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Two-neutrino double β decay of ^{96}Zr to excited 2^+ state of ^{96}Mo

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Abstract. The two-neutrino double beta decay of ^{96}Zr isotope for $0^+ \rightarrow 2^+$ transition has been studied in the PHFB model. In our earlier work, the reliability of the intrinsic wave functions of ^{96}Zr and ^{96}Mo isotopes has been established by obtaining an overall agreement between a number of theoretically calculated spectroscopic properties as well as half-lives of 2ν $\beta\beta$ decay for $0^+ \rightarrow 0^+$ transition and the available experimental data. In the present work, the half-life of 2ν $\beta\beta$ decay for $0^+ \rightarrow 2^+$ transition $T_{1/2}^{2\nu}(0^+ \rightarrow 2^+)$ has been calculated using the same set of intrinsic wave functions.

Keywords. 2ν $\beta\beta$ decay; $0^+ \rightarrow 2^+$ transition; PHFB model; spectroscopic properties; half-life $T_{1/2}^{2\nu}(0^+ \rightarrow 2^+)$.

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1. Introduction

The implications of present studies about the two modes of nuclear double beta ($\beta\beta$) decay, namely, the two-neutrino double beta (2ν $\beta\beta$) decay and the neutrinoless double beta (0ν $\beta\beta$) decay are far reaching in nature. The former is a convenient tool to test the validity of models employed to study the nuclear $\beta\beta$ decay whereas the latter has important ramifications *vis-á-vis* constraints on the mass and charge conjugation properties of neutrinos. In 2ν $\beta\beta$ decay, the total angular momentum of four S-wave leptons can be 0, 1 or 2 and is equal to the total angular momentum transferred between the parent and daughter nuclei. The lowest 1^+ state in the final nucleus of any $\beta\beta$ decay candidate lies much higher than the first excited 2^+ state. Thus, the $0^+ \rightarrow 1^+$ transition is much less probable than the $0^+ \rightarrow 0^+$ and $0^+ \rightarrow 2^+$ transitions. As the $0^+ \rightarrow 2^+$ transition is not observed experimentally so far, the present theoretical predictions can be checked against the $0^+ \rightarrow 0^+$ transition of 2ν $\beta\beta$ decay only. Once the $0^+ \rightarrow 2^+$ transition of 2ν $\beta\beta$ decay is observed, it would play a crucial role in testing the validity of different nuclear models employed to study $\beta\beta$ decay. Further, the observation of 0ν $\beta\beta$ decay for $0^+ \to 2^+$ transition will help in discriminating finer issues like dominance of Majorana neutrino mass or the right handed current. The theoretical implications and experimental aspects of ground to excited 2^+ state transition of nuclear $\beta\beta$ decay have been excellently reviewed over the past years [1–8].

The physics of nuclear $\beta\beta$ decay for $0^+ \to 2^+$ transition is quite interesting. The half-life $T_{1/2}^{2\nu}(2^+)$ of 2ν $\beta\beta$ decay for $0^+ \to 2^+$ transition is a product of accurately calculable phase space factor $G_{2\nu}(2^+)$ and the appropriate nuclear transition matrix element (NTME) $M_{2\nu}(2^+)$. Compared to the $0^+ \to 0^+$ transition, the $G_{2\nu}(2^+)$ is smaller due to low $Q_{\beta\beta}$. However, to the first approximation, there is a compensating factor of extra $Q_{\beta\beta}^4$. Thus, $G_{2\nu}(2^+)$ is suppressed by a factor of 8.57 in the case of ^{96}Zr. Further, in contrast to ground state to ground state transition, the NTME $M_{2\nu}(2^+)$ is suppressed due to the cubic power in the energy denominator. Hence, it had been rightly realized from the very beginning that the half-lives $T_{1/2}^{2\nu}(2^+)$ are usually larger and in the unobservable range, which initially discouraged the experimentalists to study this particular mode. Subsequently, the experimental activities were resumed due to the availability of modern low background facilities using Ge-detectors [9]. The NTMEs $M_{2\nu}(0^+)$ and $M_{2\nu}(2^+)$ together will constrain the validity of different nuclear models employed to calculate the 2ν $\beta\beta$ decay rate. Further, a reliable theoretical prediction of the half-life of $0^+ \to 2^+$ transition will supplement the experimental planning and designing to study this particular mode of the 2ν $\beta\beta$ decay.

The NTMEs $M_{2\nu}$ of 2ν $\beta\beta$ decay for $0^+ \to 0^+$ as well as $0^+ \to 2^+$ transitions are mainly calculated in three types of models, namely, shell-model and its variants, QRPA and its extensions and alternative models namely OEM, Wigner's spin-isospin $SU(4)_{\sigma\tau}$ model, TVRPA, SSDH, group theoretical methods and PHFB model [6]. The advantages as well as shortcomings of these models have been discussed in detail by Suhonen and Civitarese [6] and Faessler and Simkovic [7] excellently. Over the past years, experimental studies involving in-beam γ-ray spectroscopy have yielded a vast amount of data concerning the level energies as well as electromagnetic properties. The availability of data permits a rigorous and detailed critique of the ingredients of the microscopic framework that seeks to provide a description of nuclear $\beta\beta$ decay. However, most of the calculations of $\beta\beta$ decay matrix elements performed so far do not satisfy this criterion.

The success of the PHFB model [10] in conjunction with the PPQQ interaction [11] in explaining the yrast spectra, reduced transition probabilities $B(E2{:}0^+ \to 2^+)$, static quadrupole moments $Q(2^+)$, g-factors $g(2^+)$ of some nuclei in the mass range $94 \leq A \leq 110$, namely ^{94,96}Zr, ^{94,96,98,100}Mo, ^{98,100,104}Ru, ^{104,110}Pd and ^{110}Cd as well as the 2ν $\beta\beta$ decay rate of ^{94,96}Zr,^{98,100}Mo,^{104}Ru and ^{110}Pd for $0^+ \to 0^+$ transition [12] has motivated us to apply the same set of wave functions to study the 2ν $\beta\beta$ decay of ^{96}Zr for $0^+ \to 2^+$ transition. The 2ν $\beta\beta$ decay of ^{100}Mo for $0^+ \to 2^+$ transition along with the spectroscopic properties has been already studied using the PHFB model [13] using closure approximation [1]. It is commonly believed that the closure energy calculated using the prescription of Haxton *et al* is usually large. In the present work, we have extracted the energy denominator from the observed average half-life $T_{1/2}^{2\nu}$ for $0^+ \to 0^+$ transition for a more reliable prediction.

The present paper has been organized as follows. The detailed theoretical formalism to calculate the half-life of 2ν $\beta\beta$ decay mode for $0^+ \to 2^+$ transition has been given by Chaturvedi *et al* [13]. In §2, the results are presented and discussed. The final conclusions are given in §3.

2. Results and discussions

The model space, single particle energies (SPEs) and two-body effective interactions are the same as our earlier calculation on 2ν $\beta\beta$ decay of ^{94,96}Zr, ^{98,100}Mo, ^{104}Ru and ^{110}Pd isotopes for $0^+ \to 0^+$ transition [12]. We have shown that from the overall agreement between the observed and calculated electromagnetic properties, the PHFB wave functions generated by fixing χ_{pn} to reproduce the yrast spectra are quite reliable. Further, the observed 2ν $\beta\beta$ decay rate of ^{96}Zr and ^{100}Mo for $0^+ \to 0^+$ transition are well reproduced. In the following, we briefly present the results for ^{96}Zr and ^{96}Mo isotopes for convenience. We have tabulated the calculated as well as the experimentally observed values of the yrast spectra, reduced transition probabilities $B(E2{:}0^+ \to 2^+)$ [15], static quadrupole moments $Q(2^+)$ and the gyromagnetic factors $g(2^+)$ [16] of ^{96}Zr and ^{96}Mo in table 1. We have presented only some experimentally observed representative values in the case of $B(E2{:}0^+ \to 2^+)$. The calculated and observed $B(E2{:}0^+ \to 2^+)$ values are in excellent agreement in the case of ^{96}Zr and ^{96}Mo nuclei for $e_{\text{eff}} = 0.50$. The theoretically calculated $Q(2^+)$ are tabulated for the same effective charges as given above. No experimental $Q(2^+)$ result is available for ^{96}Zr. The theoretical $Q(2^+)$ result is quite off from the observed value for ^{96}Mo. The $g(2^+)$ values are calculated with g_l^π=1.0, $g_l^\nu = 0.0$, $g_s^\pi = g_s^\nu = 0.60$. No experimental result is available for ^{96}Zr and ^{96}Mo. Further, the calculated half-lives $T_{1/2}^{2\nu}$ of ^{96}Zr for $0^+ \to 0^+$ transition are 1.56×10^{19} and 3.8×10^{19} yr for $g_A = 1.254$ and 1.0 respectively in comparison to the experimentally observed half-life $2.1^{+0.8}_{-0.4} \times 10^{19}$ yr.

In the present version of the PHFB model, it is not possible to study the structure of intermediate odd–odd nuclei. Hence, the average energy $\langle E_N \rangle$ cannot be calculated exactly. It is usually calculated by using either the closure approximation [1] or the summation method [17]. These methods are always valid by definition. However, both the closure approximation and summation method have limitations in practice. The former approximation has been shown to work badly in case the $M_{GT}^{2\nu}$ are predominantly of one sign for some lower $\langle E_N \rangle$ and of opposite sign for a larger $\langle E_N \rangle$ resulting in unreliable DGT due to overcancellation [18,19]. Further, it is generally agreed that the average energy $\langle E_N \rangle$ turns out to be large in Haxtons's prescription [20]. Hence, we have taken the energy denominator $E_{\text{d}}(0^+)$ for $0^+ \to 0^+$ transition as a free parameter and extracted it from the observed average half-life $T_{1/2}^{2\nu}$ for $0^+ \to 0^+$ transition [21] to make a reliable prediction of $T_{1/2}^{2\nu}(0^+ \to 2^+)$. The energy denominator $E_{\text{d}}(2^+)$ for $0^+ \to 2^+$ transition is given by

$$E_{\text{d}}(2^+) = E_{\text{d}}(0^+) - \frac{1}{2} E_{2^+}. \tag{1}$$

The observed average half-lives $T_{1/2}^{2\nu}$ of ^{96}Zr is 2.1×10^{19} yr. The extracted energy denominators $E_{\text{d}}(2^+)$ is 8.1693 MeV in comparison to the obtained value 10.5846 MeV using the closure approximation.

Table 1. Comparison of calculated and experimentally observed yrast energies E_{2^+} in MeV, reduced transition probabilities $B(E2{:}0^+ \to 2^+)$ in e^2 b^2, static quadrupole moments $Q(2^+)$ in e b and g-factors $g(2^+)$ in nuclear magneton. Here $B(E2)$ and $Q(2^+)$ are calculated for effective charge e_p =1+$e_{\rm eff}$ and $e_n = e_{\rm eff}$. The $g(2^+)$ has been calculated for g_l^π =1.0, g_l^ν =0.0 and $g_s^\pi = g_s^\nu = 0.60$.

Nucleus	χ_{pn}	E_{2^+}		$B(E2{:}0^+ \to 2^+)$		$Q(2^+)$		$g(2^+)$	
		Theor.	Expt.[a]	Theor.	Expt.[b]	Theor.	Expt.[c]	Theor.	Expt.
^{96}Zr	0.01717	1.7570	1.7507	0.060	0.055±0.022	−0.015		0.254	
^{96}Mo	0.02557	0.7779	0.7782	0.335	0.310±0.047	−0.524	−0.20±0.08	0.563	
					0.302±0.039				
					0.288±0.016				

[a][14]; [b][15]; [c][16]

Table 2. Experimental half-lives $T_{1/2}^{2\nu}(2^+)$ and corresponding matrix element $M_{2\nu}(2^+)$ for $0^+ \to 2^+$ transition of ^{96}Zr and along with the theoretically calculated $M_{2\nu}(2^+)$ in different models. The numbers corresponding to (a) and (b) are calculated for g_A =1.254 and 1.0 respectively.

Ref.	Projects	Experiment $T_{1/2}^{2\nu}(2^+)$ (yr)		$M_{2\nu}(2^+)$	Ref.	Models	Theory $M_{2\nu}(2^+)$		$T_{1/2}^{2\nu}(2^+)$ (yr)
[9]	†	>7.9×10^{19}	(a)	<0.078	*	PHFB	1.34×10^{-5}	(a)	2.64×10^{27}
			(b)	<0.122				(b)	6.52×10^{27}
[22]		>1.5×10^{20}	(a)	<0.056	[25]	RQRPA			(1.1–1.4)×10^{21}
			(b)	<0.089	[24]	RQRPA(WS)	0.011	(a)	3.94×10^{21}
								(b)	9.73×10^{21}
					[24]	RQRPA(AWS)	0.010	(a)	4.76×10^{21}
								(b)	1.18×10^{22}
					[9]	QRPA(AWS)	0.005–0.038	(a)	(0.33–19.05)×10^{21}
								(b)	(0.82–47.10)×10^{21}
					[23]	SRPA(WS)	8.1×10^{-5}	(a)	7.26×10^{25}
								(b)	1.80×10^{26}

* and † denote present work and (0ν+2ν) mode respectively.

The phase space factors $G_{2\nu}(2^+)$ have been calculated using $g_A = 1.254$. However, in heavy nuclei it is more justified to use the nuclear matter value of $g_A \approx 1.0$. Hence, the experimental $M_{2\nu}(2^+)$ as well as the theoretical $T^{2\nu}_{1/2}(2^+)$ are calculated for g_A=1.0 and 1.254. In table 2, we have compiled all the available experimental and the theoretical results along with our calculated $M_{2\nu}(2^+)$ and corresponding half-lives $T^{2\nu}_{1/2}(2^+)$ of ^{96}Zr isotope for $0^+ \to 2^+$ transition. The NTMEs $M_{2\nu}(2^+)$ extracted from the experimentally observed $T^{2\nu}_{1/2}(2^+)$ have also been presented in column 5 of the same table. We have presented only the theoretical $T^{2\nu}_{1/2}(2^+)$ for those models for which no direct or indirect information about $M_{2\nu}(2^+)$ is available. The calculated $T^{2\nu}_{1/2}(2^+)$ of the order of 10^{21-22} yr in QRPA(AWS) by Barabash *et al* [9], RQRPA by Toivanen and Suhonen [24] and Schwiger *et al* [25] are close to the present experimental limit $T^{2\nu}_{1/2}(2^+) > 1.5\times10^{20}$ yr [22]. The NTMEs $M_{2\nu}(2^+)$ predicted in the PHFB and SRPA(WS) [23] differ by a factor of 6 approximately. Further, the predicted $T^{2\nu}_{1/2}(2^+)$ are larger by a factor of 10^{6-7} approximately in the PHFB and SRPA(WS) models than the present experimental limits.

In our earlier work [13], the half-life of ^{100}Mo for $0^+ \to 2^+$ transition has been already calculated in the PHFB model. The calculated half-lives of 2ν $\beta\beta$ decay, $T^{2\nu}_{1/2}$, of ^{100}Mo for $0^+ \to 2^+$ transition are 6.47×10^{27} and 1.58×10^{28} yr for $g_A =$ 1.254 and 1.0 respectively. However, the HFB wave functions of ^{100}Mo and ^{100}Ru have been generated with improved accuracy by Chandra *et al* [12] and the values of $T^{2\nu}_{1/2}$ of ^{100}Mo for $0^+ \to 2^+$ transition are 5.09×10^{27} and 1.24×10^{28} yr for $g_A = 1.254$ and 1.0 respectively. It is nice to see that the results remain almost unchanged. Using the observed average half-life of ^{100}Mo $=8.0\times10^{18}$ yr, the extracted energy denominators $E_{\rm d}(2^+)$ is 6.9275 MeV in comparison to the closure value 10.9302 MeV. The calculated $T^{2\nu}_{1/2}$ of ^{100}Mo for $0^+ \to 2^+$ transition is 1.47×10^{27} and 3.64 $\times10^{27}$ yr respectively.

To summarize, it is observed that although in different nuclear models, the half-lives of 2ν $\beta\beta$ decay for $0^+ \to 0^+$ transition are close [12], the predicted $T^{2\nu}_{1/2}(2^+)$ are quite different. For example, the PHFB and SRPA(WS) have $T^{2\nu}_{1/2}$ differing by a factor of 7 approximately for $0^+ \to 0^+$ transition in the case of ^{96}Zr. However, the half-lives for $0^+ \to 2^+$ transition differ by a factor of 36 approximately. Moreover, the predicted $T^{2\nu}_{1/2}$ for $0^+ \to 0^+$ transition in different nuclear models differ by a factor of 77 approximately in the case of ^{96}Zr [12]. The same models predict a $T^{2\nu}_{1/2}(2^+)$ differing by order of 10^6. Hence, it is quite clear from the above discussions that the validity of different nuclear models employed to study nuclear $\beta\beta$ decay can be established through the $0^+ \to 2^+$ transition of 2ν $\beta\beta$ decay only after the observation of this particular mode.

3. Conclusions

We have tested the quality of HFB wave functions by comparing the theoretically calculated results for a number of spectroscopic properties, namely, the yrast spectra, reduced $B(E2{:}0^+ \to 2^+)$ transition probabilities, quadrupole moments $Q(2^+)$ and g-factors $g(2^+)$ of ^{96}Zr and ^{96}Mo nuclei with the available experimental data.

Further, reliability of the intrinsic wave functions has been tested by calculating $M_{2\nu}$ and corresponding $T_{1/2}^{2\nu}$ for $0^+ \to 0^+$ transition [12]. A reasonable agreement between the calculated and observed spectroscopic properties as well as the 2ν $\beta\beta$ decay rate for $0^+ \to 0^+$ transition, makes us confident to predict the half-lives $T_{1/2}^{2\nu}(2^+)$ to be 2.64–6.52$\times 10^{27}$ yr for ^{96}Zr.

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