PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 65, No. 3
September 2005
pp. 425–436

Induced focusing and conversion of a Gaussian beam into an elliptic Gaussian beam

MANOJ MISHRA and SWAPAN KONAR
Department of Applied Physics, Birla Institute of Technology, Mesra,
Ranchi 835 215, India
E-mail: skonar@bitmesra.ac.in

MS received 19 January 2004; revised 23 January 2005; accepted 23 May 2005

Abstract. We have presented an investigation of the induced focusing in Kerr media of two laser beams, the pump beam and the probe beam, which could be either Gaussian or elliptic Gaussian or a combination of the two. We have used variational formalism to derive relevant beam-width equations. Among several important findings, the finding that a very week probe beam can be guided and focused when power of both beams are well below their individual threshold for self-focusing, is a noteworthy one. It has been found that induced focusing is not possible for laser beams of any wavelength and beam radius. In case both beams are elliptic Gaussian, we have shown that when power of both beams is above a certain threshold value then the effective radius of both beams collapses and collapse distance depends on power. Moreover, it has been found that induced focusing can be employed to convert a circular Gaussian beam into an elliptic Gaussian beam.

Keywords. Self-focusing; induced focusing; Kerr nonlinearity; nonlinear wave propagation; Gaussian laser beam.

PACS Nos 42.65.-k; 42.65.Jx; 42.65.Tg

1. Introduction

Since its first prediction by Chiao *et al* [1], the self-focusing of laser beams in nonlinear optical media has been a very fascinating research topic [1–11] because of its technological relevance in optical communication, signal processing and all optical switching. It was realized that a high-power optical beam can create a waveguide through a nonlinear medium which can then focus the beam. The balance of self-diffraction of the beam by self-focusing in such a medium leads to the formation of self-trapped spatial solitons [9,10]. It has been established that, self-trapping in one transverse dimension is stable in Kerr law nonlinear media. However, beams which are self-guided in more than one transverse dimensions are unstable and will either diffract away or self-focus catastrophically [2–4]. Kelley [8] showed that self-trapped beams in two transverse dimensions are unstable and could collapse if the intensity was above a critical limit. This has been considered as a means of production of high-strength electric field [2].

The main thrust of the theoretical investigations of self-focusing of laser beams in nonlinear media has been confined to cylindrical Gaussian beams. In a few recent publications, cylindrical off-axis mode [12], spiral self-trapping [13], elliptical Gaussian beam [14,15], higher order self-trapped modes [16], self-trapped vector waves [17] and self-trapping of Bessel beams [18] have been considered. In addition, self-trapped beams in nonlinear optical fibers with radial dependence of refractive index profile have also attracted considerable interest [19,20]. Besides these, Mannassah *et al* [21] pointed out that, owing to the nonlinearity of the medium, it is possible to guide a weak probe and significantly alter its spectral intensity distribution in the waveguide created by the intensity profile of another strong beam. In a Kerr medium, it has been found that the diameter, radius of curvature and the phase of the probe pulse are functions of the pump pulse characteristics. Moreover, it has been predicted that a probe pulse that is co-propagating in an enhanced induced phase modulation nonlinear two-dimensional medium in the presence of a pump may be transformed into induced superspikes in space and time [22]. Therefore, it appears that the phenomenon of induced focusing may be very useful in controlling and manipulating light by light. However, induced focusing of elliptic Gaussian laser beams has escaped serious attention. Therefore, in this paper, we investigate induced focusing of two laser beams which could be either Gaussian or an elliptic Gaussian or a combination of the two.

This paper is arranged as follows: In §2, relevant mathematical model has been developed. Employing variational formalism, governing equations for the widths of both the beams have been established. Result and discussions are incorporated in §3. A brief conclusion is presented in §4.

2. Mathematical model

We consider two co-propagating CW laser beams of nearly equal frequencies ω_1 and ω_2 in a Kerr nonlinear medium. These beams are linearly polarized and assumed to be polarized identically. The propagation equation governing slowly varying amplitudes A_1 and A_2 of two laser beams can be written as

$$-2i\delta_1\frac{\partial A_1}{\partial Z}+\delta_1^2\left[\frac{\partial^2 A_1}{\partial x^2}+\frac{\partial^2 A_1}{\partial y^2}\right]+\Delta(|A_1|^2+2|A_2|^2)A_1=0, \tag{1}$$

$$-2i\delta_2\frac{\partial A_2}{\partial Z}+\delta_2^2\left[\frac{\partial^2 A_2}{\partial x^2}+\frac{\partial^2 A_2}{\partial y^2}\right]+\Delta(|A_2|^2+2|A_1|^2)A_2=0, \tag{2}$$

where $\delta_j = 1/k_j$, $\Delta = 2n_2$, k_j is the linear propagation constant, n_2 is the Kerr coefficient and $j = 1, 2$. Equations (1) and (2) have been written ignoring four-wave mixing process. This is justified since we are considering a medium whose response is instantaneous. In the absence of four-wave mixing term, there is no exchange of energy between two laser beams. Hence, energy of the two beams should be separately conserved. This can be easily verified by integrating eqs (1) and (2) separately, which admits the following invariants:

$$\iint |A_1|^2 \, \mathrm{d}x \, \mathrm{d}y = N_1, \tag{3}$$

and

$$\iint |A_2|^2 \, \mathrm{d}x \, \mathrm{d}y = N_2, \tag{4}$$

where N_1 and N_2 are two constants. The above relationships signify that the beam power or energy flow is separately conserved for both beams.

Equations (1) and (2) are the well-known coupled nonlinear Schrödinger equations which can be solved analytically. One very important method to solve these coupled equations is the variational method [23]. This method is based on the trial functions and Rayleigh Ritz optimization. A detailed justification of this method has been given by Witham [24] and applied elegantly and extensively by several authors [25–28] to address different nonlinear optical problems involving nonlinear Schrödinger equation and its modified form. This formalism rely on the construction of a field Lagrangian density for the propagating beams with a number of slowly varying free parameters which may describe the beam amplitude, width and chirp, and we can increase the number of free parameters for more accurate description of the physical phenomenon. With the help of the field Lagrangian density and the prescribed beam profile, we may obtain a set of ordinary differential equations (ODE) for slowly varying free parameters. This system of coupled ODE's is in general convenient to solve analytically or otherwise numerically. The main advantage of the variational method is its simplicity and capacity to provide clear qualitative picture and good quantitative result. This has motivated us to use this method in the present investigation.

The field Lagrangian density for two propagating beams can be obtained keeping in mind the requirement that $(\delta L/\delta A_1) = (\delta L/\delta A_1^*) = (\delta L/\delta A_2) = (\delta L/\delta A_2^*) = 0$ should reproduce eqs (1) and (2) and their complex conjugate. After identification of the field Lagrangian, the solution can be obtained from the following variational problem:

$$\delta \iiint L \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}Z = 0. \tag{5}$$

Inserting suitable chosen trial functions in the variational principle, we can obtain a reduced variational problem

$$\delta \iint \langle L \rangle \, \mathrm{d}Z = 0, \tag{6}$$

where

$$\langle L \rangle = \iint L_{\mathrm{c}} \, \mathrm{d}x \, \mathrm{d}y. \tag{7}$$

L_c denotes the result of inserting the chosen trial functions into the Lagrangian L. The Euler–Lagrange equations corresponding to eq. (6) yield the desired set of coupled ordinary differential equations mentioned earlier.

The appropriate field Lagrangian density L is given by

$$\begin{aligned} L = & i\delta_1 \left(A_1^* \frac{\partial A_1}{\partial Z} - A_1 \frac{\partial A_1^*}{\partial Z} \right) + i\delta_2 \left(A_2^* \frac{\partial A_2}{\partial Z} - A_2 \frac{\partial A_2^*}{\partial Z} \right) \\ & + \delta_1^2 \left(\left| \frac{\partial A_1}{\partial x} \right|^2 + \left| \frac{\partial A_1}{\partial y} \right|^2 \right) + \delta_2^2 \left(\left| \frac{\partial A_2}{\partial x} \right|^2 + \left| \frac{\partial A_2}{\partial y} \right|^2 \right) \\ & - \frac{\Delta}{2} (|A_1|^4 + |A_2|^4) - 2\Delta |A_1|^2 |A_2|^2 , \end{aligned} \tag{8}$$

where the asterisk denotes complex conjugate. We take the following trial functions.

$$\begin{aligned} A_1(x,y,z) = & \theta_1(z) \exp\left[-\frac{x^2}{2a_1^2 r_1^2} - \frac{y^2}{2b_1^2 r_1^2} \right] \\ & \times \exp\left[i \left(\alpha_1(z) x^2 + \beta_1(z) y^2 + \Phi_1(z) \right) \right] , \end{aligned} \tag{9}$$

$$\begin{aligned} A_2(x,y,z) = & \theta_2(z) \exp\left[-\frac{x^2}{2a_2^2 r_2^2} - \frac{y^2}{2b_2^2 r_2^2} \right] \\ & \times \exp[i(\alpha_2(z) x^2 + \beta_2(z) y^2 + \Phi_2(z))], \end{aligned} \tag{10}$$

where θ_j, $r_j a_j$, $r_j b_j$, $r_j \alpha_j$ and $r_j \beta_j$ are respectively real amplitude, pulse width in x direction, pulse width in y direction, radius of curvature in x and y directions, r_j is the constant for respective beams and Φ_j is the longitudinal phase. Values of these quantities at $z = 0$ can be written respectively as $\theta_j(0)$, $a_j(0)$, $b_j(0)$, $\alpha_j(0)$, $\beta_j(0)$ and $\Phi_j(0)$. For convenience we have assumed $a_1(0)b_1(0) = a_2(0)b_2(0) = 1$, which is no restriction. Clearly for circular Gaussian beams $a_1(0) = b_1(0) = 1$ and $a_2(0) = b_2(0) = 1$. The reduced Lagrangian turns out to be

$$\begin{aligned} \langle L \rangle = & \int_{-\infty}^{\infty} L \, \mathrm{d}x \, \mathrm{d}y \\ = & -2\pi\delta_1 a_1 b_1 \theta_1^2 \left[\frac{\partial \Phi_1}{\partial Z} + a_1^2 \frac{\partial \alpha_1}{\partial Z} + b_1^2 \frac{\partial \beta_1}{\partial Z} \right] \\ & -2\pi\delta_2 a_2 b_2 \theta_2^2 \left[\frac{\partial \Phi_2}{\partial Z} + a_2^2 \frac{\partial \alpha_2}{\partial Z} + b_2^2 \frac{\partial \beta_2}{\partial Z} \right] \\ & + \frac{\pi}{2} \delta_1^2 a_1 b_1 \theta_1^2 \left[\left(\frac{1}{a_1^2} + 4a_1^2 \alpha_1^2 \right) + \left(\frac{1}{b_1^2} + 4b_1^2 \beta_1^2 \right) \right] \\ & + \frac{\pi}{2} \delta_2^2 a_2 b_2 \theta_2^2 \left[\left(\frac{1}{a_2^2} + 4a_2^2 \alpha_2^2 \right) + \left(\frac{1}{b_2^2} + 4b_2^2 \beta_2^2 \right) \right] \\ & - \frac{\pi}{2} \Delta \left(a_1 b_1 \theta_1^4 + a_2 b_2 \theta_2^4 \right) - 2\pi\Delta \frac{a_1 b_1 a_2 b_2 \theta_1^2 \theta_2^2}{(a_1^2 + a_2^2)^{1/2} (b_1^2 + b_2^2)^{1/2}} . \end{aligned} \tag{11}$$

At this stage it is possible to derive variational equations with respect to Φ_j, θ_j, a_j, b_j, α_j and β_j. These equations after some rearrangement can be written as follows:

$$\pi r_1^2 a_1 b_1 \theta_1^2 = N_1 \tag{12}$$

$$\pi r_2^2 a_2 b_2 \theta_2^2 = N_2 \tag{13}$$

$$\alpha_1 = -\frac{k_1}{2\delta_1 a_1}\frac{\mathrm{d}a_1}{\mathrm{d}\xi} \tag{14}$$

$$\alpha_2 = -\frac{k_2}{2\delta_2 a_2}\frac{\mathrm{d}a_2}{\mathrm{d}\xi} \tag{15}$$

$$\beta_1 = -\frac{k_1}{2\delta_1 b_1}\frac{\mathrm{d}b_1}{\mathrm{d}\xi} \tag{16}$$

$$\beta_2 = -\frac{k_2}{2\delta_2 b_2}\frac{\mathrm{d}b_2}{\mathrm{d}\xi} \tag{17}$$

$$M_1\frac{\mathrm{d}^2 a_1}{\mathrm{d}\xi^2} = \frac{1}{a_1^3} - \frac{p_1}{a_1^2 b_1} - \frac{8(\lambda_2/\lambda_1)^2 p_2 a_1}{X_1^3 Y_1}, \tag{18}$$

$$M_1\frac{\mathrm{d}^2 b_1}{\mathrm{d}\xi^2} = \frac{1}{b_1^3} - \frac{p_1}{a_1 b_1^2} - \frac{8(\lambda_2/\lambda_1)^2 p_2 b_1}{X_1 Y_1^3}, \tag{19}$$

$$M_2\frac{\mathrm{d}^2 a_2}{\mathrm{d}\xi^2} = \left(\frac{\lambda_1}{\lambda_2}\right)^2\left[\frac{1}{a_2^3} - \frac{p_2}{a_2^2 b_2}\right] - 8p_1\left(\frac{r_2}{r_1}\frac{\lambda_1}{\lambda_2}\right)^4\frac{a_2}{X_1^3 Y_1}, \tag{20}$$

$$M_2\frac{\mathrm{d}^2 b_2}{\mathrm{d}\xi^2} = \left(\frac{\lambda_1}{\lambda_2}\right)^2\left[\frac{1}{b_2^3} - \frac{p_2}{a_2 b_2^2}\right] - 8p_1\left(\frac{r_2}{r_1}\frac{\lambda_1}{\lambda_2}\right)^4\frac{b_2}{X_1 Y_1^3}, \tag{21}$$

where $\xi = k_1 z$, $M_1 = k_1^4 r_1^4$, $M_2 = k_2^4 r_2^4$, $X_1 = a_1\left[1+\left(\frac{r_2}{r_1}\frac{a_2}{a_1}\right)^2\right]^{1/2}$, $Y_1 = b_1\left[1+\left(\frac{r_2}{r_1}\frac{b_2}{b_1}\right)^2\right]^{1/2}$, p_1 is the power of the first beam normalized with the threshold power for self-focusing if the beam is circular Gaussian ($a_1(0) = b_1(0) = 1$) with radius r_1, p_2 is the power of the second beam normalized with the threshold power of self-focusing if the beam is circular Gaussian ($a_2(0) = b_2(0) = 1$) with radius r_2 and λ_1 and λ_2 are respectively wavelengths of the first and second beam. Equations (18)–(21) can now be solved and analyzed to investigate induced self-focusing and the conversion process.

3. Result and discussion

Induced focusing, when both beams are circular Gaussian: First we consider the case of focusing when both beams are circular Gaussian. Threshold power of two beams at which both beams remain self-trapped can be written respectively as

$$p_{1\mathrm{th}} = \left[1 - \frac{8(\lambda_2/\lambda_1)^2}{[1+(r_2/r_1)^2]}\right] \Big/ \left[1 - \frac{64(r_2/r_1)^4}{[1+(r_2/r_1)^2]}\right] \tag{22}$$

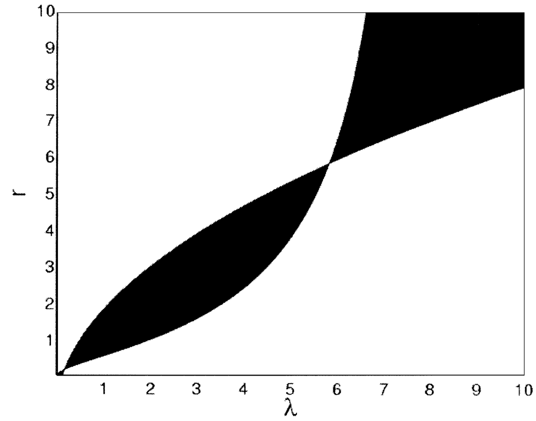

Figure 1. Allowed values of wavelength and beam radius for which induced focusing is possible. Induced focusing is possible in shaded region and impossible in unshaded region. $r = (r_2/r_1)^2$ and $\lambda = (\lambda_2/\lambda_1)^2$.

and

$$p_{2\text{th}} = 1 - \frac{8p_{1\text{th}}(\lambda_1/\lambda_2)^2(r_2/r_1)^4}{[1 + (r_2/r_1)^2]^2}. \tag{23}$$

An important issue is whether a much stronger beam called the pump beam, can induce a weak beam to focus. If this is possible, then we would be able to design a pump probe experiment in which an intense pump beam can induce a weak probe beam to focus due to cross-phase modulation when they co-propagate simultaneously in a self-focusing medium. In order to examine the above aspect, we note that $p_{1\text{th}}$ and $p_{2\text{th}}$ are less than 1 for both beams to be self-trapped, which is less than the threshold required (i.e. $p_1 = 1$ and $p_2 = 1$) for individual beams to be self-trapped. As an example, take the special case $\lambda_1 = \lambda_2$ and $r_1 = r_2$, which corresponds to two co-propagating laser beams of equal frequencies and beam-widths. We find $p_{1\text{th}} = p_{2\text{th}} = 1/3$, which is just 33% of the threshold required for a single beam self-trapping. Thus, in general when both beams are present, they can be self-trapped with much less power in comparison to individual beam self-trapping. This seems to be an interesting finding. Another very important issue is to examine whether cross-phase modulation automatically leads to the induced focusing at any wavelength and any beam radius. We have identified a region of wavelength and beam radius where induced focusing is possible. This has been depicted in figure 1 in which it has been shown that induced focusing is possible in the shaded region. Therefore, it is clear that induced focusing is not possible with two laser beams of any wavelength and beam-width. This is another important finding of the present investigation. Both beams collapse when p_1 and p_2 are greater than the corresponding self-focusing threshold. In figure 2, we have displayed induced collapse of two circular Gaussian laser beams of different wavelength. It is obvious that collapse length decreases with the decrease in wavelength. The behavior of induced collapse at different beam-width ratio is depicted in figure 3. It is obvious that one of the beams starts oscillating as the beam-width ratio increases before it finally collapses.

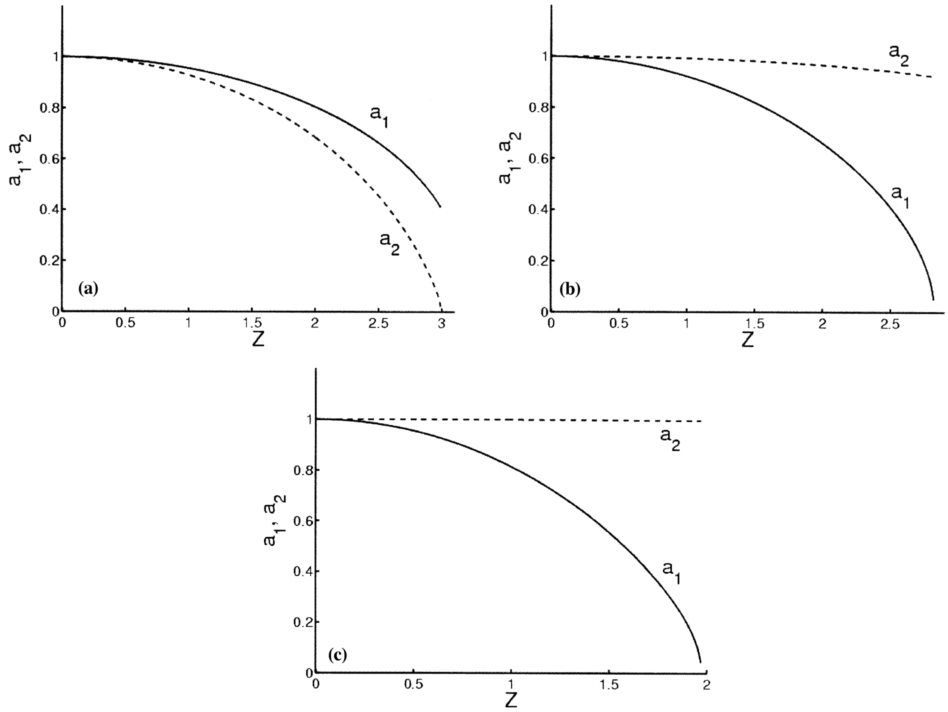

Figure 2. Induced collapse of two circular Gaussian laser beams at different wavelengths. $(r_2/r_1) = 1$, $p_1 = 6.7$, $p_2 = 1.5$. (**a**) $\lambda_2/\lambda_1 = 1$, (**b**) $\lambda_2/\lambda_1 = \sqrt{10}$, (**c**) $\lambda_2/\lambda_1 = 10$.

Induced focusing when both beams are elliptical: In general, it is not possible to solve eqs (18)–(21) analytically. However, solution can be obtained for a special case of $\lambda_1 = \lambda_2$, $r_1 = r_2$ and both beams are of equal power. For such a special case, eqs (18)–(21) reduce to

$$M\frac{\mathrm{d}^2 a_1}{\mathrm{d}\xi^2} = \frac{1}{a_1^3} - \frac{p}{a_1^2 b_1} - \frac{8pa_1}{X^3 Y}, \tag{24}$$

$$M\frac{\mathrm{d}^2 b_1}{\mathrm{d}\xi^2} = \frac{1}{b_1^3} - \frac{p}{a_1 b_1^2} - \frac{8pb_1}{XY^3}, \tag{25}$$

$$M\frac{\mathrm{d}^2 a_2}{\mathrm{d}\xi^2} = \frac{1}{a_2^3} - \frac{p}{a_2^2 b_2} - \frac{8pa_2}{X^3 Y}, \tag{26}$$

$$M\frac{\mathrm{d}^2 b_2}{\mathrm{d}\xi^2} = \frac{1}{b_2^3} - \frac{p}{a_2 b_2^2} - \frac{8pb_2}{XY^3}, \tag{27}$$

where $M = k^4 r^4$, $p = p_1 = p_2$ is the power of each beam, $X = a_1[1 + (a_2/a_1)^2]^{1/2}$ and $Y = b_1[1 + (b_2/b_1)^2]^{1/2}$. Required condition for equilibrium is $a_1(0) = b_1(0)$, and $a_2(0) = b_2(0)$, i.e., both beams are circular Gaussian. Therefore, stationary

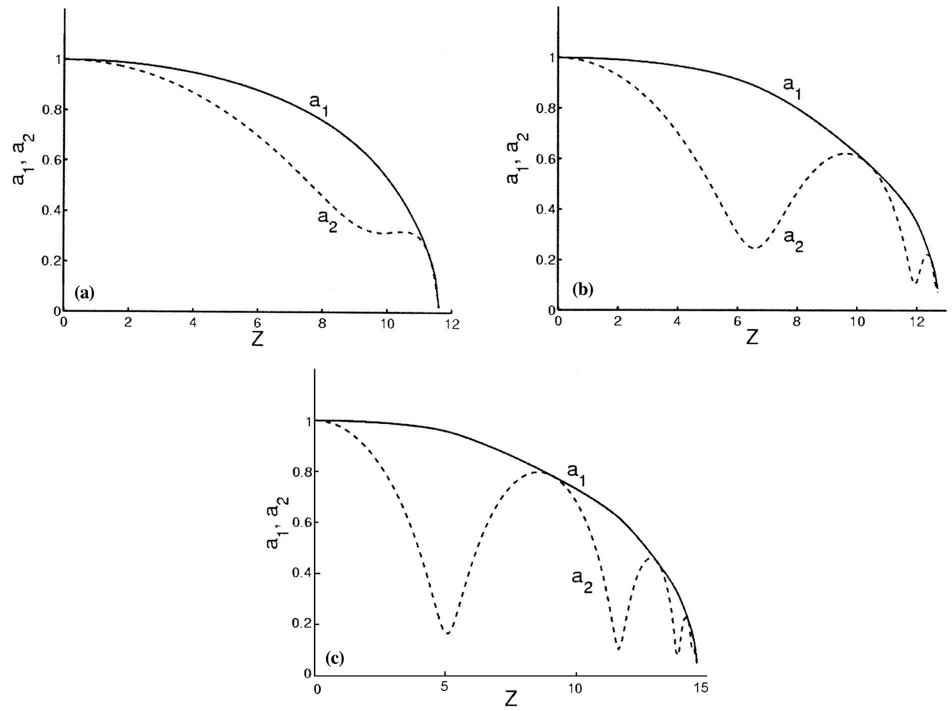

Figure 3. Induced collapse of two circular Gaussian laser beams at different beam radii. $(\lambda_2/\lambda_1) = 1$, $p_1 = 1.2$, $p_2 = 0.2$. (**a**) $r_2/r_1 = 1$, (**b**) $r_2/r_1 = 2$, (**c**) $r_2/r_1 = 4$.

propagation in which transverse width a_1, b_1, a_2 and b_2 of both beams remain constant is not possible with elliptical Gaussian beams. We can easily show that the total energy of the system is conserved and the above set of equations can be derived from a potential

$$V(a_1, b_1, a_2, b_2) = \frac{1}{2a_1^2} + \frac{1}{2b_1^2} + \frac{1}{2a_2^2} + \frac{1}{2b_2^2} - \frac{p}{a_1 b_1} - \frac{p}{a_2 b_2} - \frac{8p}{XY}. \quad (28)$$

The total energy of the system, which is conserved, can be written as

$$H = \frac{M}{2}(\dot{a}_1^2 + \dot{a}_2^2 + \dot{b}_1^2 + \dot{b}_2^2) + V(a_1, b_1, a_2, b_2), \quad (29)$$

where the quantity with dot signifies derivative with respect to ξ. Equations (24)–(27) can be recasted in the following form:

$$M\frac{\mathrm{d}^2\rho^2}{\mathrm{d}\xi^2} = \frac{4H}{M}, \quad (30)$$

where $\rho^2 = \rho_1^2 + \rho_2^2$ is the effective radius of the beam $\rho_1^2 = a_1^2 + b_1^2, \rho_2^2 = a_2^2 + b_2^2$. The above equation can be solved to yield

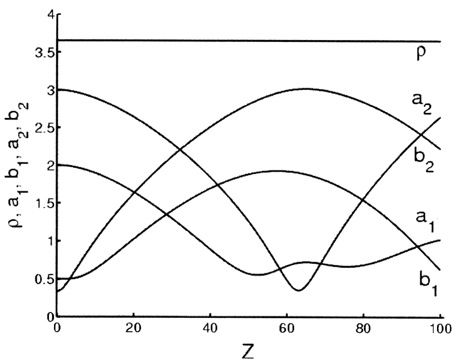

Figure 4. Stationary propagation of two elliptical Gaussian laser beams in which effective radius remains constant. $a_1(0) = 3.0$, $b_1(0) = 0.33$, $a_2(0) = 2.0$ and $b_2(0) = 0.5$. $p_{\text{th}} = 1.1736$.

$$\rho^2 = \frac{2H}{M}\xi^2 + 2(\rho_1(0)\dot{\rho}_1(0) + \rho_2(0)\dot{\rho}_2(0))\xi + \rho^2(0). \tag{31}$$

For the initial parallel beams $\dot{\rho}_1(0) = \dot{\rho}_2(0) = 0$, and therefore $\rho^2 = (2H/M)\xi^2 + \rho^2(0)$. The condition for stationary effective radius is $H = 0$. For $H < 0$, the effective radius decreases and finally collapse will take place at a distance $\xi_c = [-(\rho^2(0)M/2H)]^{1/2}$. For $H > 0$, the effective radius increases quadrically with ξ. The threshold power required for stationary effective radius can be evaluated as

$$p_{\text{th}} = \left[\left(e_1 + \frac{1}{e_1}\right) + \left(e_2 + \frac{1}{e_2}\right)\right] \Bigg/ \left[4 + \frac{16}{\sqrt{(1+a_{21}^2)(1+b_{21}^2)}}\right], \tag{32}$$

where e_1 and e_2 are respectively the ellipticity of the first and second beams, i.e. $e_1 = a_1(0)/b_1(0)$, $e_2 = a_2(0)/b_2(0)$, $a_{21} = a_2(0)/a_1(0)$ and $b_{21} = b_2(0)/b_1(0)$. Note that for $H = 0$, though ρ^2 remains stationary, all other beam-width parameters such as a_1, a_2, b_1 and b_2 oscillate as the beam propagates. In figure 4 we have depicted this behavior. For the general case where two laser beams are of different wavelengths, beam-widths and power, the phenomenon of induced focusing can be investigated only by numerically solving eqs (18)–(21).

Conversion process: We have earlier shown that the condition for equilibrium is $a_1(0) = b_1(0)$ and $a_2(0) = b_2(0)$. Therefore, an elliptic Gaussian beam cannot propagate without oscillation of its two widths. This property can be used to convert a circular Gaussian beam into elliptic Gaussian beam with the help of another elliptic Gaussian beam. At equilibrium, second derivative of the beam-widths of each beam should be zero. Therefore, from eqs (18) and (19) we get

$$\frac{a_2(0)}{a_1(0)} = \frac{b_2(0)}{b_1(0)}. \tag{33}$$

From the above relationship, we can conclude that in case the first beam is circular Gaussian, i.e. $a_1(0) = b_1(0) = 1$, then the equilibrium can be obtained only if the

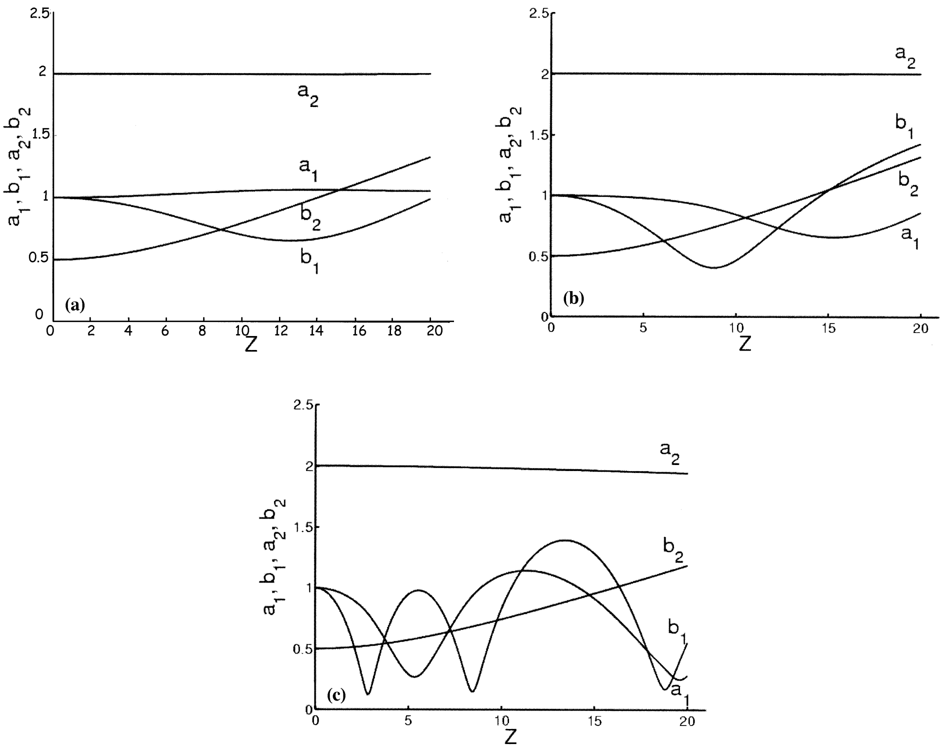

Figure 5. Conversion of a circular Gaussian beam into an elliptic Gaussian beam. Initially the first laser beam is circular Gaussian ($a_1(0) = b_1(0) = 1$) and the second laser beam is elliptical Gaussian ($a_2(0) = 2.0, b_2(0) = 0.5$). (**a**) $p_1 = 0.5$, $p_2 = 0.05$, (**b**) $p_1 = 0.5$, $p_2 = 0.10$, (**c**) $p_1 = 0.5$, $p_2 = 0.8$.

second beam is also circular Gaussian, i.e. $a_2(0) = b_2(0)$. Therefore, using an elliptic Gaussian beam we can convert a circular Gaussian beam into an elliptical beam. Such conversion may be useful in sensor design and signal processing. In figure 5 we have displayed this conversion process, in which we have assumed initially the first beam to be circular Gaussian and the second beam to be elliptic Gaussian. From the figure it is obvious that the first beam has been converted into elliptic Gaussian beam as it propagates in the nonlinear medium. One interesting point is that such conversion is possible even if the power of the converting beam is very small in comparison to individual beam self-focusing threshold.

4. Conclusion

We have presented an investigation of the induced focusing in Kerr media of two laser beams, the pump beam and the probe beam, which could be either Gaussian or elliptic Gaussian or a combination of the two. We have used variational formalism to derive relevant beam-width equations. Extensive numerical simulation has been performed to investigate the influence of the pump beam on the probe beam. Among several important findings, a noteworthy one is that, a very week probe beam can be guided and focused when power of both beams are well below their individual threshold for self-focusing. It has been found that induced focusing is not possible for laser beams of any wavelength and beam radius. In case both beams are elliptic Gaussian, it has been found that when power of both beams are above a certain threshold value then the effective radius of both beams collapses and collapse distance depends on power. Moreover, it has been found that induced focusing can be employed to convert a circular Gaussian beam into elliptic Gaussian beam. Such conversion is possible with a converting beam of very small power.

Acknowledgements

Authors would like to thank the reviewer for the valuable comments and suggestions. This work is supported by the Department of Science & Technology (DST), Government of India, through the R&D grant No. SP/S-2/L-21/99 and this support is acknowledged with thanks. One of the authors M Mishra would like to thank DST for providing Junior Research Fellowship.

References

[1] R Y Chiao, E Garmire and C H Townes, *Phys. Rev. Lett.* **13**, 479 (1964)
[2] Y Silberbarg, *Opt. Lett.* **15**, 1282 (1990)
[3] M Desaix, D Anderson and M Lisak, *J. Opt. Soc. Am.* **B8**, 2082 (1991)
[4] M S Sodha, A K Ghatak and V K Tripathi, in *Progress in optics* edited by E Wolf (North Holland, Amsterdam, 1976) Vol. XIII, p. 171
[5] A Upadhyay, V K Tripathi and H C Pant, *Phys. Scr.* **63**, 326 (2001)
[6] N C Kothari and S C Abbi, *Prog. Theor. Phys.* **83**, 414 (1990)
[7] A Upadhyay, V K Tripathi, A K Sharma and H C Pant, *J. Plasma Phys.* **68**, 75 (2002)
[8] P L Kelley, *Phys. Rev. Lett.* **15**, 1005 (1965)
[9] M Karlsson and D Anderson, *J. Opt. Soc. Am.* **B9**, 1558 (1992)
[10] A W Snyder, D J Mitchell, L Poladian and F Ladouceur, *Opt. Lett.* **16**, 21 (1991)
[11] Magnus Karlsson, *Phys. Rev.* **A46**, 2726 (1992)
[12] R R Jones, W C Mead, S V Coggeshal, C H Aldrich, L Norton, G D Pollak and J M Wallace, *Phys. Fluids* **31**, 1249 (1988)
[13] V I Kruglov and R A Vlasav, *Phys. Lett.* **A111**, 401 (1985)
[14] F Cornolti, M Lucchesi and B Zambar, *Opt. Commun.* **75**, 129 (1990)
[15] Y Li, *J. Opt. Soc. Am.* **B17**, 555 (2000)
[16] H A Hauss, *Appl. Phys. Lett.* **8**, 128 (1996)
[17] D Pohl, *Opt. Commun.* **2**, 305 (1970)
[18] P Johannisson, D Anderson, M Lisak and M Marklund, *Opt. Commun.* **222**, 107 (2003)
[19] R A Sammut and C Pask, *J. Opt. Soc. Am.* **B8**, 395 (1991)
[20] A W Snyder, Y Chen, L Poladian and D J Mitchell, *Electron. Lett.* **26**, 643 (1990)
[21] J T Mannassah, P L Baldeck and R R Alfano, *Appl. Opt.* **27**, 3586 (1988)

[22] J T Mannassah, *Opt. Lett.* **16**, 563 (1991)
[23] D Anderson, *Phys. Rev.* **A27**, 3135 (1983)
[24] G B Witham, *Linear and nonlinear waves* (Wiley, New York, 1974)
[25] J N Kutz and S G Evangelides, Jr., *Opt. Lett.* **23**, 685 (1998)
[26] T M Quirogo and H Michinal, *J. Opt. Soc. Am.* **B14**, 2004 (1997)
[27] S Raghavan and G P Agrawal, *Opt. Commun.* **180**, 377 (2000)
[28] S Konar and Anjan Biswas, *Optical and quantum electronics* **36**, 905 (2004)