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Optical model potential of 800 MeV/c K^+ meson for ^{12}C and ^{40}Ca by the method of inversion

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Abstract. The elastic scattering differential cross-sections of 800 MeV/c K^+ mesons from ^{12}C and ^{40}Ca have been analyzed using the Ericson's parametrization for the phase shift. It is found that the parameter values obtained by our analysis are significantly different from those obtained from the closed expression for K^+- nucleus amplitude derived by the strong absorption approximation. Next, using the phase shift obtained from the present analysis we calculate the K^+ optical model potentials for ^{12}C and ^{40}Ca by the method of inversion. The calculated potentials are compared with the recently determined phenomenological ones.

Keywords. K^+-nucleus scattering; diffraction model; optical potential by inversion.

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1. Introduction

The study of the scattering of K^+ mesons from nuclei in the momentum range of about 500–800 MeV/c (see refs [1–7]) has attracted a lot of attention over the past two decades. Reasons for the interest are well-known. In this momentum range, the K^+ meson is the weakest of all hadronic probes. It has a mean free path of about 5–6 fm in nuclear matter, and the K^+N scattering amplitude varies fairly smoothly. These characteristics imply that corrections to the first-order microscopic optical potential are small and the conventional '$t\rho$' model with the free K^+N amplitude (impulse approximation) should provide a satisfactory description of the experimental data. However, in practice it has been found that the '$t\rho$' model, even after incorporating some well-known corrections, does not provide a satisfactory theoretical framework for the description of K^+-nucleus scattering. This theoretical situation has prompted many authors to propose that the K^+N amplitude within the nuclear medium differs from the free one in a significant way, and to suggest ways to account for the medium effect in order to get a better agreement with the experimental data [1–3,6]. At present, it may be said that despite extensive theoretical efforts the situation regarding the microscopic K^+-nucleus optical potential is not well-settled.

In parallel with the microscopic studies, phenomenological models have also been employed to analyze the K^+ elastic scattering data. Here, we will mention the work of Choudhary [5] who applied the diffraction model and the Ericson's parametrization of the elastic S matrix element (or S function) S_l to analyze the 800 MeV/c K^+ elastic scattering differential cross-section data for ^{12}C and ^{40}Ca nuclei. However, Choudhary's work needs a fresh look for two reasons. First, his analysis is based on the closed expression for the K^+-nucleus amplitude that has been derived by the strong absorption approximation. This approximation scheme might not work satisfactorily for K^+ mesons that are absorbed weakly in nuclei below about 800 MeV/c. Second, Choudhary has completely neglected the Coulomb scattering. Coulomb effect though small at higher energies, have noticeable effect in the forward direction as well as in regions of angular distribution minima, and hence have some bearings on the parameter values of the S function.

In this work we present a study of K^+ optical potentials for ^{12}C and ^{40}Ca at 800 MeV/c. The optical potential has been obtained in two steps. First, the K^+ elastic angular distribution has been fitted using the Ericson's parametrization for the S function. Second, the resulting S function is used to calculate the optical potential by the method of inversion. The last step employs the relation between the phase shift and the potential as obtained in the high-energy approximation [8]. Since, Ericson's parametrization of the S function involves only three parameters, each parameter reflecting a specific aspect of the data, it is hoped that this parametrization would give a relatively less ambiguous optical potential than would be obtained by the conventional six-parameter optical model phenomenology [9].

2. Theoretical considerations

The elastic scattering amplitude for the scattering of a charged nuclear particle from a target nucleus of mass number A, and charge number Z may be written as

$$F_{\mathrm{el}}(\theta) = F_{\mathrm{c}}(\theta) + \frac{l}{2k}\sum_{l=0}^{\infty}(2l+1)\mathrm{e}^{2i\sigma_l}[1-S_l]P_l(\cos\theta), \tag{1}$$

where $F_{\mathrm{c}}(\theta)$ is the point Coulomb scattering amplitude, k the c.m. momentum, σ_l the Coulomb phase shift, $P_l(\cos\theta)$ the Legendré polynomial, and S_l is the elastic S-matrix element. The last one is related to the nuclear phase shift δ_l through the relation

$$S_l = \exp[2i\delta_l]. \tag{2}$$

In Ericson's parametrization, it is assumed that the quantity S_l is of the form:

$$S_l = \frac{1}{1+\exp[(L_R - l - i\mu/\Delta]}, \tag{3}$$

where L_R, μ and Δ are the parameters. Using the relations $b = (l+1/2)/k$ and $R = (L_R + 1/2)/k$, $a = \Delta/k$, and $\mu' = \mu/k$, expression (3) may also be written as

$$S(b) = \frac{1}{1 + \exp[(R - b - i\mu')/a]}. \tag{4}$$

In the above formulation b is the impact parameter, R is the effective radius, and a is the effective surface diffuseness. With regard to the parameter μ' it is connected with the real part of the nuclear phase shift and hence it describes the refractive effects. Neglecting the Coulomb effects in eq. (1) and making a series of approximations the following expression for the elastic scattering amplitude may be obtained [5].

$$F_{\mathrm{el}}(\theta) = ig(\theta, \Delta)\frac{(kR - i\mu)}{k\theta}J_1[(kR - i\mu)\theta], \tag{5}$$

where the nuclear form factor $g(\theta, \Delta)$ is the same as given in [5].

3. Elastic scattering differential cross-sections

Using the elastic scattering amplitude given by eq. (5), Choudhary [5] has fitted elastic scattering differential cross-sections for 800 MeV/c K$^+$ mesons for ^{12}C. He finds that a fairly satisfactory agreement with the experimental data is obtained with $R = 2.08$ fm, $a = 0.61$ fm and $\mu = 0.98$ as shown by the dotted curve in figure 1. In the figure we also show by the solid and dashed curves the predictions of the exact expression (1) with and without the Coulomb scattering respectively with the same set of parameter values. It is seen that the predictions of the exact expression with or without the Coulomb scattering are in great disagreement with the experimental data as well as with the calculation of ref. [5]. Similar results (not shown) have been obtained for K$^+$-^{40}Ca scattering also. This implies that expression (5) is not a good approximation for K$^+$-nucleus scattering in the energy range under consideration. In other words the parameter values of S_l as deduced by fitting with the approximate expression are not accurate enough to be used for the determination of the optical potential by inversion, which is the main aim of the present work.

In figures 2 and 3 we show our best-fit results for Ericson's parametrization for S_l using the exact expression for the scattering amplitude as given by eq. (1). The corresponding parameter and per point χ^2 values are: $R = 1.47$ fm, $a = 0.78$ fm,

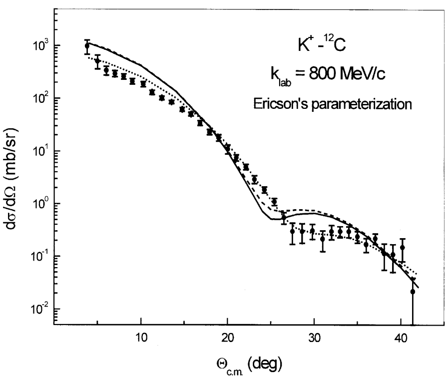

Figure 1. Elastic scattering differential cross-sections for 800 MeV/c K$^+$ mesons on ^{12}C. The dotted curve shows the predictions of the approximate expression (5). The dashed and the dotted curves show the predictions of expression (1) with and without the Coulomb scattering. In each case the parameter values are [5]: $R = 2.08$ fm, $a = 0.61$ fm and $\mu = 0.98$. The experimental data are of ref. [1].

$\mu = -0.16$, $\chi^2 = 3.3$ for ^{12}C, and $R = 3.19$ fm, $a = 0.74$ fm, $\mu = -0.29$, $\chi^2 = 0.44$ for ^{40}Ca respectively. It is seen that the parametrization works exceeding well for ^{40}Ca but not so well for ^{12}C. In the latter case the quality of fit is poor especially at smaller scattering angles. This poor fit is also reflected by the large χ^2-value for K$^+$-^{12}C system. The relatively good working of Ericson's parametrization for ^{40}Ca is not unexpected. This parametrization is motivated by the form of the two-parameter density distribution which is more suited for medium and heavier nuclei than for lighter nuclei.

It must be mentioned that in the fitting process the parameter μ was constrained to be negative. This was done to ensure that the real part of the K$^+$-nucleus phase shift be negative so that the corresponding real potential be repulsive as suggested by the microscopic theories. However, it was found that acceptable fits may also be obtained even with a positive value of μ with reasonable values for the parameters R and a. In fact the existing data are not sensitive to the sign of μ, though it was noted that for ^{40}Ca negative μ gives the lowest χ^2-value.

4. Inversion optical potential and discussion

Having determined the parameters of $S(b)$, the optical potential $V_{\mathrm{op}}(r)$ for K^+-nucleus system at intermediate and high energies may be calculated from the relation

$$V_{\mathrm{op}}(r) = \frac{\hbar\nu}{\pi}\frac{\mathrm{d}}{r\mathrm{d}r}\int_r^\infty \frac{\chi(b)b\mathrm{d}b}{\sqrt{b^2-r^2}}, \tag{6}$$

where $\chi(b)(= -i\ln S(b))$ denotes the phase-shift function [8].

Using Ericson's parametrization for $S(b)$ and the parameter values as determined by us in §3, we have calculated the real and imaginary parts of the optical potential from eq. (6). The results of our calculation are shown by the solid curves in figures 4 and 5. The dashed curves in the figures show the six-parameter Saxon-Woods

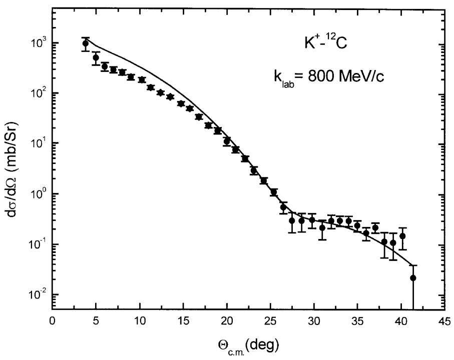

Figure 2. Elastic scattering differential cross-sections for 800 MeV/c K$^+$ mesons on ^{12}C. The solid curve shows the results of our fit with the parameter values given in the text.

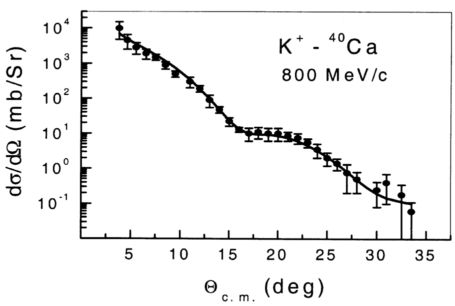

Figure 3. Same as figure 2 but for ^{40}Ca.

phenomenological optical potentials determined by Ebrahim and Khallaf [9] by fitting the elastic scattering differential cross-sections. These authors find several sets of potential parameter values that give acceptable fits to the experimental data. However, in the paper, they have given values of only four sets. Of these sets only one has repulsive real part. Since, as stated earlier, the microscopic theories suggest a repulsive K^+ potential we have chosen this set of values to calculate the phenomenological potential shown by the dashed curves in the figures.

From figures 4 and 5 it is seen that both the real and imaginary parts of the inversion optical potential differ greatly from those of the phenomenological potential in the interior region. Such large disagreements between the potentials as obtained by the two different phenomenologies are hardly surprising. It is generally known that the elastic S-matrix element (S-function) obtained from the phenomenological potential happens to be not the same as the phenomenological S-function resulting from the strong absorption model analysis of the same data, and that the potential obtained from the corresponding inverse scattering problem are found to be different from the phenomenological potential [10–12]. This is due to serious ambiguities, in the shape as well as parameter values, present in both the phenomenological models. It has been already mentioned that Ebrahim and Khallaf [9] have found several sets of potential parameter values that give acceptable fits to the 800 MeV/c K^+-^{12}C data. The extent of ambiguities should be judged from the fact that the best-fit parameter values include both repulsive and attractive potentials. (In figures 4 and 5 we have shown only repulsive potentials for reasons discussed earlier.) As an example of the large disagreements generally found between the phenomenological Woods-Saxon and the inversion potentials obtained from the strong absorption model we refer to the paper by Eldebawi and Simbel [13]. In figure 2 of the paper

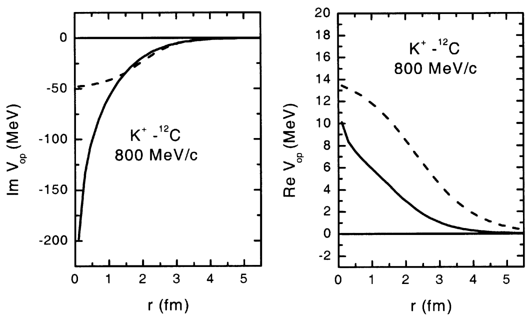

Figure 4. Real and imaginary parts of the K^+-^{12}C optical potential at 800 MeV/c. The solid curve shows the results of the present calculation, while the dashed curve shows the phenomenological potential of ref. [9].

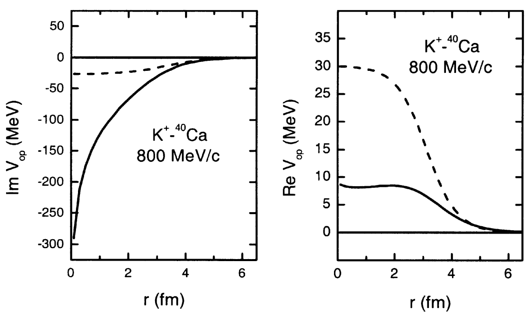

Figure 5. Same as figure 4 but for ^{40}Ca.

these authors have compared their calculated inversion potentials with the phenomenological Woods-Saxon potentials for ^{12}C–^{12}C system at several energies. It may be seen that at almost all the energies large disagreements are present for the real as well as imaginary potentials. Coming to the present study, it is satisfying to find that except for the Re $V_{\mathrm{op}}(r)$ for ^{12}C (figure 4), the radial extensions and the radial behaviors of the optical potentials in the surface region obtained by the two approaches are similar. With regard to the disagreement in the surface behavior in case of Re $V_{\mathrm{op}}(r)$ for ^{12}C as seen in the upper panel of figure 4, it should be noted that the Ericson's parametrization does not provide a satisfactory fit to the ^{12}C data as already discussed in §3. We are of the opinion that the disagreement in the surface behavior in this case is mostly a reflection of this fact.

5. Concluding remarks

In this work we have presented a study of the efficacy of the Ericson's parametrization of the S function for describing elastic scattering of K^+ meson from ^{12}C and ^{40}Ca at intermediate energies. We have found that (i) the predictions of the closed expression for the scattering amplitude derived using Ericson's S function and the strong absorption approximation deviate much with the results of realistic calculation and (ii) Ericson's parametrization works exceedingly well for ^{40}Ca but not so well for ^{12}C nucleus. This indicates that the parametrization is more appropriate for medium and heavy nuclei than for light nuclei. This is not totally unexpected. The Ericson's parametrization is motivated by the form of the two-parameter Fermi density which in general works better for heavier nuclei. Our calculation of the inversion potential gives reasonable results except for the real part of the optical potential for ^{12}C which is very likely due to poor working of the Ericson's parametrization. Finally, it may be added that since K^+-^{40}Ca data is very nicely fitted with the Ericson's S function which has only three parameters, the corresponding inversion potential may be considered to be more realistic than the one obtained by the six-parameter optical model phenomenology.

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