathrm{dq}_2} \oplus |15_{as}\rangle_{\mathrm{dq}_1\mathrm{dq}_2},$$

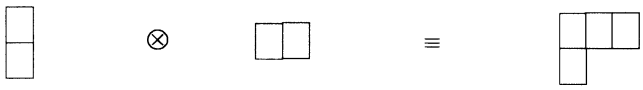
where the suffixes dq_1 and/or dq_2 denote a diquark state of $u(1)d(2)$ and/or $u(3)d(4)$. Similarly, in the spin space of the two diquarks, we have

(i) 

$$|1, S_1 = 0\rangle_{\mathrm{dq}_1} \otimes |1, S_2 = 0\rangle_{\mathrm{dq}_2} \equiv |1_a, S = 0\rangle_{\mathrm{dq}_1\mathrm{dq}_2};$$

(ii) 

$$|3, S_1 = 1\rangle_{\mathrm{dq}_1} \otimes |3, S_2 = 1\rangle_{\mathrm{dq}_2} \equiv |5, S = 2\rangle_{\mathrm{dq}_1\mathrm{dq}_2} \oplus |3_s, S = 1\rangle_{\mathrm{dq}_1\mathrm{dq}_2} \oplus |1_s, S = 0\rangle_{\mathrm{dq}_1\mathrm{dq}_2};$$

(iii) 

$$|1, S_1 = 0\rangle_{\mathrm{dq}_1} \otimes |3, S_2 = 1\rangle_{\mathrm{dq}_2} \equiv |3_s, S = 1\rangle_{\mathrm{dq}_1\mathrm{dq}_2}.$$

We, therefore, get several possibilities of coupling the color and spin states of the two-diquark systems. Labeling the color states by $\underline{C}$ and spin states by $\underline{S}$, the interaction energy between two-diquark systems is calculated by taking the expectation value of the Hamiltonian, H_{int}, viz.,

$$\langle(\underline{C}, \underline{S})_{\mathrm{dq}_1\mathrm{dq}_2}| - A \sum_{(i,j)=(13),(23),(14),(24)} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j)|(\underline{C}, \underline{S})_{\mathrm{dq}_1\mathrm{dq}_2}\rangle. \tag{13}$$

Here, we recognize that the existence of diquark dq_1 consisting of quarks $u(1)$ and $d(2)$ is already taking care of the interaction between quarks 1 and 2 and in the same way the diquark dq_2 composite of quarks 3 and 4 has included the pair interaction (3,4). That is why we have not included such pairs, namely (i, j) = (1, 2), (3, 4) in the summation index above.

The results of the calculations of the interaction energies are presented in table 1. In table 1, the two-diquark clusters representing the color states are specified along the column while the corresponding spin states are indicated along the row. From the table it is clear that the effective dq–dq interaction is essentially repulsive in nature – the strength varying from A to $34A/3$ sensitively dependent on the color and spin states of the dq–dq system. There is only one state characterized by spin 2 and color $|15_s\rangle$, which has an overall attraction. This is presumably because for the color symmetric state $|15_s\rangle$ the q–q interaction operates, in accordance with the Pauli principle, and for the color antisymmetric pairs it gives rise to an effective attraction.

To establish a connection with the coupling parameter λ in the ϕ^4-theory, we follow the procedure prescribed in [2]. Thus, considering the diquarks confined to a cavity of radius R, the ground state wave function is written as

$$\phi(x) = N j_0(x) \mathrm{e}^{-iEt}, \quad \text{where} \quad x = pr, \quad E = (p^2 + m^2)^{1/2}$$

and the normalization constant N is given by

$$N = \frac{1}{(4\pi R^3)^{1/2}} \, \frac{1}{\sqrt{E} j_1(pR)}.$$

Defining the interaction energy between two diquarks as $(\Delta E)_{\mathrm{dq_1dq_2}}$, we have from eq. (13)

$$(\Delta E)_{\mathrm{dq_1dq_2}} \equiv \langle (\underline{C}, \underline{S})_{\mathrm{dq_1dq_2}} | H_{\mathrm{int}} | (\underline{C}, \underline{S})_{\mathrm{dq_1dq_2}} \rangle. \tag{14}$$

Following DS, we relate this result with the one derived in the bag model as

$$(\Delta E)_{\mathrm{dq_1dq_2}} = 2\lambda \int \mathrm{d}^3x |\phi(x)\phi^\dagger(x)|^2$$
$$= \frac{0.34\lambda}{E^2R^3}.$$

Thus the coupling parameter λ can be expressed by the equation,

$$\lambda = \frac{(\Delta E)_{\mathrm{dq_1dq_2}} E^2 R^3}{0.34}.$$

Table 1. Interaction energies between two-diquark clusters in various color and spin state configurations. The color states are given in the column, while the spin states are represented along the row.

$\underline{C}\backslash\underline{S}$	$\lvert 1_a\rangle$	$\lvert 5\rangle$	$\lvert 3_s\rangle$	$\lvert 1_s\rangle$	$\lvert 3_a\rangle$
$\lvert 3_a\rangle$	$3A$	$4A$	$2A$	A	$(14/3)A$
$\lvert 6_{as}\rangle$	$(5/2)A$	$(2/3)A$	$(13/3)A$	$(37/6)A$	$3A$
$\lvert 15_s\rangle$	$2A$	$-(8/3)A$	$(20/3)A$	$(34/3)A$	$(4/3)A$
$\lvert 15_{sa}\rangle$	$3A$	$4A$	$2A$	A	$(14/3)A$
$\lvert 6_s\rangle$	$(5/2)A$	$(2/3)A$	$(13/3)A$	$(37/6)A$	$3A$
$\lvert 3_s\rangle$	$3A$	$4A$	$2A$	A	$(14/3)A$
$\lvert 15_{as}\rangle$	$(5/2)A$	$(2/3)A$	$(13/3)A$	$(37/6)A$	$3A$

The values of $(\Delta E)_{\mathrm{dq_1 dq_2}}$ for different color and spin configurations, as listed in table 1, can now be used to evaluate the corresponding values of λ. In particular, in the case of $\{\overline{3},0\}$ state, which has been exclusively considered in ref. [2], we find by substituting $(\Delta E)_{\mathrm{dq_1 dq_2}} = 3A$ (see second column and second row of table 1) the value of λ to be 6.95, which, when compared with that obtained by DS, i.e., 27.8, turns out to be much smaller.

It may be noted that in our case the interaction energy is only $3A$ whereas it turns out to be $24A$ (see eq. (11) of ref. [2]). This is due to the fact that in ref. [2] the diquark–diquark interaction energy has been calculated in the spirit of ϕ^4-field theory, which happens to be attractive in nature and turns out to be $-8A$. By excluding the interaction energy of each of the diquarks calculated from eqs (7)–(10), the effective interaction has been obtained by DS as repulsive, i.e., $-8A + 32A = 24A$. In the present approach, however, considering the pairwise interactions between different quarks of the diquark systems and excluding the q–q interaction of the diquarks, the interaction Hamiltonian of ref. [2] predicts a net repulsion in a natural way. In other words, the interaction energies not only in the color anti-triplet spin singlet, i.e., $|\bar{3}, 0\rangle$ states, are important but also other states like $|\bar{3}, 1\rangle$, $|6, 0\rangle$, and $|6, 1\rangle$ have an important role to play. Moreover, our approach also accounts for the extended character of diquarks contrary to the point structure of diquarks as considered by DS.

4. Concluding discussion

In view of the fact that multidiquark states can play an important role in different situations such as in the studies of quark–gluon, diquark–gluon or quark–diquark–gluon plasmas and also in the studies of elementary particle reactions involving baryons, in the present work we have tried to construct these states within the framework of a constituent quark model in conjunction with the generalized Pauli principle. For this purpose, the interactions arising from color, spin and isospin degrees of freedom of two-flavor quark system, as consisting of u and d quarks are considered. As a matter of fact it is the generalized Pauli principle that guides us in picking up the suitable terms out of the lot in a composite state characterized by color, spin and isospin. In the prescription followed here and outlined in Appendix A while the role of color and spin degrees of freedom of quarks is considered in an explicit manner, the account of isospin, however, remains implicit in the sense that for the u–d quark system the construction of such states is straightforward. As a matter of fact, the results obtained here have been derived in the framework of $SU(3)_{\mathrm{color}} \otimes SU(2)_{\mathrm{spin}}$.

To be more specific, with regards to these results the following remarks are in order:

1. These estimates are done by ignoring not only the dynamic interactions (i.e., the role of gluons) between the quarks and subsequently between the diquarks but also the three- and higher-body effects. The presence of gluons will, however, bring in the concept of dynamic interactions between the quarks and diquarks. This can be carried out possibly through potential models [16].

To that extent the present results, although model independent, have some limitations.

2. It may be added that in this investigation, for simplicity of calculations, we have not taken account of strange quarks. From the point of view of group theoretical considerations, we need to extend the present approach for $SU(2)$ to $SU(3)$ (flavor symmetry). The work in this direction is in progress.
3. The values of λ calculated for different configurations can lead to several improvements in the numerical form of the equation of state for diquark–gluon gas/soup [17] and thereby affecting the value of mass and radius of diquark stars. Such studies are in progress.

Appendix A

In this appendix we illustrate in detail the procedure for evaluating the interaction energy of the three quark Δ-state using the pairwise q–q interaction Hamiltonian

$$H_{\text{int}} = -A \sum_{i<j=1,2,3} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j). \tag{A1}$$

Clearly, the interaction has only three terms. Now using the standard result for the expressions of the projection operators, P_{ij}, for the symmetric and antisymmetric pairs in color, spin, isospin and configuration spaces (see eqs (3)–(5)), we employ the generalized Pauli's principle to write

$$P^c_{ij} P^\sigma_{ij} = -P^\tau_{ij} P^x_{ij} \tag{A2}$$

or

$$\left(\frac{1}{2}\vec{\lambda}_i \cdot \vec{\lambda}_j + \frac{1}{3}\right)\left(\frac{1+\vec{\sigma}_i \cdot \vec{\sigma}_j}{2}\right) = -\left(\frac{1+\vec{\tau}_i \cdot \vec{\tau}_i}{2}\right) P^x_{ij}. \tag{A3}$$

Here it may be pointed out that even when we consider the pair of (ud) quarks from the point of view of isospin, it has equal probability of existing in the isospin $I = 1$ (symmetric) and $I = 0$ (antisymmetric) states, viz.,

$$|ud\rangle = (1/\sqrt{2})\left[\left(\frac{|ud\rangle + |du\rangle}{\sqrt{2}}\right)(I=1,\text{symmetric}) + \left(\frac{|ud\rangle - |du\rangle}{\sqrt{2}}\right)(I=0,\text{antisymmetric})\right]. \tag{A4}$$

Now in the Δ-state, all the three quark pairs are in the symmetric isospin state giving the total isospin as 3/3. Similarly, as the total spin is also 3/2 the quark pairs in the spin state are also in the symmetric state. Assuming that in the configuration x-space the quarks are in the symmetric state, to satisfy the generalized Pauli's principle, the quark pairs have to be in the antisymmetric color state ($P^c_{ij} = -1$). With these considerations, we rewrite eq. (A3) as

$$\frac{1}{4}(\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) + \frac{1}{6}(\vec{\sigma}_i \cdot \vec{\sigma}_j) + \frac{1}{4}(\vec{\lambda}_i \cdot \vec{\lambda}_j) + \frac{1}{6} = -\left(\frac{1 + \vec{\tau}_i \cdot \vec{\tau}_j}{2}\right) P_{ij}^x$$

or

$$\begin{aligned}(\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) &= -(\vec{\lambda}_i \cdot \vec{\lambda}_j) - \frac{2}{3}(\vec{\sigma}_i \cdot \vec{\sigma}_j) - 2(1 + \vec{\tau}_i \cdot \vec{\tau}_j)P_{ij}^x - \frac{2}{3} \\ &= \frac{8}{3} - \frac{2}{3} - 4 - \frac{2}{3} = -\frac{8}{3}.\end{aligned} \tag{A5}$$

Thus,

$$\langle \Delta | H_{\text{int}} | \Delta \rangle = -3A(-8/3) = 8A, \tag{A6}$$

the result given in eq. (11).

Note that in the paper of DS, this energy for the Δ-state has been evaluated as $16A$ (eq. (2a) of DS). This, in our opinion, is due to the fact that the interaction Hamiltonian H_{int} given by eq. (1) (DS) has in its sum six terms (12, 21, 13, 31, 23, 32), since the indices i and j are running over from 1 to 3 without any constraint (such as $i < j$). This is double counting. This double counting has been avoided in evaluating the matrix elements in our results contrary to the results of DS (see eqs (2)–(4)). Similarly the matrix element for the N-state works out to be

$$\langle N | H_{\text{int}} | N \rangle = -8A, \tag{A7}$$

instead of $-16A$ as obtained by DS (see eq. (2b)). Clearly, therefore in our case

$$M_\Delta - M_N = 16A = 300 \text{ MeV},$$

giving the value of A as $A = 75/4$ MeV. This is just twice the value of A obtained by DS.

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