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A correspondence between IBA-1 and IBA-2 models and electromagnetic transitions in the decay of some erbium isotopes

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Abstract. The interacting boson approximation IBA-1 model space, in which neutron and proton degrees of freedom are not distinguished, can be considered as a subspace of the IBA-2 model space. Using the microscopic background of the IBA-2 model, a correspondence can be established between IBA-1 and IBA-2 model space. Since the space of the IBA-1 model can be regarded as a subspace of the IBA-2 model there is a unique way to 'Project' the operators of the IBA-2 model onto those of IBA-1. This projection can be carried out using the F-spin formalism. In the IBA-2 model, the lowest states are indeed fully symmetric, and using the calculations with the help of this projection, we explore the energy levels and the electric quadrupole transition probabilities $B(E2; I_i \rightarrow I_f)$ and γ-ray $E2/M1$ mixing ratios for selected transitions of ^{162,164,166,168,170}Er. Owing to admixtures of non-fully-symmetric states in IBA-2, we renormalized the parameters (ε) and (κ). This is the first time we show that this projection can be applied to some heavier isotopes and the results obtained for ^{162,164,166,168,170}Er isotopes are reasonably in good agreement with the previous experimental values.

Keywords. Interacting boson approximations; the electric quadrupole transition probability; mixing ratios.

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1. Introduction

Detailed work has been done on the structure of erbium nucleus in recent years; Gill *et al* [1] studied the (n, γ) reaction for ^{168}Er and obtained a number of new levels for the first time, Alfter *et al* [2] determined $M1/E2$ multipole mixing ratios of erbium isotopes by experiment, Chen [3] studied the negative parity high-spin states of even–odd erbium nucleus with mass number $159 < A < 165$ within the framework of the interacting boson fermion model, Barrett *et al* [4] calculated the multipole mixing ratios of ^{168}Er within the framework of the interacting boson approximation and Gau *et al* [5] calculated the energies of the excited states and the values of $B(E2)$ of $^{155-165}$Er using the interacting boson fermion model.

The interacting boson approximation represents a significant step towards our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques which have also recently found application to problems in atomic, molecular, and high-energy physics [6,7]. The application of this model to deformed nuclei is currently a subject of considerable interest and controversy [8].

In the first version, IBA-1, no distinction is made between neutron and proton degrees of freedom. An unsatisfactory aspect of this model is that there is no clear connection with a microscopic structure of the nucleus. The microscopic theory strongly suggests that it is important to treat the neutron and proton degrees of freedom independently. This has led to the introduction of the second, generalized, version of the IBA-model, called the IBA-2 model. In the second version, the neutron and proton degrees of freedom are treated explicitly. In this model the nucleus is described explicitly in terms of neutron (s_ν, d_ν) and proton (s_π, d_π) bosons. From the calculations in the IBA-2 model, it appears that the lowest levels are symmetric under the interchange of neutrons and protons. This symmetry is most easily discussed in terms of a variable called F-spins [9]. In the case of boson systems, F-spin plays a role similar to that of isospin in the case of fermion systems.

The relation between the IBA-1 and IBA-2 models is obtained by identifying the states of the former to be fully symmetric, i.e. maximal F-spin states of the latter model. Since the space of the IBA-1 model can be regarded as the subspace of the IBA-2 model, there is a unique way to 'Project' the operators of the IBA-2 model onto those of IBA-1. This projection can be carried out using the F-spin formalism [10].

From these considerations it follows that IBA-1 and IBA-2 models can be related to each other and the states of the IBA-1 model can be identified with the fully symmetric states in the IBA-2 model. The purpose of this work is to study this relation and apply it to $^{162-170}$Er isotopes.

The project approximation used in this study has been extensively described by Olaf Scholten for neodymium, samarium and gadolinium isotopes [10]. We shall present here only the results of calculation and refer the reader to the work of the project approximation for details. In §2 we study the positive parity spectra of the $^{162-170}$Er isotopes. In the same section $E2$ and $M1$ transition probabilities and electric quadrupole transition probabilities $B(E2; I_i \rightarrow I_f)$ are analysed. Finally, the work is summarized in §3.

2. Theory and method of calculation

In IBA-2 the neutron and proton degrees of freedom are treated explicitly. This has the advantage of being closer to a microscopic theory. The matrices that have to be diagonalized are, however, much larger. One can regard the IBA-1 model space, in which neutron and proton degrees of freedom are not distinguished, as a subspace of the IBA-2 Hamiltonian and one can thus project out its IBA-1 pieces [10]. In the present work the relevant terms in the IBA-2 Hamiltonian are

$$H = \varepsilon(n_{d_\nu} + n_{d_\pi}) + \kappa(Q_\rho \cdot Q_\pi) + V_{\nu\nu} + V_{\pi\pi}, \tag{1}$$

Table 1. IBM-2 parameters. All parameters are in MeV except χ_ν, χ_π.

	ε	κ	χ_ν	χ_π	$C_{\pi\nu L}(0,2,4)$		
^{162}Er	0.28	−0.06	−0.45	−0.55	−0.15	−0.14	0.15
^{164}Er	0.26	−0.05	−0.46	−0.55	−0.15	−0.13	0.15
^{166}Er	0.23	−0.04	−0.49	−0.59	−0.15	−0.12	0.15
^{168}Er	0.20	−0.02	−0.61	−0.71	−0.18	−0.18	0.18
^{170}Er	0.18	−0.02	−0.64	−0.74	−0.17	−0.17	0.17

where the dot denotes a scalar product. The first term represents the single-boson energies for proton and neutron bosons and n_{d_ρ} is the number of d-bosons where ρ corresponds to π (proton) or ν (neutron) bosons. The second term denotes the main part of the boson–boson interaction, i.e. the quadrupole–quadrupole interaction between neutron and proton bosons with strength κ. The quadrupole operator is

$$Q_\rho = [d_\rho^+ s_\rho + s_\rho^+ \tilde{d}_\rho]^{(2)} + \chi_\rho [d_\rho^+ \tilde{d}_\rho]^{(2)}, \quad (2)$$

where χ_ρ determines the structure of the quadrupole operator and is determined empirically. The square brackets in (2) denote angular momentum coupling.

The terms $V_{\pi\pi}$ and $V_{\nu\nu}$ correspond to interactions between like-bosons. They are of the form

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_L^\rho ([d_\rho^+ d_\rho^+]^{(L)} \cdot [\tilde{d}_\rho \tilde{d}_\rho]^{(L)}). \quad (3)$$

The isotopes $^{162-170}$Er have $N_\pi = 7$, and N_ν varies from 6 to 9, while the parameters κ, χ_ν, χ_π and ε were treated as free parameters and their values were estimated by fitting to the measured level energies. This procedure was made by selecting the 'traditional' values of the parameters and then allowing one parameter to vary while keeping the others constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. The best fit values for the Hamiltonian parameters are given in table 1.

In the present work IBA-2 Hamiltonian parameters are normalized with the help of IBA-1 model Hamiltonian. In the IBA-2 calculation the lowest states are indeed fully symmetric and the calculation with the help of this projection gave good results for the excitation energies. Because of the admixtures of non-fully-symmetric states in IBA-2 model space, the projection gave some difficulties and we had to renormalize the parameters (ε) and (κ) (as shown in figure 1). The IBA-2 Hamiltonian is non-linear in the parameters. To obtain the values of the parameters which give the best fit we have to calculate for each energy level the difference between its experimental and calculated values. Then we have to sum over the squares of all these differences and to find a local minimum to this summation and therefore, in particular, a minimum where the ε_π and ε_ν parameters are equal to the experimental values in the appropriate semi-magic nuclei. The least square fit procedure was used to find the best fit to the three lowest bands (ground state, γ-state and β-state bands) of the erbium isotopes under consideration. The best fit obtained for these isotopes is shown in figures 2a–e. Only two calculated energy

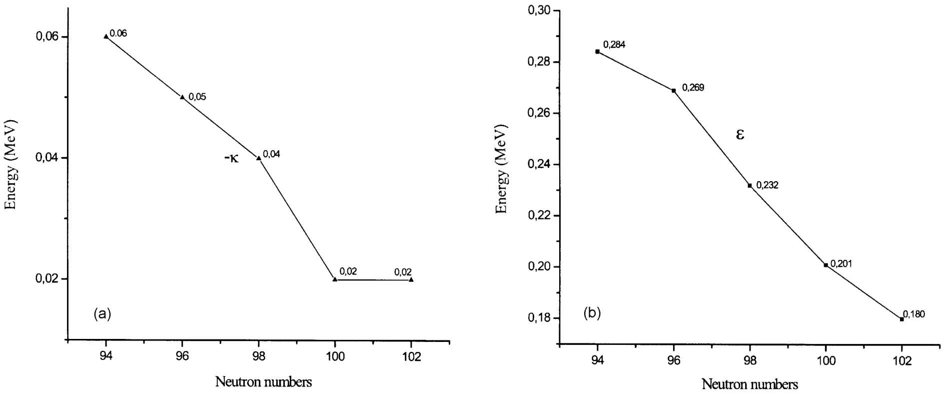

Figure 1. The parameters (**a**) κ and (**b**) ε employed for the IBA-2 calculations for erbium isotopes with even neutron numbers 94 up to 102.

states deviate from the corresponding experimental ones: the γ-states and β-states of the nuclei ^{162}Er and ^{164}Er.

The numerical diagonalization of Hamiltonian has been carried out by using the PHINT code [11]. The values of the main parameters of the Hamiltonian are given in table 1. The calculated excitation energies for ^{162,164,166,168,170}Er isotopes as well as the experimental ones are compared in figures 2a–e. The general agreement between experiment and model is quite good.

3. Results for $B(E2)$'s and $E2/M1$ mixing ratios

A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties. The most important electromagnetic features are the $E2$ transitions. The $B(E2)$ values were calculated using the $E2$ operator

$$E2 = e_\pi Q_\pi + e_\nu Q_\nu, \tag{4}$$

where the Q_π and Q_ν operators are defined in eq. (2) and e_π and e_ν are the 'effective charges' for the proton bosons and the neutron bosons. For simplicity the 'effective charges' e_π and e_ν were taken as equal ($e = 0.120eb$). Some calculated $B(E2)$ values from the ground state band and $B(E2)$ ratios are given in table 2.

Since erbium nucleus has a rather rotational character, taking into account the dynamic symmetry location of the even–even erbium nuclei at the IBM phase triangle where their parameter sets are at the $O(6)$–$SU(3)$ transition region and closer to $SU(3)$ rotational character and possessing good rotational states, we used the multiple expansion form of the Hamiltonian for our approximation. In order to find the value of the effective charge we have fitted the calculated absolute strengths $B(E2)$ of the transitions within the ground state band to the experimental ones. The best agreement is obtained with the value $e_\pi = e_\nu = e = 0.120eb$, as shown in table 2. The $B(E2)$ values depend quite sensitively on the wave functions, which suggests that the wave functions obtained in this work are reliable.

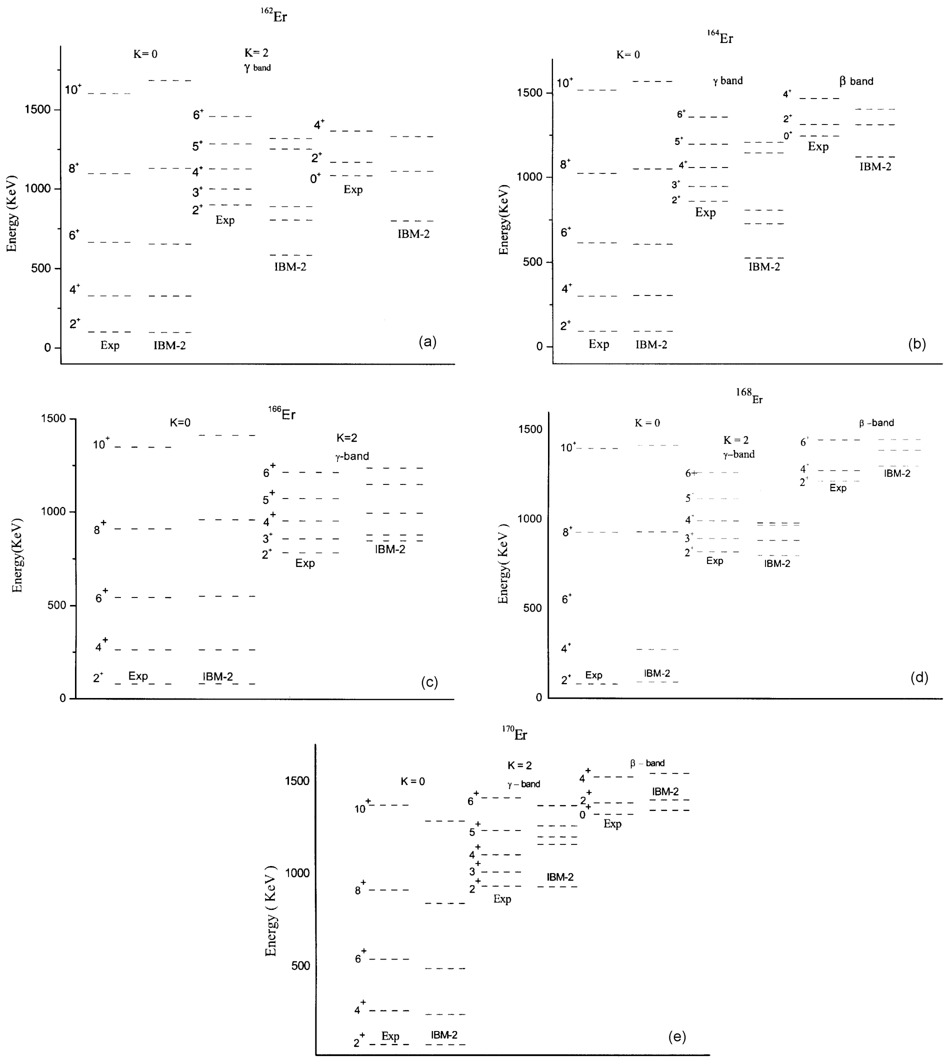

Figure 2. The three lowest rotational bands in the spectra of (**a**) ^{162}Er, (**b**) ^{164}Er, (**c**) ^{166}Er, (**d**) ^{168}Er and (**e**) ^{170}Er. In each band the experimental data are plotted on the left and the calculated values on the right.

$E2 : M1$ *multipole mixing ratios*: Arima and Iachello in their original interacting boson approximation (IBA-1) gave the M1 operator in the restricted case of $SU(5)$ dynamic symmetry [13] as well as the general case [10]. However, even when starting with the general operator, they derived the $E2/M1$ mixing ratio by neglecting the term which break the $SU(3)$ symmetry [14]. It follows that the reduced mixing ratio is given by the same simple formula for both $SU(5)$ and $SU(3)$ symmetries. The formula contains only one parameter and the initial and final spins. Warner *et al* [15] have developed an IBA description of the $E2/M1$ mixing ratio and their

Table 2. $B(E2;\, I \rightarrow 1\text{–}2)$ values for the ground bands of Er isotopes.

	$B(E2)$ values (e^2b^2)				$B(E2)$ ratios	
	$4_g \rightarrow 2_g$		$2_g \rightarrow 0_g$		$(4_g \rightarrow 2_g)/(2_g \rightarrow 0_g)$	
N	Theory	Exp. [12]	Theory	Exp. [12]	Theory	Exp. [12]
94	1.62		1.13	1.16	1.43	
96	1.58	1.39	1.10	1.12	1.43	1.24
98	1.62	1.63	1.12	1.16	1.44	1.40
100	1.71	1.71	1.15	1.18	1.48	1.44
102	1.49	1.55	1.01	1.04	1.47	1.49

point of departure was essentially the same as that of Scholten *et al* [16]. Up to now, several systematic studies [17,18] have been performed within the framework of the IBA.

In the nucleus, an electromagnetic exchange connecting a state of spin I_1 to I_2 can carry an angular momentum L between $|I_1 + I_2|$ and $|I_1 - I_2|$. In the rotation–vibration model, pioneered by Bohr and Mottelson [19], the low-lying, even-parity states of even–even nuclei are ascribed to the collective quadrupole motion of the nucleus as a whole.

The $M1$-$E2$ mixing parameter δ is defined as

$$\delta = \pm\sqrt{\frac{T(E2)}{T(M1)}} = \pm\frac{\sqrt{3}}{10}\frac{w}{c}\sqrt{\frac{B(E2|I \rightarrow I')}{B(M1|I \rightarrow I')}}, \tag{5}$$

where the $\pm$ sign must be chosen depending on the relative sign of the reduced matrix element [20]. The electric quadrupole and magnetic dipole transition probabilities $T(E2)$ and $T(M1)$ are, respectively,

$$T(E2|I \rightarrow I') = \frac{4\pi}{75}\frac{1}{\hbar}\left(\frac{w}{c}\right)^5 B(E2|I \rightarrow I'), \tag{6}$$

$$T(M1|I \rightarrow I') = \frac{16\pi}{9}\frac{1}{\hbar}\left(\frac{w}{c}\right)^3 B(M1|I \rightarrow I'), \tag{7}$$

and $B(E2|I \rightarrow I')$ is the reduced $E2$ transition probability,

$$B(E2|I \rightarrow I') = \frac{1}{2I+1}\sum_{\mu,M,M'} |\langle\psi^{I'M'}|(E2,\mu)|\psi^{I'M'}\rangle|^2. \tag{8}$$

The reduced transition probability $M1$ is given by

$$B(M1|I \rightarrow I') = \frac{3}{4\pi}\left(\frac{e\hbar}{2Mc}\right)^2 \frac{1}{2I+1}\sum_{\mu,M,M'} |\langle\psi^{I'M'}|\mu_\sigma|\psi^{I'M'}\rangle|^2. \tag{9}$$

The δ-mixing ratios for some selected transitions in ^{162,164,166,168,170}Er isotopes are calculated from the useful equations as above and with the help of $B(E1)$ and

Table 3. Experimental and theoretical $\delta(E2/M1)$ multipole mixing ratios of ^{162}Er.

I_i^π (E_γ MeV) I_s^π	This work	Experimental	Previous work
$3^+[0.6730]4^+$	−5.62	–	−0.04 [17,19]
$4^+[0.8000]4^+$	−6.68	–	–
$5^+[0.9570]4^+$	−13.00	−8.0 [23]	−7.90 [17,19]
$5^+[0.6200]6^+$	−3.54	0.0 [23]	0.00 [17,19]
$6^+[0.7930]6^+$	−4.52	–	−3.50 [17,19]
$6^+[0.1730]5^+$	−1.17	–	−2.65 [17,19]
$7^+[1.0003]6^+$	−9.71	−8.0 [23]	−7.90 [17,19]

Table 4. Experimental and theoretical $\delta(E2/M1)$ multipole mixing ratios of ^{164}Er.

I_i^π (E_γ MeV) I_s^π	This work	Experimental	Previous work
$3^+[0.6964]4^+$	−2.63	–	–
$3^+[0.6469]2^+$	−4.68	–	–
$4^+[0.7588]4^+$	−3.86	2.4 [23]	−1.1 [17]
$3^+[0.8549]2^+$	−6.18	−2.8 [23]	−7.7 [17]
$7^+[0.9299]6^+$	−5.40	−2.4 [23]	−6.5 [17]

Table 5. Experimental and theoretical $\delta(E2/M1)$ multipole mixing ratios of ^{166}Er.

I_i^π (E_γ MeV) I_s^π	This work	Experimental	Previous work
$2^+[0.7053]2^+$	17.61	16.01 [24]	16.84 [17]
$3^+[0.7788]2^+$	19.11	19.00 [24]	18.41 [15]
$3^+[0.5943]4^+$	8.97	8.00 [25]	17.61 [17]
$4^+[0.6912]4^+$	9.32	7.50 [26]	9.06 [15]
$5^+[0.1190]4^+$	1.40	1.46 [27]	0.23 [17]
$5^+[0.5298]6^+$	5.38	5.00 [26]	5.40 [26]
$7^+[0.1601]6^+$	1.23	1.39 [27]	0.20 [17]

$B(M1)$ values which are obtained from FBEM (computer code which is subroutine of PHINT package program) [21]; the results are given in tables 3–7. In general, the calculated electromagnetic properties of the erbium isotopes do not differ significantly from those calculated in the experimental and previous theoretical work [22,23].

4. Discussion and conclusions

The strongly deformed even–even erbium isotopes have been described by IBA-2 Hamiltonian. In these calculations no truncation has been put in the huge neutron–

Table 6. Experimental and theoretical $\delta(E2/M1)$ multipole mixing ratios of ^{168}Er.

I_i^π (E_γ MeV) I_s^π	This work	Experimental	Previous work
2^+[0.7413]2^+	16.14	16.00 [28]	16.39 [15]
3^+[0. 0747]2^+	1.21	1.42 [29]	1.76 [29]
3^+[0. 6317]4^+	3.50	9.30 [24]	6.60 [17]
4^+[0.7306]4^+	8.94	5.70 [28]	8.42 [15]
5^+[0.8535]4^+	2.43	3.64 [29]	10.13 [15]
5^+[0.5695]6^+	4.95	25.00 [26]	5.66 [15]
6^+[0.7150]6^+	3.25	2.99 [30]	4.06 [30]
3^+[0.8159]2^+	13.26	17.40 [24]	17.03 [15]

Table 7. Experimental and theoretical $\delta(E2/M1)$ multipole mixing ratios of ^{170}Er.

I_i^π (E_γ MeV) I_s^π	This work	Experimental	Previous work
2^+[0.8812]2^+	19.79	$2.2 < \delta < 4$ [31]	–
3^+[0.9312]2^+	15.69	$10 < \delta < 20$ [31]	–
3^+[0.9570]4^+	15.51	–	–
4^+[0.8432]4^+	9.10	$\delta \geq 8$ [30]	–
3^+[1.1385]2^+	19.19	$\delta \geq 3$ [30]	–
4^+[0.4960]3^+	8.03	–	–
5^+[0.8547]4^+	12.34	$3 < \delta < 10$ [31]	–

proton boson spaces. This is the first time that IBA-2 Hamiltonian parameters are obtained by the projection that we have developed by using the F-spin formalism from the operator of IBA-2 model over the operator of IBA-1 model space. The single d-boson energies were determined from the experimental data – the 0-2 spacing in the appropriate semi-magic nuclei, where these data are known. It was found that although Hamiltonian yielded a good description of the energy levels of the ^{166,168,170}Er isotopes they failed in ^{162,164}Er isotopes.

For totally symmetric states, the description of the nuclear properties is approximately equal in going from IBA-1 to IBA-2. However, IBA-2 model which distinguished neutrons and protons, has a clear microscopic connection with the spherical shell model while the IBA-1 has not. The present work demonstrates that IBA-2 Hamiltonian parameters based on IBA-1 model gave good results for the excitation energies and the electric quadrupole transition probability $B(E2; I_i \rightarrow I_f)$ of ^{162,164,166,168,170}Er isotopes. For the non-fully-symmetric states, we renormalized the parameters (ε) and (κ) and obtained good results. In the present calculations we have shown the ability of the projection in correlating different properties in erbium isotope in terms of a few parameters.

We have also examined the mixing ratio $\delta(E2/M1)$ of transitions linking the γ and ground state bands. The transitions which link low-spin states and were obtained in the present work are in good agreement and show a little bit irregularities. We find that the transitions which link low-spin states and which were obtained in the present work are largely consistent with this requirement although some may be considered to show irregularities.

In the treatments of the IBA-2 Hamiltonian mentioned above, few IBA-2 interactions were used. In the IBA-2 model there are additional interactions with (or without) microscopic basis. It is possible that by adding some of the interactions to our IBA-2 Hamiltonians, the wave functions will be altered such that the agreement with the mixing ratios could be improved. A deeper understanding of the microscopic basis of the IBA-2 model will certainly help one to find the interactions that must be included in the IBA-2 Hamiltonians in order to provide better description of the strongly deformed even–even nuclei.

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