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Discriminating neutrino mass models using Type-II see-saw formula

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Abstract. An attempt has been made to discriminate theoretically the three possible patterns of neutrino mass models,viz., degenerate, inverted hierarchical and normal hierachical models, within the framework of Type-II see-saw formula. From detailed numerical analysis we are able to arrive at a conclusion that the inverted hierarchical model with the same CP phase (referred to as Type [IIA]), appears to be most favourable to survive in nature (and hence most stable), with the normal hierarchical model (Type [III]) and inverted hierarchical model with opposite CP phase (Type [IIB]), follow next. The degenerate models (Types [IA,IB,IC]) are found to be most unstable. The neutrino mass matrices which are obtained using the usual canonical see-saw formula (Type I), and which also give almost good predictions of neutrino masses and mixings consistent with the latest neutrino oscillation data, are re-examined in the presence of the left-handed Higgs triplet within the framework of non-canonical see-saw formula (Type II). We then estimate a parameter (the so-called discriminator) which may represent the minimum degree of suppression of the extra term arising from the presence of left-handed Higgs triplet, so as to restore the good predictions on neutrino masses and mixings already acquired in Type-I see-saw model. The neutrino mass model is said to be favourable and hence stable when its canonical see-saw term dominates over the non-canonical (perturbative) term, and this condition is used here as a criterion for discriminating neutrino mass models.

Keywords. Neutrino mass models; neutrino mixings; see-saw mechanism; solar neutrinos; atmospheric neutrinos.

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1. Introduction

Recent neutrino oscillation experiments [1] have provided important information on the nature of neutrino masses and mixings, and have also tremendously strengthened our understanding of neutrino oscillation. However, we are still far from a complete understanding of the nature of neutrinos. An important question that still remains open is the pattern of the three neutrino masses [2], though some reactor-based experiments [3] will be able to help us understand it in the near future. We summarise in table 1 [1] the most recent results of the three-flavour neutrino oscillation parameters from global data including solar [4], atmospheric [5], reactor (KamLAND [6] and CHOOZ [7]) and accelerator (K2K [8]).

At present the LSND data [9] fail to agree with the rest of the global data, and a further confirmation of the LSND signal by the MiniBooNE experiment [10] is very desirable. There are also some complementary information from other sources. The recent analysis of the WMAP Collaboration [11,12] gives the bound $\sum_i |m_i| < 0.69$ eV (at 95% CL). The bound from the $0\nu\beta\beta$-decay experiment is $|m_{ee}| < 0.2$ eV [13,14]. However the value of the $|m_{ee}|$ from the recent claim [15] for the discovery of the $0\nu\beta\beta$ process at 4.2σ level, is $|m_{ee}| \sim (0.2–0.6)$ eV.

Since the above data on solar and atmospheric neutrino oscillation experiments give only the mass square differences, we usually have three models [15a] of neutrino mass levels [16]:

Degenerate (*Type* [*I*]): $m_1 \simeq m_2 \simeq m_3 \simeq 0.4$ eV $\gg \Delta m^2_{21}$.

Inverted hierarchical (*Type* [*II*]): $m_1 \simeq m_2 \gg m_3$ with $\Delta m^2_{23} = m^2_3 - m^2_2 < 0$ and $m_{1,2} \simeq \sqrt{\triangle m^2_{23}} \simeq 0.052$ eV.

Normal hierarchical (*Type* [*III*]): $m_1 \ll m_2 \ll m_3$, and $\Delta m^2_{23} = m^2_3 - m^2_2 > 0$; and $m_3 \simeq \sqrt{\Delta m^2_{23}} \simeq 0.052$ eV, $m_2 \simeq 0.009$ eV.

(Appendix A presents a classification list of the zeroth-order left-handed Majorana neutrino mass matrices which can explain the above three patterns of neutrino masses when appropriate perturbations are added as in Appendix B).

The result of $0\nu\beta\beta$ decay experiment [15], if confirmed, would be able to rule out Type [II] and Type [III] neutrino mass models right away, and points to Type [I] or to models with more than three neutrinos [1]. Again, the WMAP limit [11] (at least for three degenerate neutrinos), $|m| < 0.23$ eV also would rule out Type [I] neutrino model, or at least it could lower the parameter space for the degenerate model [1]. It also gives further constraint on $|m_{ee}|$. However, a final choice among these three models is a difficult task. At the moment we are in a very confusing state [2,3]. The work in the present paper is a modest attempt from a theoretical point of view to discriminate the three neutrino mass models using the Type-II see-saw formula (non-canonical see-saw formula) for neutrino masses.

The paper is organized as follows. In §2, we outline the main points of the Type-II see-saw formula and a criterion for discriminating the neutrino mass models. We

Table 1. Summary of the most recent observation data of the neutrino oscillation parameters.

Parameter	3σ level
Δm^2_{21} (10^{-5} eV^2)	7.2–9.5
Δm^2_{23} (10^{-3} eV^2)	1.28–4.17
$\tan^2\theta_{12}$	0.27–0.59
$\sin^2 2\theta_{23}$	0.86–1.00
$\sin\theta_{13}$	$\leq$0.22

carry out numerical computations in §3 and present our main results. Section 4 concludes with a summary and discussion.

2. Type-II see-saw formula and neutrino mass matrix

The canonical see-saw mechanism (generally known as Type-I see-saw formula) [17] is the simplest and the most appealing mechanism for generating small neutrino masses and lepton mixings. There is also another type of non-canonical see-saw formula where a left-handed Higgs triplet Δ_L picks up a vacuum expectation value (VEV) in the left–right symmetric GUT models such as $SO(10)$. This is expressed as

$$m_{LL} = m_{LL}^{\rm II} + m_{LL}^{\rm I}, \tag{1}$$

where the usual Type-I see-saw formula is given by the expression

$$m_{LL}^{\rm I} = -m_{LR} M_{RR}^{-1} m_{LR}^T. \tag{2}$$

The sum of two terms $m_{LL}^{\rm II}$ and $m_{LL}^{\rm I}$ in eq. (1) is widely referred to as the Type-II see-saw formula in [18]. We follow this convention in the present paper. However, such convention is no longer unique in [19] as some authors prefer to use the Type-II see-saw formula as simply the first term $m_{LL}^{\rm II}$ arising from the coupling to the left-handed triplet-Higgs field. This ambiguity is partially removed in [20] by adopting the terminology such as 'mixed Type-II see-saw formula' to represent the sum of the two terms and 'pure Type-II see-saw formula' to represent only the first term. Type-II see-saw formula is also different from Type-III see-saw formula which contains $SO(10)$ singlet neutrinos [20a].

In eq. (2) m_{LR} is the Dirac neutrino mass matrix in the left–right convention and the right-handed Majorana neutrino mass matrix $M_{RR} = v_R f_R$ with v_R being the vacuum expectation value (VEV) of the Higgs fields imparting mass to the right-handed neutrinos and f_R is the Yukawa coupling matrix. The second term $m_{LL}^{\rm II}$ in eq. (1) is due to the $SU(2)_L$ Higgs triplet, which can arise, for instance, in a large class of $SO(10)$ models in which the $(B-L)$ symmetry is broken by a 126 Higgs field. In the usual left–right symmetric theories, $m_{LL}^{\rm II}$ and M_{RR} are proportional to the vacuum expectation values (VEVs) of the electrically neutral components of scalar Higgs triplets, i.e., $m_{LL}^{\rm II} = f_L v_L$ and $M_{RR} = f_R v_R$, where $v_{L,R}$ denotes the VEVs and $f_{L,R}$ is a symmetric 3×3 matrix. By acquiring the VEV v_R, breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to $SU(2)_L \times U(1)_Y$ is achieved. The left–right symmetry demands the presence of both $m_{LL}^{\rm II}$ and M_{RR}, and in addition, it holds $f_R = f_L = f$. The induced VEV for the left-handed triplet v_L is given by $v_L = \gamma M_W^2/v_R$, where the weak scale $M_W \sim 82$ GeV such that $|v_L| \ll M_W \ll |v_R|$ [21,22,22a]. In general γ is a function of various couplings, and without fine tuning γ is expected to be of the order of unity ($\gamma \sim 1$). Type-II see-saw formula in eq. (1) can now be expressed as

$$m_{LL} = \gamma (M_W/v_R)^2 M_{RR} - m_{LR} M_{RR}^{-1} m_{LR}^T. \tag{3}$$

In the light of the above Type-II see-saw formula in eq. (3), the neutrino mass matrices, m_{LL} in the literature, are constructed in view of the following three assumptions: (a) $m_{LL}^{\rm II}$ is dominant over $m_{LL}^{\rm I}$, (b) both terms are contributing with comparable amounts, and (c) m_{LL}^{I} is dominant over $m_{LL}^{\rm II}$. In recent times, Case (a) has gathered momentum because in certain $SO(10)$ models, large atmospheric neutrino mixing and $b-\tau$ unification are the natural outcomes of this dominance [23,24]. In some models this leads to degenerate model [25] which imparts bimaximal mixings, as well as extra contribution to leptogenesis [25–27]. However all these cases are not completely free from certain assummptions and ambiguities. It can be stressed here that the two terms $m_{LL}^{\rm I}$ and $m_{LL}^{\rm II}$ in eq. (1) are not completely independent. The term $m_{LL}^{\rm II}$ is heavily constrained through the definition of v_R as seen in eq. (3). Usually the value of v_R is fixed through the definition $M_{RR} = v_R f$ present in the canonical term $m_{LL}^{\rm I}$. There is no ambiguity in the definition of v_R with the first term, and it also does not affect $m_{LL}^{\rm I}$ as long as M_{RR} is taken as a whole in the expression. However, it severely affects the second term $m_{LL}^{\rm II}$ where v_R is entered alone, and different choices of v_R in M_{RR} would lead to different values of $m_{LL}^{\rm II}$. This ambiguity is seen in the literature where different choices of v_R are made according to convenience [22,23,26,28]. However, in the present paper we shall always take v_R as the heaviest right-handed Majorana neutrino mass eigenvalue M_{N3} obtained after the diagonalization of the mass matrix M_{RR}. This is true for the physical right-handed Majorana mass matrix. Once we adopt this convention, there is little freedom for the second term $m_{LL}^{\rm II}$ in eq. (3) to have arbitrary value of v_R. We also assume that the $SU(2)_R$ gauge symmetry breaking scale v_R is the same as the scale of the breakdown of parity [29].

The present work is carried out in the line of Cases (b) and (c) cited above, but the choice of which term is dominant over other, is not arbitrary any more. We carry out a complete analysis of the three models of neutrino mass matrices (see Appendix B for the expressions of M_{RR} and $m_{LL}^{\rm I}$ generated in Type-I see-saw formula) where the (already acquired) good predictions of neutrino masses and mixings in the canonical term $m_{LL}^{\rm I}$, are subsequently spoiled by the presence of second (non-canonical) term $m_{LL}^{\rm II}$ when $\gamma = 1$ in m_{LL}. We make a search programme for finding the values of the 'minimum departure' of γ from the canonical value of one, i.e., $\gamma < 1.0$, in which the good predictions of neutrino masses and mixing parameters can again be restored in m_{LL}. We propose here a bold hypothesis which acts as a sort of 'natural selection' for the survival of neutrino mass models which enjoy the 'least value of deviation' of γ from unity. In other words, the value of γ is just enough to suppress the perturbation effect arising from Type-II see-saw formula. Nearer the value of γ to one, better the chance for the survival of the model in question. Thus the value of γ is an important parameter for the proposed natural selection of the neutrino mass models in question. The condition $\gamma > 1$ implies that $m_{LL}^{\rm I}$ is dominant over $m_{LL}^{\rm II}$ and hence the model is favourable to survive under the present hypothesis.

The above criterion for the favourable selection imposes certain constraints on the neutrino mass models which one can obtain in the following way, at least for the heaviest neutrino mass eigenvalue (without considering mixings). If the neutrino masses are solely determined from the second term of eq. (3), then the first term must be less than the order which is dictated by the particular pattern of neutrino mass spectrum. In this view, the largest contribution of neutrino mass from the first term must be less than about 0.05 eV for both normal hierarchical and inverted hierarchical models; and about 0.5 eV for degenerate model as the data suggest [1]. Thus we have the bound for the natural selection:

$$m_{LL}^{\mathrm{I}} > v_L f. \tag{4}$$

Denoting the heaviest right-handed neutrino mass as v_R and taking $M_W \sim 82$ GeV [22] in the expression of v_L, the following lower bounds on v_R for the natural selection are obtained:

For normal hierarchical and inverted hierarchical model:

$$v_R > \gamma 1.345 \times 10^{14} \text{ GeV}. \tag{5}$$

For degenerate model:

$$v_R > \gamma 1.345 \times 10^{13} \text{ GeV}. \tag{6}$$

The above bounds just indicate the approximate measure of the degree of natural selection, but a fuller analysis will take both the terms of the Type-II see-saw formula in the 3×3 matrix form. This will give all the three mass eigenvalues as well as mixing angles. This numerical analysis will be carried out in the next section. It is clear from eqs (5) and (6) that any amount of arbitrariness in fixing the value of v_R in M_{RR} will distort the conclusion.

3. Numerical calculations and results

For a full numerical analysis we refer to our earlier papers [30] where we performed the investigations on the origin of neutrino masses and mixings which can accommodate LMA MSW solution for solar neutrino anomaly and the solution of atmospheric neutrino problem within the framework of Type-I see-saw formula. Normal hierarchical, inverted hierarchical and quasi-degenerate neutrino mass models were constructed from the non-zero textures of the right-handed Majorana mass matrix M_{RR} along with diagonal form of m_{LR} being taken as either the charged lepton mass matrix (Case i) [28] or the up-quark mass matrix (Case ii) [30]. However, a general form of the Dirac neutrino mass matrix is given by

$$m_{LR} = \begin{pmatrix} \lambda^m & 0 & 0 \\ 0 & \lambda^n & 0 \\ 0 & 0 & 1 \end{pmatrix} m_f, \tag{7}$$

where m_f corresponds to $m_\tau \tan\beta$ for $(m,n) = (6,2)$ in the case of charged lepton (Case i) and m_t for $(m,n) = (8,4)$ in the case of up-quarks (Case ii). Here λ can pick up any value between 0.2 and 0.3 for the Dirac neutrino mass matrix. The main assumption is that neutrino mass mixings can have the origin from the texture of right-handed neutrino mass matrix only through the interplay of see-saw mechanism [31]. This can be understood from the following operation [32,33] where M_{RR} can be transformed in the basis in which m_{LR} is approximately diagonal [33a]. Using the diagonalization relation, $m_{LR}^{\mathrm{diag}} = U_L m_{LR} U_R^\dagger$, we have,

$$m^{\mathrm{I}}_{LL} = -m_{LR} M_R^{-1} m_{LR}^T \simeq -m_{LR}^{\mathrm{diag}} M_{RR}^{-1} m_{LR}^{\mathrm{diag}},$$

where $U_R M_R^{-1} U_R^T = M_{RR}^{-1}$ and $U_L m^{\mathrm{I}}_{LL} U_L^T \simeq m^{\mathrm{I}}_{LL}$ by considering a simple assumption, $U_L \simeq 1$, since the Dirac neutrino mass matrices are hierarchical in nature and the CKM mixing angles of the quark sector are relatively small. In such a situation U_L slightly deviates from 1, i.e., $U_L \simeq V_{\mathrm{CKM}}$, and it hardly affects the numerical accuracy [32] for practical purposes. Here M_{RR} is the new RH matrix defined in the basis of diagonal m_{LR} matrix. We thus express M_{RR} in the most general form as its origin is quite different from those of the Dirac mass matrices in an underlying grand unified theory. As usual the neutrino mass eigenvalues and neutrino mixing matrix known as MNS leptonic mixing matrix [30] are obtained through the diagonalization of m_{LL},

$$m_{LL}^{\mathrm{diag}} = V_{\nu L} m_{LL} V_{\nu L}^T = \mathrm{Diag}(m_1, m_2, m_3),$$

and the neutrino mixing angles are then extracted from the MNS leptonic mixing matrix defined by $V_{\mathrm{MNS}} = V_{\nu L}^{\dagger}$ in the basis where charged lepton mass matrix is diagonal.

An example: Normal hierarchical model (Type [III])

We then perform a detailed numerical analysis to search for the (discriminator) parameter γ which measures the least perturbation effects arising from the Type-II see-saw term. As a simple example, we take up the case for the normal hierarchical model (Type [III]) while the expressions for other models are relegated to Appendix B. Using the general expression for m_{LR} given in eq. (7) and the following texture for M_{RR} [30]:

$$M_{RR} = \begin{pmatrix} \lambda^{2m-1} & \lambda^{m+n-1} & \lambda^{m-1} \\ \lambda^{m+n-1} & \lambda^{m+n-2} & 0 \\ \lambda^{m-1} & 0 & 1 \end{pmatrix} v_0, \tag{8}$$

we get the neutrino mass matrix of Type [III] through eq. (2),

$$-m^{\mathrm{I}}_{LL} = \begin{pmatrix} -\lambda^4 & \lambda & \lambda^3 \\ \lambda & 1-\lambda & -1 \\ \lambda^3 & -1 & 1-\lambda^3 \end{pmatrix} m_0. \tag{9}$$

Here we have $m_0 = m_f^2/v_0 = 0.03$ eV. For Case (i) we have fixed the value of v_0 as 8.92×10^{13} GeV, taking (m, n) as $(6, 2)$ and the input values $m_\tau = 1.292$ GeV, $\tan\beta = 40$ and $\lambda = 0.3$. The diagonalization of M_{RR} gives the three corresponding RH Majorana neutrino masses $M_{RR}^{\mathrm{diag}} = (5.74\times10^9, 7.04\times10^{10}, 8.92\times10^{13})$ GeV. As already stated, the mass matrix in eq. (9) predicts correct neutrino mass parameters and mixing angles consistent with recent data [30]: $\Delta m^2_{21} = 9.04 \times 10^{-5}$ eV^2, $\Delta m^2_{23} = 3.01 \times 10^{-3}$ eV^2, $\tan^2\theta_{12} = 0.55$, $\sin^2 2\theta_{23} = 0.98$, $\sin\theta_{13} = 0.074$.

In the next step we take up the additional contribution arising from the second term $m^{\mathrm{II}}_{LL} = \gamma (M_W/v_R)^2 M_{RR}$ in Type-II see-saw formula in eq. (3). When $\gamma = 1$, all the good predictions of neutrino masses and mixings already had in m^{I}_{LL}, are spoiled. This means that m^{II}_{LL} dominates over m^{I}_{LL}, and we have to explore values of $\gamma < 1$. The value of γ for the 'least deviation from canonical value of one', which could restore the good predictions in m_{LL}, is again obtained through a search programme. The predictions are: $\gamma \simeq 0.007$ leading to $\Delta m^2_{21} = 9.41 \times 10^{-5}$ eV^2, $\Delta m^2_{23} = 2.98 \times 10^{-3}$ eV^2, $\tan^2\theta_{12} = 0.54$, $\sin^2 2\theta_{23} = 0.98$, $\sin\theta_{13} = 0.09$. Here the solar mixing angle in terms of $\tan^2\theta_{12}$ falls in the 'light side', $\tan^2\theta_{12} < 1$, for the usual sign convention $\Delta m^2_{21} = m^2_2 - m^2_1 > 0$ [34,35].

For Case (ii) when $(m, n) = (8, 4)$ in eq. (7), we take the input value $m_t = 82.43$ GeV at the high scale. We have again the final predictions from m_{LL}: $\gamma \simeq 0.007$, $M^{\rm diag}_{RR} = (1.18 \times 10^8, 1.45 \times 10^9, 2.267 \times 10^{14})$ GeV, $\Delta m^2_{21} = 9.18 \times 10^{-5}$ eV^2, $\Delta m^2_{23} = 2.80 \times 10^{-3}$ eV^2, $\tan^2\theta_{12} = 0.55$, $\sin^2 2\theta_{23} = 0.98$, $\sin\theta_{13} = 0.07$.

In Appendix B we list other textures of $m^{\rm I}_{LL}$ along with the corresponding M_{RR} textures for degenerate (Types [I(A,B,C)]) and inverted hierarchy (Types [II(A,B)]) [30]. We repeat the same procedure described above for all these cases and find out the corresponding values of γ.

We present here the main results of the analysis. In table 2 we present the predictions of the neutrino mass parameters and mixings in Type-I see-saw formula ($\gamma = 0$), taking mass matrices from Appendix B. Neutrino mass models of Type [IA] and Type [IIB] give inherently maximal solar angles compared to observed values, whereas Types [IB,IC] give lesser values of solar mixings, but within experimental bounds. Types [IIA] and [III] models predict very good solar mixing angles, $\tan^2\theta_{12} \simeq 0.50$. All other predictions are in excellent agreement with the recently observed data. We then calculate the right-handed (RH) neutrino masses in table 3 for both Cases (i) and (ii). The heaviest RH Majorana mass eigenvalue is taken as v_R scale for calculation of $m^{\rm II}_{LL}$. It is interesting to see in table 3 that only Type [II(A)] strongly satisfies the bounds given in eqs (5) and (6) when $\gamma = 0.1$. This roughly implies that inverted hierarchical model with the same CP phases, may lead to the best choice for nature and hence most stable in the presence of Higgs triplet, though a fuller analysis needs the full matrix form when all terms are present. This is followed by the normal hierarchical model (Type [III]) and inverted hierarchical model with opposite CP phase (Type [IIB]). $\gamma \simeq 10^{-2}$ for both of them.

Our main results on neutrino masses and mixings in Type-II see-saw formula are presented in table 4 for Case (i) and in table 5 for Case (ii). One particularly important parameter is the predicted value of γ in each case. From tables 4 and 5,

Table 2. Predicted values of the solar and atmospheric neutrino mass-squared differences and three mixing parameters extracted from $m^{\rm I}_{LL}$ (Type-I see-saw models) given in Appendix B.

Type	Δm^2_{21} (10^{-5} eV^2)	Δm^2_{23} (10^{-3} eV^2)	$\tan^2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin\theta_{13}$
IA	8.80	2.83	0.98	1.0	0.0
IB	7.91	2.50	0.27	1.0	0.0
IC	7.91	2.50	0.27	1.0	0.0
IIA	8.36	2.50	0.44	1.0	0.0
IIB	9.30	2.50	0.98	1.0	0.0
III	9.04	3.01	0.55	0.98	0.074

Table 3. The three right-handed Majorana neutrino masses for both Case (i) and Case (ii) in three patterns of neutrino mass models given in Appendix B. The $B - L$ symmetry breaking scale v_R is taken as the heaviest right-handed Majorana neutrino mass eigenvalue in the calculation (The values of v_0 are given in Appendix B.)

Type	Case (i): $\|M_{RR}^{\text{diag}}\|$ GeV	Case (ii): $\|M_{RR}^{\text{diag}}\|$ GeV
IA	1.33×10^7, 1.99×10^8, 3.33×10^{12}	7.95×10^4, 1.188×10^6, 8.456×10^{12}
IB	8.5×10^4, 1.54×10^{10}, 6.6×10^{12}	5.10×10^2, 9.26×10^7, 1.68×10^{13}
IC	8.5×10^4, 3.07×10^{11}, 3.33×10^{11}	5.10×10^2, 1.85×10^{10}, 8.41×10^{10}
IIA	2.87×10^7, 8.54×10^{11}, 5.95×10^{15}	5.85×10^5, 1.76×10^{10}, 1.49×10^{16}
IIB	5.0×10^9, 5.0×10^9, 4.8×10^{15}	1.02×10^8, 1.02×10^8, 1.18×10^{16}
III	5.74×10^9, 7.04×10^{10}, 8.92×10^{13}	1.18×10^8, 1.45×10^9, 2.27×10^{14}

Table 4. Predicted values of the solar and atmospheric neutrino mass-squared differences and three mixing parameters extracted from m_{LL} using the values of parameters given in table 3 and Appendix B, for Case (i) (choosing γ for best predictions has been explained in the text).

Type	γ	Δm_{21}^2 (10^{-5} eV^2)	Δm_{23}^2 (10^{-3} eV^2)	$\tan^2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin\theta_{13}$
IA	10^{-5}	8.45	2.73	0.98	1.00	0.0
IB	10^{-4}	7.97	2.30	0.28	1.00	0.0
IC	10^{-5}	7.93	2.47	0.27	1.00	0.0
IIA	0.1	8.20	2.50	0.49	1.00	0.0
IIB	0.009	9.40	2.40	0.98	1.00	0.01
III	0.007	9.41	2.98	0.54	0.98	0.09

Table 5. Predicted values of solar and atmospheric neutrino mass-squared differences, and three mixing parameters extracted from m_{LL} using the values of parameters given in table 3 and Appendix B for Case (ii) (choosing γ for best predictions has been explained in the text).

Type	γ	Δm_{21}^2 (10^{-5} eV^2)	Δm_{23}^2 (10^{-3} eV^2)	$\tan^2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin\theta_{13}$
IA	10^{-5}	8.56	2.74	0.98	1.00	0.0
IB	10^{-4}	7.69	2.30	0.27	1.00	0.0
IC	10^{-5}	7.69	2.54	0.29	1.00	0.0
IIA	0.1	8.3	2.5	0.47	1.00	0.0
IIB	0.02	9.40	2.40	0.98	1.00	0.0
III	0.007	9.18	2.80	0.55	0.98	0.07

we wish to draw a few conclusions that inverted hierarchical model with even CP phase (Type [IIA]) having $\gamma = 0.1$ is the most favourable model under the presence of $SU(2)_L$ triplet term $m_{LL}^{\rm II}$ in the Type-II see-saw formula. On such ground we can discriminate other models in favour of it. Next to it is the normal hierarchy model (Type [III]) with $\gamma = 0.007$ and inverted hierarchical model with odd CP phase (with $\gamma = 0.009$–0.02). In the present analysis the three degenerate models (Type [I(A,B,C)]) are not favourable at all as they predict $\gamma \sim 10^{-4}$ or lesser.

We also note the stability of these models under radiative corrections in MSSM for both neutrino mass splittings and mixing angles. For large $\tan\beta = 55$ where the effect of radiative corrections are relatively large, only two models, namely, inverted hierarchy [36] of Type [IIB] and normal hierarchy [37] of Type [III] are found stable under radiative corrections [38]. Following this result, the inverted hierarchy of Type [IIA] is less favourable than its counterpart, Type [IIB]. Type [IIA] is again having excellent solar mixing angle and Type [IIB] for maximal value. If one consider all these factors, normal hierarchical model (Type [III]) is free from any shortcoming and it may also represent the only natural choice [2] if we take into account the radiative corrections and correct solar mixings.

4. Summary and discussion

We summarize the main points of this work. We first generate the three neutrino mass matrices, namely, degenerate (Types [I(A,B,C)]), inverted hierarchical (Types [II(A,B)]) and normal hierarchical (Type [III]) models, by taking the diagonal form of the Dirac neutrino mass matrix and a non-diagonal form of the right-handed Majorana mass matrix in the canonical see-saw formula (Type I). We then examine whether these good predictions are spoiled or not in the presence of the left-handed Higgs triplet in Type-II see-saw formula; and if so, we find out the 'least perturbation' for retaining good predictions which have been previously obtained. We make use of a simple hypothesis for discriminating the neutrino mass models based on the dominance of the canonical see-saw term over the non-canonical term. Under such hypothesis we arrive at the conclusion that inverted hierarchical model with even CP phase (Type [IIA]) is the most favourable one in nature. Next to it is the normal hierarchical model. Degenerate models are badly spoiled by the presence of non-canonical term in Type-II see-saw formula. Our conclusion also nearly agrees with the calculations using the mass matrices m_{LR} and M_{RR} predicted by other authors in $SO(10)$ models [21,39]. It can be stressed that the method adopted here is also applicable to any neutrino mass matrix obtained using a general non-diagonal texture of Dirac mass matrix.

As a remark we also point out that unlike Types [IIB] and [III] [36], Type [IIA] is unstable under quantum radiative corrections in MSSM [35,38]. As emphasized before, the present analysis is based on the hypothesis that those models of neutrinos where the canonical see-saw term is dominant over the perturbative term arising from Type-II see-saw, are favourable in nature. The present work is a modest attempt to understand the correct model of neutrino mass pattern. Future reactor-based experiments [2,3] will be able to decide the correct form [40] of neutrino mass pattern.

Appendix A

We list here for ready reference [16], the zeroth-order left-handed Majorana neutrino mass matrices with texture zeros, m_{LL}^{I}, corresponding to three models of neutrinos, viz., degenerate (Type [I]), inverted hierarchical (Type [II]) and normal hierarchical (Type [III]). These mass matrices are compatible with the LMA MSW solution as well as maximal atmospheric mixings.

Type	m_{LL}	m_{LL}^{diag}
[IA]	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\mathrm{Diag}(1,-1,1)m_0$
[IB]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0$	$\mathrm{Diag}(1,1,1)m_0$
[IC]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$\mathrm{Diag}(1,1,-1)m_0$
[IIA]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\mathrm{Diag}(1,1,0)m_0$
[IIB]	$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0$	$\mathrm{Diag}(1,-1,0)m_0$
[III]	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\mathrm{Diag}(0,0,1)m_0$

Appendix B

Here we list the textures of the right-handed neutrino mass matrix M_{RR} along with the left-handed Majorana mass matrix m_{LL}^{I} generated through the canonical see-saw formula (Type I) (eq. (2)), for three different models of neutrinos presented in Appendix A. The Dirac neutrino mass matrix is given in eq. (7) where $m_f = m_\tau \tan\beta$ for Case (i) and $m_f = m_t$ for Case (ii). For normal hierarchical model the corresponding matrices are given in the main text. These are collected from ref. [30] for ready reference.

Degenerate model (Type [IA]):

$$M_{RR} = \begin{pmatrix} -2\delta_2\lambda^{2m} & (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m+n} & (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m} \\ (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m+n} & (\frac{1}{2}+\delta_1-\delta_2)\lambda^{2n} & (-\frac{1}{2}+\delta_1-\delta_2)\lambda^{n} \\ (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m} & (-\frac{1}{2}+\delta_1-\delta_2)\lambda^{n} & (\frac{1}{2}+\delta_1-\delta_2) \end{pmatrix} v_0$$

$$-m_{LL}^{\mathrm{I}} = \begin{pmatrix} (-2\delta_1+2\delta_2) & (\frac{1}{\sqrt{2}}-\delta_1) & (\frac{1}{\sqrt{2}}-\delta_1) \\ (\frac{1}{\sqrt{2}}-\delta_1) & (\frac{1}{2}+\delta_2) & (-\frac{1}{2}+\delta_2) \\ (\frac{1}{\sqrt{2}}-\delta_1) & (-\frac{1}{2}+\delta_2) & (\frac{1}{2}+\delta_2) \end{pmatrix} m_0.$$

Degenerate model (Type [IB])

$$M_{RR} = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{2m} & \delta_1\lambda^{m+n} & \delta_1\lambda^{m} \\ \delta_1\lambda^{m+n} & (1+\delta_2)\lambda^{2n} & \delta_2\lambda^{n} \\ \delta_1\lambda^{m} & \delta_2\lambda^{n} & (1+\delta_2) \end{pmatrix} v_R$$

$$-m_{LL}^{\mathrm{I}} = \begin{pmatrix} (1-2\delta_1-2\delta_2) & -\delta_1 & -\delta_1 \\ -\delta_1 & (1-\delta_2) & -\delta_2 \\ -\delta_1 & -\delta_2 & (1-\delta_2) \end{pmatrix} m_0.$$

Degenerate model (Type [IC])

$$M_{RR} = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{2m} & \delta_1\lambda^{m+n} & \delta_1\lambda^{m} \\ \delta_1\lambda^{m+n} & \delta_2\lambda^{2n} & (1+\delta_2)\lambda^{n} \\ \delta_1\lambda^{m} & (1+\delta_2)\lambda^{n} & \delta_2 \end{pmatrix} v_0$$

$$-m_{LL}^{\mathrm{I}} = \begin{pmatrix} (1-2\delta_1-2\delta_2) & -\delta_1 & -\delta_1 \\ -\delta_1 & -\delta_2 & (1-\delta_2) \\ -\delta_1 & (1-\delta_2) & -\delta_2 \end{pmatrix} m_0.$$

Invereted hierarchical model (Type [IIA])

$$M_{RR} = \begin{pmatrix} \eta(1+2\epsilon)\lambda^{2m} & \eta\epsilon\lambda^{m+n} & \eta\epsilon\lambda^{m} \\ \eta\epsilon\lambda^{m+n} & \frac{1}{2}\lambda^{2n} & -(\frac{1}{2}-\eta)\lambda^{n} \\ \eta\epsilon\lambda^{m} & -(\frac{1}{2}-\eta)\lambda^{n} & \frac{1}{2} \end{pmatrix} \frac{v_0}{\eta}$$

$$-m_{LL}^{\mathrm{I}} = \begin{pmatrix} (1-2\epsilon) & -\epsilon & -\epsilon \\ -\epsilon & \frac{1}{2} & (\frac{1}{2}-\eta) \\ -\epsilon & (\frac{1}{2}-\eta) & \frac{1}{2} \end{pmatrix} m_0.$$

Inverted hierarchical model (Type [IIB])

$$M_{RR} = \begin{pmatrix} \lambda^{2m+7} & \lambda^{m+n+4} & \lambda^{m+4} \\ \lambda^{m+n+4} & \lambda^{2n} & -\lambda^{n} \\ \lambda^{m+4} & -\lambda^{n} & 1 \end{pmatrix} v_0$$

$$-m_{LL}^{\mathrm{I}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -(\lambda^3-\lambda^4)/2 & -(\lambda^3+\lambda^4)/2 \\ 1 & -(\lambda^3+\lambda^4)/2 & -(\lambda^3-\lambda^4)/2 \end{pmatrix} m_0.$$

The values of the parameters used are: Type [IA]: $\delta_1 = 0.0061875$, $\delta_2 = 0.0031625$, $m_0 = 0.4$ eV; Types [IB] and [IC]: $\delta_1 = 7.2 \times 10^{-5}$, $\delta_2 = 3.9 \times 10^{-3}$, $m_0 = 0.4$ eV; Type [IIA]: $\eta = 0.0045$, $\epsilon = 0.0055$, $m_0 = 0.05$ eV; for Type [IIB]: $m_0 = 0.035$ eV and $\lambda = 0.3$. The expressions for m_0 in all cases except for Type [IIB] is defined as $m_0 = m_f^2/v_0$ and for Type [IIB], $m_0 = (\frac{m_f^2}{v_0})(\frac{1}{2\lambda^4})$. The corresponding values of v_0 for Cases (i) and (ii) are estimated below, while v_R is defined as the heaviest eigenvalues of M_{RR} as listed in table 3.

Values of v_0 in M_{RR} (in GeV).

Type	Case (i)	Case (ii)
IA	6.6×10^{12}	1.681×10^{13}
IB	6.6×10^{12}	1.681×10^{13}
IC	6.6×10^{12}	1.681×10^{13}
IIA	5.34×10^{13}	1.3448×10^{14}
IIB	4.71×10^{15}	1.198×10^{16}
III	8.92×10^{13}	2.27×10^{14}

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