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The incompatibility between local hidden variable theories and the fundamental conservation laws

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Abstract. I discuss in detail the result that the Bell's inequalities derived in the context of local hidden variable theories for discrete quantized observables can be satisfied only if a fundamental conservation law is violated on the average. This result shows that such theories are physically nonviable, and makes the demarcating criteria of the Bell's inequalities redundant. I show that a unique correlation function can be derived from the validity of the conservation law alone and this coincides with the quantum mechanical correlation function. Thus, any theory with a different correlation function, like any local hidden variable theory, is incompatible with the fundamental conservation laws and space-time symmetries. The results are discussed in the context of two-particle singlet and triplet states, GHZ states, and two-particle double slit interferometry. Some observations on quantum entropy, entanglement, and nonlocality are also discussed.

Keywords. Quantum correlation functions; conservation laws; Bell's inequality; entanglement; quantum entropy; nonlocality.

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1. Introduction

This paper deals with a generalization and a detailed discussion of the implications of the recent result that the quantum mechanical correlation function is a unique consequence of a fundamental conservation law (that arises from a space-time symmetry, like the conservation of angular momentum), and therefore all local hidden variable and local realistic theories are incompatible with the fundamental conservation laws [1]. The basis of this result was the discovery that the angular correlation function is simply an average over the angular momenta of one of the particles *conditional* on specific values of the projection of angular momentum ($+\hbar/2$ for example) for the other particle. The relation between the correlation functions and the conservation laws goes to the core of the theories of quantized observables and reveals the fundamental flaw in local realistic theories, making any experimental tests on their viability unnecessary. This statement is in the same spirit as stating that theories or models allowing perpetual motion are ruled out on first principles. If conservation laws are not applicable on the average, perpetual motion becomes possible since mechanical quantities can be transferred from quantum ensembles to classical systems. In experimentally interesting cases (like the spin-singlet and its photonic equivalent) the Bell's inequalities can be obeyed only by violating a fundamental conservation law, and therefore any expectation that they might be obeyed in nature is physically naive.

The apparent indeterminism and quantum jumps during quantum measurements are striking non-classical features of quantum mechanics. Attempts to understand these features, especially in measurements of correlations, have lead to considerable progress in the understanding of quantum correlations and entanglement, including many applications involving processing and transmission of quantum information. Yet, the fundamental issues that served as motivations for these developments remain, by and large, as they were. A local hidden variable theory or a local realistic theory is a *classical* statistical theory meant to replace quantum mechanics [2]. Such theories were proposed with the hope that one could preserve classical notions like locality, and reality of events and history in space and time, and yet reproduce the statistical results of quantum mechanics. The randomness in the measured results of an observable in such theories is mapped to the random values taken by certain hidden or unobservable classical variables. These were brought back into discussion in mainstream physics by the observations by Bohm, Bell and others that von Neumann's no-go theorem for hidden variables was not universally valid. John Bell's analysis of local hidden variable theories resulted in the celebrated Bell's inequalities [3]. These represent an upper limit on the correlation expected in such theories between results of measurements on separated correlated quantum systems. In the standard formulations, the magnitude of the correlation and its upper limit in a local hidden variable theory are typically smaller than what are predicted in the quantum mechanical description, for a wide range of settings of the measurement apparatus. Thus, quantum mechanical correlations violate the Bell's inequalities.

Several experiments have been done in the past, and several are in planning and execution to test the Bell's inequalities, and to thereby test the viability of local hidden variable theories [4]. Essentially all experiments to date find that the Bell's inequalities are violated, and that the measured correlations are in fact larger than the upper limit specified by the Bell's inequalities. The experiments also confirm with high precision that the quantum mechanical prediction for the correlation is what is favoured. The fact that many of these experiments are often discussed, and even more new experiments are being planned and performed means that somehow the issue of viability of a local realistic theory is not closed, and that the surprise over the violation of Bell's inequalities is strong. Modern experiments are attempting to perfect the tests so that various experimental loopholes arising in imperfections in the experiments are removed. However, if it is generally known that such experiments are testing quantum mechanics against theories that are grossly incompatible with fundamental conservation laws, these tests will become irrelevant. The importance of the result found in ref. [1] is that it unveils the fundamental physical constraint that makes the experimental observations in conformity with the quantum mechanical correlations, and shows clearly that no such experiments will ever show the Bell's inequality to be obeyed. In fact, the result suggests that the more perfect the experiment is, the better will be the violation of the Bell's inequalities; the conservation constraint is more stringent and the conservation law is perfect when there are no random decohering interactions that can perturb quantities like angular momentum and when the experimental ensemble is complete and the detection efficiency is perfect.

2. Correlation function from conservation law

Now I discuss the main result, its proof and its implications in detail. I start the discussion in the context of two-valued discrete observables, like spin projections of systems for which the total angular momentum is zero. Typical experimental configurations can be mapped to this case. Thus the starting configuration is characterized by the total spin, $S_{\rm tot} = 0$. Subsequently the system splits into n particles on which spin-projection measurements are done at different locations in arbitrary directions. Since typical measurements are done with two particles, we often take $n = 2$. Our aim is to see whether there are definite predictions for the correlation function of these measurements if the only assumption or constraint we make is the conservation of angular momentum on the average. Such a correlation function will then be applicable to all theories that respect the validity of the conservation law on the average. Further, if such a correlation function is a unique implication of the conservation laws, then the converse is true – a theory that has a different correlation function will be incompatible with the fundamental conservation laws.

2.1 *Spin-1/2 singlet*

First I will derive and discuss the result that the quantum correlation function for the two-particle singlet state is a unique consequence of the conservation law of angular momentum. I will then derive the general result for higher spin singlet states, and for certain other systems relevant and important for the experimental studies.

The maximally entangled state of two spin-1/2 particles (or polarization entangled photons) often employed in discussions and experiments on the Bell's inequalities is described by the wave function

$$\Psi_S = \frac{1}{\sqrt{2}}\{|1,-1\rangle - |-1,1\rangle\}, \tag{1}$$

where the state $|1,-1\rangle$ is the short form for $|1\rangle_1 |-1\rangle_2$, and represents a definite value for spin projection of $+\hbar/2$ for the first particle and $-\hbar/2$ for the second particle if measured in any particular direction. The state is a superposition of two such product states and it is an entangled state. Two observers A and B make measurements on these particles individually at space-like separated regions with time information such that these results can be correlated later through a classical channel.

The local hidden variable description of the same system starts with the functional restrictions on the outcomes A and B of measurements at the two locations [3].

$$A(\mathbf{a},\mathbf{h}) = \pm 1, \quad B(\mathbf{b},\mathbf{h}) = \pm 1. \tag{2}$$

A and B denote the outcomes $+1$ or -1 of measurements A and B, $\mathbf{a}$ and $\mathbf{b}$ denote the settings of the analyzer or the measurement apparatus for the first particle and the second particle respectively and $\mathbf{h}$ are the hidden variables associated with the outcomes. The local results should depend only on local settings and values of the hidden variables. The Bell correlation function is of the form

$$P(\mathbf{a},\mathbf{b}) = \int \mathrm{d}\mathbf{h}\rho(\mathbf{h})A(\mathbf{a},\mathbf{h})B(\mathbf{b},\mathbf{h}), \quad \text{where} \quad \int \mathrm{d}\mathbf{h}\rho(\mathbf{h}) = 1, \tag{3}$$

where $\rho(\mathbf{h})$ is the normalized statistical distribution of the hidden variables.

This is an average over the product of the measurement results. $P(\mathbf{a},\mathbf{b})$ is similar to the classical correlation function of the outcomes defined by

$$P(\mathbf{a},\mathbf{b}) = \frac{1}{N}\sum(A_i B_i). \tag{4}$$

The observed correlation is calculated using this formula, with observed outcomes A_i and B_i.

The quantum mechanical correlation for the same experiment is given by

$$P(\mathbf{a},\mathbf{b})_{\mathrm{QM}} = -\mathbf{a}\cdot\mathbf{b}. \tag{5}$$

This is the expectation value of the operator $\sigma_1\cdot\mathbf{a}\otimes\sigma_2\cdot\mathbf{b}$ for the singlet state when the projection is expressed in units of $\hbar/2$. (In terms of values of the projections of spin, the correlation function is $P(\mathbf{a},\mathbf{b})_{\mathrm{QM}} = -\mathbf{a}\cdot\mathbf{b}\ \hbar^2/4$.) The essence of the Bell's theorem is that the function $P(\mathbf{a},\mathbf{b})$ has distinctly different dependences on the relative angle between the analyzers for a local hidden variable description and for quantum mechanics. In the local realistic model, A (and B) can have simultaneous definite values for various directions $\mathbf{a}$ (and $\mathbf{b}$) in the set $\{+1,-1\}$, unlike the case in quantum theory where a definite value is manifested only in a measurement for a particular direction without any way of assigning values in the other unmeasured directions. Then the combination of joint measurements

$$AB + A'B - AB' + A'B' = A(B-B') + A'(B+B') = \pm 2 \tag{6}$$

because each observable takes values ± 1, and the simultaneously assigned values for A and A' (or B and B') can only be the combinations $(+1,+1)$, $(+1,-1)$, $(-1,+1)$ and $(-1,-1)$. So the specific combination of the Bell correlation functions $P(\mathbf{a},\mathbf{b}) + P(\mathbf{a}',\mathbf{b}) - P(\mathbf{a},\mathbf{b}') + P(\mathbf{a}',\mathbf{b}')$ is an average of ± 2, and lies between $+2$ and -2. Its magnitude is bounded from above by 2. This is the Bell's inequality. Looked at this way, it is clear that the inequality arises from ignoring the fundamental premise of quantum mechanics – superposition of states. Allowing the possibility of simultaneously assigning definite values (reality) for the spin projection in two different and even orthogonal directions for the same particle ($B = B' = 1$, for example, for 25% of the particles in this case) makes *the subensemble violate quantum*

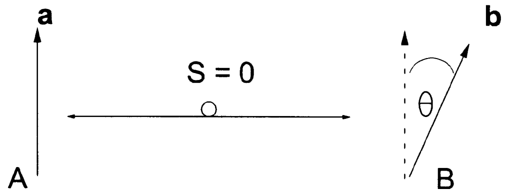

Figure 1. The settings of the analyzers in a two-particle spin correlation experiment.

superposition as well as the conservation constraint. The Bell's theorem refers to a classical statistical alternative to quantum theory. Our result goes a step ahead and shows that such theories do not respect fundamental conservation laws, thereby taking them out of contention on first principles.

Our aim is to derive the correlation function *independent of any particular theory*, from the only assumption that angular momentum is conserved. I will show that the quantum expectation value, $P(\mathbf{a},\mathbf{b})_{\rm QM} = -\mathbf{a}\cdot\mathbf{b}$, follows uniquely from the conservation law for angular momentum. (When $\mathbf{a}=\mathbf{b}$, $P(\mathbf{a},\mathbf{b})_{\rm QM} = -1$, and as the angle between $\mathbf{a}$ and $\mathbf{b}$ increases, the correlation drops as the vector projection of the two directions, exactly as expected from the conservation of angular momentum after averaging.)

2.2 *Assumptions and clarifications*

Apart from the fact that we are dealing with measurements that result in discrete quantized outcomes, $+1$ and -1 in units of $\hbar/2$, there is only one physical assumption used in deriving the main result: the total angular momentum is a conserved quantity *on the average*. Thus, the average total angular momentum measured on the sub-systems (particles) should add up to the initial value of the starting configuration.

We discuss this requirement in the context of the singlet state for clarity. We are considering a set of measurements in which the direction at A is fixed as $\mathbf{a}$ and that at B is fixed as $\mathbf{b}$ (see figure 1). Each individual measurement in any direction gives either $+1$ or -1. Some important clarifications are essential at this stage. Since the only constraint on the initial state is that it is a singlet state with $S_{\rm tot}=0$, the individual particles at different measurement locations can show both $+1$ and -1 as results with equal probability. If measured in the same direction a pair of particles will always show the result $\{+1,-1\}$, in conformity with the angular momentum conservation (in quantum theory, this can be read off from the state since the singlet state retains its form when written in any basis). If the directions are different, the vector components of the average angular momentum in the same direction should add up to zero. Clearly the average over the *total ensemble* comprising of all the particles at A or B will show zero values.

$$\sum_1^N A_i = \sum_1^N B_i = 0. \tag{7}$$

Therefore their sum is always zero in a trivial way. This does not test the conservation of angular momentum for the *two-particle system*. To test the validity of the conservation of angular momentum on the average, it is essential that all sufficiently large correlated sub-ensembles also obey the conservation law. For example, the two sub-ensembles in which the measured values at A are $+1$ and -1, along with corresponding correlated values measured at B, should separately conserve the average angular momentum. I will prove this point independent of any particular theory before proceeding further, to establish the generality of the final results.

The elementary detection event in a correlation experiment is the detection of the values of a specific observable for two or more particles in the multiparticle event. A statistically large sub-ensemble for our purpose is that in which the statistical fluctuations are much smaller than the average value itself, and this is the case when $|\Delta Q|/\langle Q\rangle \simeq \sqrt{N}/N$ is sufficiently small, where Q is the relevant observable. In the context of the Bell's inequalities, we need only $1/\sqrt{N} \ll 2(\sqrt{2}-1)$, satisfied by any good experiment. Consider a set of elementary detection events $\{1, 2, ..., N\}$. This consists of observations $\{(A_1B_1), (A_2B_2)...(A_iB_i)...(A_NB_N)\}$. The average value of the correlation is *independent of arbitrary reordering* of the index i. The ensembles are taken to be large enough for the statistical errors to remain small in the sub-ensembles. The reordering of the record of the events can be done such that $A_i = +1$ for events $\{i = 1, ..., N/2\}$ and $A_i = -1$ for events $\{i = N/2 + 1, ..., N\}$. The distribution of B_i in each sub-ensemble depends on the angle between the analyzers. (The situation is symmetric. We could classify the sub-ensembles based on the values of B_i being $+1$ and -1.)

The theory independent correlation function is

$$P(\mathbf{a}, \mathbf{b}) = \frac{1}{N}\sum_{1}^{N}(A_iB_i). \tag{8}$$

For the singlet state, we have the condition that *if the correlation is measured in the same direction* at A and B, $P(\mathbf{a}, \mathbf{b}) = -1$. (This follows from the fact that for a zero-angular momentum system breaking up into two particles, each showing only ± 1 as projections, the only combinations of *projections in the same direction* allowed by angular momentum conservation are $(+1, -1)$ and $(-1, +1)$. For other systems, there are similar 'boundary' conditions. For example, for the triplet state, $P(\mathbf{a}, \mathbf{b})_Z = -1$, and $P(\mathbf{a}, \mathbf{b})_X = +1$, when both are measured along the z-axis and x-axis respectively.) Then

$$A_i + B_i = 0 \tag{9}$$

for every event. Therefore, for the sub-ensembles,

$$\langle A\rangle_1 \equiv \frac{1}{N/2}\sum_{i=1}^{N/2} A_i = -\frac{1}{N/2}\sum_{i=1}^{N/2} B_i \equiv -\langle B\rangle_1 = +1, \tag{10}$$

$$\langle A\rangle_2 \equiv \frac{1}{N/2}\sum_{i=\frac{N}{2}+1}^{N} A_i = -\frac{1}{N/2}\sum_{i=\frac{N}{2}+1}^{N} B_i \equiv -\langle B\rangle_2 = -1. \tag{11}$$

Each of the terms above is the average angular momentum for that sub-ensemble. Therefore, in this case the average angular momentum is conserved for each two-particle sub-ensembles separately, $\langle A\rangle_1 + \langle B\rangle_1 = 0$, and $\langle A\rangle_2 + \langle B\rangle_2 = 0$.

For each of the sub-ensembles the magnitude of average angular momenta, $\langle A\rangle_1$, $\langle A\rangle_2$, $\langle B\rangle_1$, $\langle B\rangle_2$ are preserved in rotations of the axis through angle θ. For example consider $\langle B\rangle_2 = 1$ in a particular direction. If measurements are made at angle θ to this direction in this sub-ensemble of spin-1/2 particles, the probabilities for +1 is $\cos^2\frac{\theta}{2}$ and for −1, it is $\sin^2\frac{\theta}{2}$. So, the average spin projection at angle θ, or equivalently the average angular momentum in the direction θ is

$$\langle B\rangle_\theta = +1 \times \cos^2\frac{\theta}{2} + (-1)\times\sin^2\frac{\theta}{2} = \langle B\rangle\cos\theta. \tag{12}$$

Therefore, if the analyzer at B is at angle θ, the average angular momentum measured at B for each sub-ensemble will be $\langle B\rangle_{\theta 1} = \langle B\rangle_1\cos\theta$ and $\langle B\rangle_{\theta 2} = \langle B\rangle_2\cos\theta$ respectively. (Effectively we can derive the equivalent of the Malus' law from the constancy of the magnitude of the angular momentum, by performing this calculation in the reverse.)

Then we get from eqs (11) and (12), after multiplying with $\cos\theta$,

$$\langle A\rangle_1\cos\theta = -\langle B\rangle_{\theta 1}, \tag{13}$$

$$\langle A\rangle_2\cos\theta = -\langle B\rangle_{\theta 2}. \tag{14}$$

Therefore, the conservation law applies to each sub-ensemble separately for arbitrary θ.

The discovery that the correlation function is determined by the average angular momentum of one particle conditional on a specific measured angular momentum projection of the other particle [1] is the basis of the results presented in the subsequent sections. Thus

$$\begin{aligned} P(\mathbf{a},\mathbf{b}) &= \frac{1}{N}\sum(A_iB_i) = \frac{1}{2}\left(P(\mathbf{a},\mathbf{b})_{A=+1} + P(\mathbf{a},\mathbf{b})_{A=-1}\right) \\ &= \frac{1}{2}\left((A_i=1)\times\frac{1}{N/2}\sum_{i=1}^{N/2}B_i + (A_i=-1)\times\frac{1}{N/2}\sum_{i=\frac{N}{2}+1}^{N}B_i\right) \\ &= \frac{1}{2}\left(1\times\langle B_i\rangle_{A=+1} + (-1)\times\langle B_i\rangle_{A=-1}\right). \end{aligned} \tag{15}$$

The notation $\langle B_i\rangle_{A=\pm 1}$ represents the average value of angular momentum at B, conditional on specific values of angular momentum projection at A.

We do not make any assumption regarding locality or reality. These concepts are not needed for the proof since we are dealing with only averages and expectation values obtained in measurements. (Assumption of locality makes the proof stronger.) No counterfactual statements are made in this discussion. At no stage we demand the validity of the conservation law for individual events. This in any case is impossible when the outcomes are discrete.

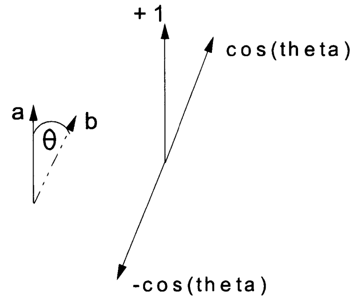

Figure 2. The vertical arrow represents the average angular momentum of the ensemble with spin projection +1 measured at A. Its vectorial component along direction $\mathbf{b}$ is $\cos(\theta)$. Then conservation requires that the average angular momentum along $\mathbf{b}$ at the location B is $-\cos(\theta)$.

2.3 *Derivation of the unique correlation function*

Consider two subsets, $A1$ and $A2$, of results at the location A; one subset containing only +1 and the other containing only −1. For the two-particle system with total zero angular momentum, there will be equal number of +1 and −1 at A and B. The average angular momentum for the first subset at A is +1 and for the second subset, it is −1. There will be two correlated subsets, $B1$ and $B2$, at B whose individual averages will depend on the angle setting at B (figure 1). Consider

$$P(\mathbf{a},\mathbf{b})_{A=+1} = \frac{1}{N/2}\sum[(A_i=+1)B_i] = \frac{1}{N/2}\sum_{B1} B_i. \tag{16}$$

This is nothing but the average angular momentum measured at B in the direction $\mathbf{b}$. From eq. (12), with $\langle A\rangle_1 = +1$, this is $-\cos(\theta)$ in unit of $\hbar/2$, where θ is the angle between the measurement directions (see figure 2).

$$P(\mathbf{a},\mathbf{b})_{A=+1} = -\cos(\theta) \tag{17}$$

Then, and only then, the average angular momentum is conserved.

Similarly, for the subset with results −1 for A,

$$P(\mathbf{a},\mathbf{b})_{A=-1} = \frac{1}{N/2}\sum[(A_i=-1)B_i] = -\frac{1}{N/2}\sum_{B2} B_i = -\cos(\theta). \tag{18}$$

Therefore the correlation for the entire set is also $-\cos(\theta)$, or $P(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}$.

Let us summarize, for clarity, the steps leading to the theory-independent correlation function that follows from the conservation law [1]. The projection of the normalized classical angular momentum vector in a direction that is rotated by angle θ is just $\cos(\theta)$. In a situation where individual measurements give discrete values +1 and −1, the *average* angular momentum still preserves this relation. A set of particles with average angular momentum +1 in a particular direction will show an average angular momentum $\cos(\theta)$ in a direction rotated by θ.

Since A_i and B_i take values of $+1$ or -1 (in units of $\hbar/2$), we group the values such that all A_i are $+1$ in one group and all A_i are -1 in another. Of course the B_i are mixed in both groups. Then the summation index, which is scrambled due to regrouping, is relabelled from 1 to N again. We get

$$\begin{aligned} P(\mathbf{a},\mathbf{b}) &= \frac{1}{N}\sum(A_iB_i) \\ &= \frac{1}{2}\left(\frac{1}{N_{A+}}\sum_{i=1}^{N/2}+1.(B_i)+\frac{1}{N_{A-}}\sum_{i=(N/2)+1}^{N}-1.(B_i)\right). \end{aligned} \tag{19}$$

The first term is the average of B_i conditional on $A_i = +1$, and the second term is the average of B_i given $A_i = -1$. $N_{A+} = N_{A-} = N/2$. Then we get, using only the conservation law for the averages,

$$P(\mathbf{a},\mathbf{b}) = \frac{1}{2}\left(-\cos(\theta)-\cos(\theta)\right) = -\cos(\theta). \tag{20}$$

This is same as the quantum mechanical correlation function $P(\mathbf{a},\mathbf{b})_{\mathrm{QM}}$. We have proved that the correlation function $-\cos(\theta)$ and thus $P(\mathbf{a},\mathbf{b})_{\mathrm{QM}}$ is a consequence of the conservation of angular momentum. The correlation function represents the conservation law, and follows uniquely and directly from it.

This result immediately implies that any other correlation function with a different dependence on the relative angle violates the law of conservation of angular momentum. Equivalently, any expectation that the experimental tests might have supported a correlation function *different* from $P(\mathbf{a},\mathbf{b})_{\mathrm{QM}} = -\mathbf{a}\cdot\mathbf{b}$ certainly had not appreciated the fact that *in order to get a deviation from* $-\mathbf{a}\cdot\mathbf{b}$, *the conservation law for angular momentum has to be violated*. The Bell's inequalities can be obeyed only when the correlation function deviates significantly from $-\mathbf{a}\cdot\mathbf{b}$. This completes the proof that the Bell's inequalities can be obeyed only in theories of discrete observables that do not respect the fundamental conservation law, invalidating any credibility for such theories. The correlations in any local hidden variable theory obeys the Bell's inequalities, and hence these theories fall in a class that do not respect the fundamental conservation laws.

Since all the measurements are done using classical apparatus there is no question of attributing this discrepancy in the angular momentum to some uncontrolled measurement interaction. In particular, the correlation is perfect when the two space-like separated analyzers are in the same direction, and this rules out any correlated influence of the hidden variables accounting for the discrepancy in any local theory.

It is important to note that *the requirement of conservation laws is much stronger than the demarcating criteria based on the Bell's inequalities*. Our result implies that even the slightest deviation from the quantum mechanical correlation function makes the theory physically nonviable due to the incompatibility with the fundamental conservation laws. For the Bell's inequalities to be obeyed, this violation has to be as much as 30% to 50% or even more, depending on the experimental system. I may also stress again, to avoid a common confusion, that the discussion so far is in the context of measurements that give discrete results, and that we are not dealing with classical observables that can take any value within some continuous interval. The results are generalized to continuous observables in a later section.

2.4 *Generalization to higher spins*

The result can be generalized to two-particle systems with larger total spin [1]. For a spin-S entangled singlet state, there will be $2S+1$ different possibilities for measurement results, $+S, +(S-1)..0..-(S-1), -S$. All of these occur with equal probability, due to rotational invariance. Since each sub-ensemble has to separately obey the conservation law, as explained earlier, we group the measurements at A in a sequence. The angle between analyzers at A and B is θ. For a specific group at A with measured results of $(S-n)$, for example, the average of the angular momentum in the direction rotated by θ is $(S-n)\cos(\theta)$. Conservation of angular momentum then dictates that the average angular momentum at B is $-(S-n)\cos(\theta)$, and the correlation for that group is

$$P(S,\theta) = -(S-n)^2\cos(\theta). \tag{21}$$

The same correlation will be observed also for the results $-(S-n)$ at A. For the '0' state, the correlation (average of the angular momentum correlation) is zero. Therefore, the average of all the two-point measurements is

$$\begin{aligned} P(\mathbf{a},\mathbf{b}) &= \frac{-\cos(\theta)\left(S^2+(S-1)^2+\cdots+0+\cdots+(S-1)^2+S^2\right)}{2S+1} \\ &= -\cos(\theta)S(S+1)/3. \end{aligned} \tag{22}$$

This is same as the quantum mechanical correlation function for the spin-S singlet state [5]. The physically valid correlation function is uniquely predicted from the conservation law. Therefore, any other functional dependence of correlation is incompatible with the conservation constraint.

Clearly, it is possible to generalize this approach to larger number of particles sharing the angular momentum of the initial state. Instead of a single correlation function, there will be a set of parametrized correlation functions representing various conditional joint probabilities. But in each case, it is the conservation of total angular momentum on the average that will decide the correlation function.

2.5 *Spin-1/2 triplet state*

Though most of the experiments related to the Bell's inequalities are done with an effective singlet state, we derive the correlation function for a triplet state to show the generality of the equivalence of the conservation of angular momentum and the correlation function. The spin-1/2 triplet state corresponds to the situation where the total angular momentum is $\sqrt{s(s+1)}$ with $s=1$. There are three discrete projections along the quantization axis possible with $m=\pm 1$, and $m=0$. Our task is to derive the correlation functions using only the conservation constraint, without using the quantum mechanical operators. If the measurement axis of spin projection of one of the particles is fixed to be the quantization axis that prepared the two-particle system, then the correlation function for the $m=0$ state is identical to the one that we derived earlier for the singlet state, because in that direction the total spin projection is constrained to be zero. Similarly, the correlation functions

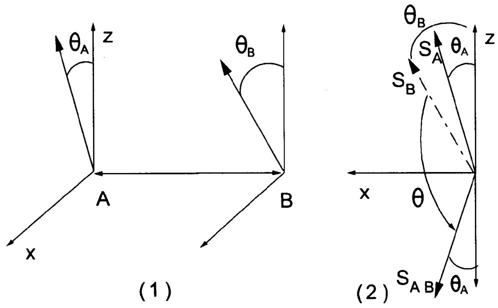

Figure 3. Diagram detailing the geometry for the calculation of two-particle correlation for a triplet state from the conservation of angular momentum. See text for a complete description.

for the $m = \pm 1$ states are simple, since it corresponds to both the spin projections aligned up for $m = +1$, and aligned down for $m = -1$. Therefore, consideration of the average angular momentum balance dictates that these correlation functions are just *products of two cosine functions* in respective angles measured from the z-axis. Here we calculate the case with $m = 0$, for general angular settings of the detectors. Of course, the condition $s = 1$, $m_z = 0$ means that classically the spins should be aligned in the x–y plane, with both pointing in the same direction. The projection of the total angular momentum along the z-direction should be zero for each measurement, since $m_z = 0$. So, for any single measurement on the pair in the x–y plane, the projections will be $(+,+)$ or $(-,-)$. But along the z-direction, the measurement on the pair will give projections $(+,-)$ or $(-,+)$, according to the two angular momentum constraints that $s = 1$, and $m_z = 0$. Quantum mechanically the measurement has a similar interpretation as can be seen by writing the state in the z-basis, and also in the x-basis (see below).

Now we derive the general correlation function from the considerations of constraints on the total angular momentum. The measurements are done at two locations A and B as in the case of the singlet, in directions $\mathbf{a}$ (angle θ_A) and $\mathbf{b}$ (angle θ_B). All results will be either $+1$ or -1 in unit of $\hbar/2$. Consider all the results with a $+1$ at A. The average angular momentum of this subset is $+1$. This is represented as vector $\mathbf{S}_A$ in figure 3. The correlated measurements at B will have an average angular momentum $\mathbf{S}_B$ that corresponds to the conservation constraint of the original state – the average angular momentum at B should be a vector that cancels the z-component ($m_z = 0$), and has identical x and y components as the average angular momentum at A. It is just the reflection of the average angular momentum at A, in the x–y plane. This is represented as vector $\mathbf{S}_{AB}$ in the figure. Only then can one satisfy the fact that the total angular momentum of the state is non-zero, requiring that the x or y component is non-zero since the z-component is zero. All vectors in the figure are measured average angular momenta, and their projection along particular directions.

Thus the magnitude of the average angular momentum at B along the direction $-\theta_A$ from the $-z$ axis is $+1$, which is identical to the angular momentum at A in the θ_A direction. The correlation function is simply the projection of this angular momentum along the θ_B direction, from the z-axis. Therefore, the correlation function for the triplet, $m = 0$ state can be deduced as

$$\begin{aligned} P(\mathbf{a},\mathbf{b}) &= +1 \times \cos\theta = +1 \times \cos(180 - (\theta_A + \theta_B)) \\ &= -\cos(\theta_A + \theta_B)\,. \end{aligned} \tag{23}$$

The subset for which all measurements at A gives -1 leads to the same $P(\mathbf{a},\mathbf{b})$. Thus the correlation function for the entire ensemble is also

$$P(\mathbf{a},\mathbf{b}) = -\cos(\theta_A + \theta_B)\,. \tag{24}$$

More specific correlation functions can be derived from this. If the measurements at A are done along the z-axis, then $\theta_A = 0$, and then the correlation is identical to that in the singlet state, $P(\mathbf{a},\mathbf{b}) = -\cos\theta_B$. If the measurements at A are done along the x-axis, then $\theta_A = \pi/2$, and the correlation function is $P(\mathbf{a},\mathbf{b}) = \sin\theta_B$. This correlation function was obtained using the fact that the total angular momentum of the state is $s = 1$, and that the total projection on the z-axis is zero. Measurements on the two particles should then reflect the same conservation constraints.

Let us now see whether this agrees with the quantum mechanical correlation function, to be obtained by the appropriate operators acting on the state. The triplet $m = 0$ state in the z-basis is

$$\Psi_T = \frac{1}{\sqrt{2}}\{|1,-1\rangle + |-1,1\rangle\}. \tag{25}$$

In the x-basis this becomes

$$\Psi_{TX} = \frac{1}{\sqrt{2}}\{|1,1\rangle_X - |-1,-1\rangle_X\}. \tag{26}$$

Clearly, a single measurement on the pair along the z-axis should give either the result $(+,-)$ or the result $(-,+)$ with equal probability. On the other hand, the measurement along x-axis (or along y-axis) will give either $(+,+)$ or $(-,-)$.

The quantum mechanical correlation function is calculated as

$$P(\mathbf{a},\mathbf{b})_{\mathrm{QM}} = \langle\Psi_T|\,(\sigma\cdot\mathbf{a})\,(\sigma\cdot\mathbf{b})\,|\Psi_T\rangle\,. \tag{27}$$

This may be done conveniently in the matrix representation of the operators and the state.

$$\sigma\cdot\mathbf{a} = \begin{pmatrix} a_3 & a_1 - ia_2 \\ a_1 + ia_2 & -a_3 \end{pmatrix}. \tag{28}$$

In the case we are considering where the measurements are in a plane normal to the y-axis, in the x–z plane, $a_2 = 0$. Similarly, $\sigma\cdot\mathbf{b}$ can be written. In the matrix representation the triplet state is

$$\Psi_T = \frac{1}{\sqrt{2}}\left\{\begin{pmatrix}1\\0\end{pmatrix}_A\begin{pmatrix}0\\1\end{pmatrix}_B + \begin{pmatrix}0\\1\end{pmatrix}_A\begin{pmatrix}1\\0\end{pmatrix}_B\right\}. \tag{29}$$

The expression for $P(\mathbf{a},\mathbf{b})_{\mathrm{QM}}$ can now be evaluated, and this gives

$$P(\mathbf{a},\mathbf{b})_{\mathrm{QM}} = (\sin\theta_A \sin\theta_B - \cos\theta_A \cos\theta_B) = -\cos(\theta_A + \theta_B)\,. \qquad (30)$$

Thus the quantum mechanical correlation function is identical to the one we obtained using the conservation constraints on the average angular momentum. This particular correlation violates the Bell's inequality for specific angles. This again is what is expected since obeying the Bell's inequality will amount to violating the conservation law.

2.6 *The three-particle GHZ state*

This is an interesting example where the quantum mechanical correlation product of the values of three-point measurement conflicts with local realistic theories even for a single measurement, and not just for averages and correlation functions. Remarkably, the analysis in terms of angular momentum conservation gives the same result. Though various correlations as a function of relative angles can be derived using conservation of angular momentum, as in the case of the two-particle states, here we discuss only the result when the measurement is done in the same direction for all the three particles.

The GHZ state [6], written in the z-basis is

$$\Psi_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}}\left(|+,+,+\rangle_Z - |-,-,-\rangle_Z\right). \qquad (31)$$

The total spin projection on the z-axis can only be $+3/2$ or $-3/2$. It is obvious that this state, if written in the eigenbasis of the x-axis measurements, cannot have terms with an odd number of $+$ values, since there is a minus sign between the two three-particle states constituting the superposition in the z-basis. Expansion of $|+,+,+\rangle_Z$ in the x-basis contains triple products of terms of the form $(|+\rangle + |-\rangle)_X$ that simply add. Terms in the expansion of $|-,-,-\rangle_Z$ contains similar products of $(|+\rangle - |-\rangle)_X$ that cancel out unless there is an odd number of $|-\rangle$ kets. Writing explicitly in the x-basis,

$$\Psi_{\mathrm{GHZ}-X} = \frac{1}{2}\left(|+,+,-\rangle_X + |+,-,+\rangle_X + |-,+,+\rangle_X + |-,-,-\rangle_X\right). \qquad (32)$$

Of course, this says that the initial state can have spin projection on the x-axis, in each measurement, of only $-3/2$ and $+1/2$. Thus, from the conservation of angular momentum, the correlation product of three separate measurements on the three particles, all performed in the x-direction is always -1. Each measurement should show an odd number of (-1) outcomes to give a net angular momentum projection of $-3/2$ or $+1/2$ along the x-axis. Any other combination of measurement results $((+,-,-)$ or $(+,+,+)$ for example) will be incompatible with the conservation of angular momentum, just as a result of the form $(+,+)$ when measured in the same direction is incompatible with the zero angular momentum of the spin-singlet state.

The local hidden variable theories predict that the product of measured values in the x-direction will be $+1$. The state Ψ_{GHZ} is an eigenstate of the commuting operators $\sigma_x^1\sigma_y^2\sigma_y^3$, $\sigma_y^1\sigma_x^2\sigma_y^3$, and $\sigma_y^1\sigma_y^2\sigma_x^3$ with eigenvalue $+1$. The product of these operators is the operator $-\sigma_x^1\sigma_x^2\sigma_x^3$, and therefore the eigenvalue of $\sigma_x^1\sigma_x^2\sigma_x^3$ in the GHZ state is -1. In a local realistic theory, the values of the spin projections represent reality as they exist, context independent, and implies that $m_i^1 m_j^2 m_k^3$ has the same value as implied by the corresponding operator equation. Then, $m_x^1 m_y^2 m_y^3 = 1$, $m_y^1 m_x^2 m_y^3 = 1$, and $m_y^1 m_y^2 m_x^3 = 1$. But each of m_i is ± 1, and the product of the three expressions involve squares of the various m_y values, and therefore the product should have been $m_x^1 m_x^2 m_x^3 = +1$. (This statement is equivalent to ignoring the noncommutativity of the operators in the product.) *This is obviously inconsistent with the restriction on the angular momentum along the x-axis.*

In contrast, the three-particle state

$$\Psi_+ = \frac{1}{\sqrt{2}}\left(|+,+,+\rangle_Z + |-,-,-\rangle_Z\right) \tag{33}$$

has the property that its spin-projections along x-axis can only be $+3/2$ or $-1/2$.

$$\Psi_{+X} = \frac{1}{2}\left(|+,+,+\rangle_X + |+,-,-\rangle_X + |-,+,-\rangle_X + |-,-,+\rangle_X\right). \tag{34}$$

The product of eigenvalues in the x-direction will be $+1$. In this case, local hidden variable theories predict -1 for the product, violating the restriction on the angular momentum along the x-axis.

The application of the angular momentum constraint allows us to also derive angle-dependent correlation functions easily for these cases, in simple situations. For example, for the original GHZ three-particle state, when two of the three particles are detected in the same state in the x-direction as $(+,+)$ or $(-,-)$, the correlation function with the third particle measured at angle θ from the x-axis is

$$P(X, X, \theta) = -\cos(\theta). \tag{35}$$

If any two are detected in different states along x-axis, the three-point correlation function is $+\cos(\theta)$. Thus the quantum predictions have a transparent and simple meaning when seen as a measurement compatible with the conservation of the angular momentum along a particular axis.

3. Continuous variables and correlations

There are several examples of non-classical, two-particle correlations that violate a Bell's inequality involving the continuous variables position and momentum in the entanglement and correlation. Here we discuss one such example to show that such correlations are also determined by the conservation law alone, and hence experimental results can obey a Bell's inequality only by violating the fundamental conservation law on the average.

The original EPR wave function has such a correlation. The EPR system consists of two particles with well-defined simultaneous values for their *total* momentum and their *relative* positions, $p_1 + p_2$ and $q_1 - q_2$, represented by commuting observables. The relevant conservation law is that the total momentum should be conserved during propagation and that independent measurements on the two particles should give results that obey this conservation law on the average.

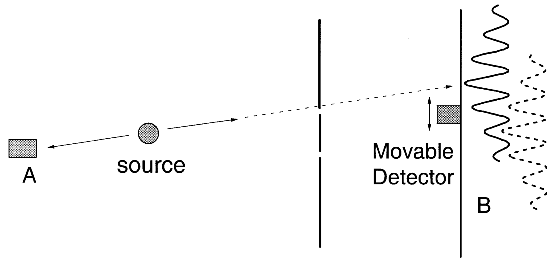

Figure 4. The two-particle quantum correlation function in this experiment is the same as the shifted one-particle two-slit interference pattern determined by the conservation of the linear momentum. The momentum of the particles detected at A dictates the momentum of the correlated particles to be along the dashed long arrow, and the two-slit interference pattern should then be centred in that direction. This is also the conditional probability or two-particle correlation. The dashed line interference pattern is the one centred at $x_A = x_B = 0$.

The experimental set-up that we consider is the two-particle, two-slit interference geometry in which at least one particle encounters a screen with two slits and then a measurement plane where there are detectors with timing information for recording the position of the particles (figure 4). Measurements on each particle separately reveal no interference pattern. But the coincidence measurements in which one detector is kept at a fixed position at A and the other scanned at B reveal a *two-particle correlation identical to a shifted single-particle two-slit interference pattern* with 100% visibility. The equivalent of the Bell's inequality in this case is that the local hidden variable theory predicts visibility less than 70%. The quantum mechanical two-particle correlation function for this set up is

$$P(x_1, x_2) = \frac{1}{2}(1 + \cos k\alpha(x_1 + x_2)), \tag{36}$$

where $k = 2\pi/\lambda$, and α is the geometrical amplification factor from the source to the screen. The detector positions x_1 and x_2 are measured in the same sense at locations A and B. This is a conditional probability. The similarity to the correlation function for spin variables is obvious. Let us see what the conservation of momentum alone predicts in this situation. Consider the sub-ensemble generated by fixing one of the detectors at some position x_A. For all the particles detected at this position the momentum direction from the source is fixed (within a small uncertainty related to the size of the source) as $\theta_A = x_A/D_A$, where D_A is the distance between the source and the detector at A. This means that the average transverse momentum for the particles detected at the detection region B has to be oppositely directed such that $\theta_B \equiv x_B/D_B = -\theta_A$. Thus the *single-particle two-slit interference pattern* detected at B should have its mean position at angle

$$\theta_B = -\theta_A \propto -x_A/D_A. \tag{37}$$

The interference pattern at B should be shifted by $x_{B0} = -D_B \times \theta_A$. Since the single-particle two-slit interference pattern with mean at $x = 0$ is $(1 + \cos \alpha k x_B)/2$, the shifted two-particle correlation pattern predicted by the conservation of linear momentum is

$$P(x_A, x_B) = \frac{1}{2}(1 + \cos k\alpha(x_B + x_A)) \tag{38}$$

if we take $D_A = D_B$. This is the same as the quantum mechanical correlation function. The condition $x_B + x_A = 0$ determines the central fringe, and this is exactly the condition that the transverse momenta are equal and opposite. We see that the quantum two-particle correlation function is just the shifted single-particle interference pattern consistent with the conservation of linear momentum. Therefore, application of the conservation constraint alone predicts a correlation function with 100% visibility, shifted by an appropriate angle that depends on the position of the detector at A. Clearly, a reduction of visibility to 70% in a good experiment can happen only by a gross violation of the conservation law.

A reduction of the visibility can happen in a non-ideal experiment where the conservation constraint is weak due to geometry or decoherence. Our analysis predicts that the wider the detector at A, the less will be the visibility of the correlation function, due to averaging over x_A in the cosine function. It also predicts the weakening of visibility when the source is very small in size due to the uncertainty in the total momentum by $\delta p \simeq h/\delta x$, where δx is the size of the source. Then the conservation constraint is uncertain by δp, and $\vec{p}_A + \vec{p}_B \simeq \delta p$ and not zero. I stress again the point that the conservation constraint is applied only for those events that are detected, and there is no assumption on the reality of values of momenta without a measurement. Also, there is no counterfactual quantities used anywhere in the analysis.

4. Discussion

4.1 *Correlation in local hidden variable theories*

From the generality of the result that the conservation law implies a unique correlation function identical to the quantum mechanical correlation function, it is clear that our fundamental attitude towards local hidden variable theories and the Bell's inequalities will require significant revision. The conservation law is both necessary and sufficient to have the correlation function of the quantum mechanical form. Local hidden variable theories are not viable physical theories because they violate the fundamental conservation laws on the average by a large margin. To be more explicit and precise about the amount of departure from the conservation law, let us see what the angular dependence of the Bell correlation function for local hidden variable theory looks like. This dependence was derived and discussed in detail in Bell's writings (see for example ref. [7]). If the angle of one of the analyzers is suddenly changed, the correlation function has to change, and since there is no information available on the instantaneous setting of the spatially separated analyzer, this change is to be composed of individual changes that depends separably on the individual angular setting of each analyzer. This shows that the correlation function changes linearly with angle (figure 5) [7]. If the correlation function obeys

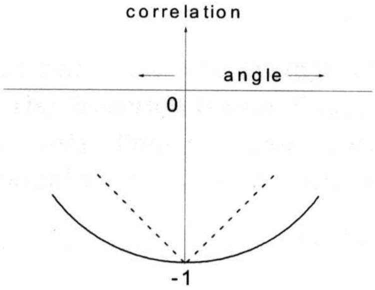

Figure 5. Diagram showing the different dependences of the correlation functions on relative angle. The solid curve represents the quadratic behaviour, near zero relative angle, of the correlation function from the conservation of angular momentum (same as the quantum mechanical correlation). The linear change in correlation (dotted lines) is a characteristic of local hidden variable theories of the type discussed by Bell. It violates the requirement of conservation of angular momentum as explained in the text.

the constraint that for $\theta = 0$ the correlation is -1 and for $\theta = \pi/2$ the correlation is zero for the spin-half case, the function is linear in the entire range. We see that such a correlation function does not respect the conservation of angular momentum of the total state on the average. Every fundamental conservation law associated with a space-time symmetry is expected to be valid in any fundamental theory of physics on the average, and this indeed is the case for conservation of energy–momentum, and angular momentum. When one tests the Bell's inequalities, one is testing whether a theory for which the conservation law is not applicable could be a valid theory of natural phenomena – obviously, the inequalities are bound to be violated.

4.2 *A comment on correlations in mixed states*

Mixed states in quantum mechanics are statistical mixtures of pure states. In such a system the average of every physical quantity is a statistical ensemble average over the various pure states with weightage specified by the mixing fractions. Therefore, in general, the correlations are less than or equal to those in pure states of multiparticle systems. Description of a mixed state in a local hidden variable theory is doubly classically statistical. Since the pure state correlations are completely equivalent to the conservation constraints as we have shown, the correlations in a mixed state is simply the statistical average of the pure state correlations, and therefore the statistical average of the conservation constraints. There is a direct *linear relation* between the correlations of the constituent pure states and that of the mixed state. For the mixture specified by

$$X = \{p_i, X_i\}, \tag{39}$$

where p_i is the classical statistical weightage for each pure state X_i constituting the mixed ensemble, the correlation is

$$P(\mathbf{a},\mathbf{b}) = \sum p_i P_i(\mathbf{a},\mathbf{b},X_i). \quad (40)$$

As the mixing increases, the correlations drop. These correlations may even degrade towards the bounds set by the Bell's inequalities. But the correlations are still equivalent to the conservation constraints because the average of any quantity is just the linear weighted average over the constituent pure states of the mixture.

4.3 *A comment on quantum entropy*

We can use the result on pure state correlations for gaining insight as to why the maximal violation of Bell's inequalities in an N-particle maximally entangled state increases exponentially. The maximal violation is in fact due to the difference in the cosine and the linear correlation functions (just the amount by which the conservation law is violated). Their ratio is maximum at $\theta = \pi/4$, and it is $\sqrt{2}$. For an N-particle system, the correlation space is multidimensional with $\theta_1, \theta_2, ...$ representing the various relative settings between apparatus. Now the violation of the inequalities is the ratio of areas in this multi-dimensional space, and its maximal value is $(\sqrt{2})^n$ where $n = N - 1$ is the dimensionality of the correlation space, and N is the number of spin-half particles. Thus the violation is given by

$$\zeta(N) = 2^{(N-1)/2}. \quad (41)$$

Our results indicate that the amount of violation depends only the number of particles in the N-particle maximally entangled state and not on the value of the spin itself. This is because the coefficient multiplying the angular dependence $-\cos(\theta)$ is just a scaling factor applicable to both the quantum correlation and the local hidden variable theory when the correlations are constrained to be equal in the two theories at $\theta = 0$ and $\theta = \pi/2$. Therefore their ratio will reflect only the difference between the cosine correlation and the linear correlation. Clearly, the source of the exponential factor is the same as that in the expressions for entropy for a classical multiparticle system. It is a measure of how many ways a fundamentally conserved quantity can be distributed among N-particles. The difference in the quantum mechanical case from the classical case is that the distribution is reflected in the relative phases between the states, and not directly in the value of the observables [8]. The essential idea is that a classical conservation constraint on any generalized momentum p_μ, like the total angular momentum or energy being a constant, directly translates to a constraint on the phase of the combined quantum mechanical system since the quantum phase depends linearly on these physical quantities (in the form $\frac{1}{\hbar} p_\mu \mathrm{d}x^\mu$). Thus a *classical constraint* on a two-particle system of the form $\vec{p}_A + \vec{p}_B = 0$ will *induce a constraint on the quantum phases* of the two-particle wave function. In a multi-particle system, the different ways of sharing the phases subject to the constraint is in fact the quantum entropy [8]. While individual phases can be arbitrary, their sum or difference will be constrained and this is how the conservation law and hence the quantum correlation are encoded in terms of local quantities. This clarifies the physical relation between quantum entropy, conservation laws and violation of the Bell's inequality.

4.4 *A comment on nonlocality*

Since the quantum mechanical correlation function is shown to be a unique consequence of a fundamental conservation law, it is clear that the correlation is encoded at source as the information on the total conserved quantity in the particles. In this paper we have not discussed the 'mechanism' in the quantum world that gives the correct correlations on spatially separated measurements despite the lack of reality for the values of the observables prior to the measurement. In the standard theory, the mechanism goes by the name quantum nonlocality. I have shown elsewhere [9,10] that this information is encoded in the phase correlation between the particles at the source itself – the classical conservation law becomes a constraint on the relative phase. Each particle has a corresponding state with an arbitrary initial phase, but the phases of the two states obey a constraint. This is sufficient to give the observed correlations. The *correlation is predetermined, and encoded in the difference between the unobservable individual phases*, without having to encode it directly as the value of the observable quantity like spin projection or momentum itself. Thus quantum correlations can be reproduced without ascribing definite reality, in the EPR sense, to observable quantities before an actual measurement. These facts indicate that there need not be any nonlocal influence violating Einstein locality.

In the context of the result discussed in this paper, an important point to be noted is that quantum formalism allows the unmeasurable initial phase to be a continuous variable associated with the system, with a hidden reality, without committing on the reality of the measurable values of physical quantities. This allows a description of EPR-like correlations without nonlocality, but with a shifted reality. The Bell's theorem prohibits ascribing reality at the level of the observables if quantum correlations are to be reproduced. Once the observables themselves are relieved of the burden of pre-assigned values, by assigning reality only to the phases, Einstein locality can be preserved and the conservation laws can be respected while reproducing the quantum correlations. Such an approach circumvents von Neumann's objections as well as Bell's theorem. This was suggested as the correct way of resolving the issues contained in the EPR problem [9,10], and some related issues and questions [11] are being investigated. It is also plausible that the true source of quantum randomness (Einstein's dice) is related to the randomness of initial phases. In this sense the quantum phase is the true hidden variable, already contained in the standard quantum formalism. A more detailed discussion of this aspect as well as of the importance of phase of individual systems in multi-particle correlations may be found in refs [9,10].

5. Summary

In this paper I discussed in detail the result that a correlation function different from what is predicted by the quantum theory, and in particular the correlation function of the local hidden variable theories, implies a violation of a fundamental conservation law related to the basic space-time symmetries, like the conservation of angular momentum. This result followed from the observation that the angular correlation function is determined by the average angular momentum of one of the particles *conditional* on specific values of the projection of angular momentum for the other particle. Similar remarks apply to other set of observables like linear momentum in appropriate entangled and correlated systems. The Bell's inequalities can be obeyed only by violating a fundamental conservation law on the average by a large margin. Validity of the relevant conservation law implies a violation of the Bell's inequalities. Hence no theory of quantized observables that respects basic conservation laws will follow the Bell correlation function. While the Bell's inequalities are good criteria for demarcating local hidden variable theories from quantum mechanics, the fact that local hidden variables do not respect fundamental conservation laws even on the average make the Bell's inequalities redundant from the point of view of viable physical theories. More importantly this finding discredits any local realistic theory as a viable alternative to quantum mechanics. They are flawed seriously even in a theoretical sense (like perpetual motion machines are), and there is no need to test their viability by sophisticated experiments; these experiments attempting to test the Bell's inequalities are in fact trying to test whether a class of theories that is *grossly incompatible* with a fundamental conservation law could be physically viable. Much experimental ingenuity and efforts have been invested in testing a theory that respects conservation laws (quantum mechanics) against others that do not. The consolation perhaps is that unexpected and remarkable technological offshoots for quantum information and computing have resulted from these efforts.

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References

[1] C S Unnikrishnan, *Europhys. Lett.* **69**, 489 (2005); *Correlation functions, Bell's inequalities, and the fundamental conservation laws*, quant-ph/0407041
[2] F J Belinfante, *Survey of hidden variable theories* (Pergamon, Oxford, 1973)
[3] J S Bell, *Physics* **1**, 195 (1965); *Speakable and unspeakable in quantum mechanics* (Cambridge University Press, 1987)
[4] See for a review, A Afriat and F Selleri, *The Einstein, Podolsky, and Rosen Paradox in atomic, nuclear and particle physics* (Plenum Press, New York and London, 1999)
[5] A Garg and N D Mermin, *Phys. Rev. Lett.* **49**, 901 (1982)
[6] D M Greenberger, M Horne and A Zeilinger, in: *Bell's theorem, quantum theory and conceptions of the universe* edited by M Kafatos (Kluwer, Dordrecht,1989) p. 73

D M Greenberger, M Horne, A Shimony and A Zeilinger, *Am. J. Phys.* **58**, 1131 (1990)

[7] J S Bell, Einstein–Podolsky–Rosen experiments (Article 10), in: *Speakable and unspeakable in quantum mechanics* (Cambridge University Press, 1987)

[8] C S Unnikrishnan, *Invited talk at the congress on quantum philosophy* (CPFS, New Delhi, January, 2004) (unpublished)

[9] C S Unnikrishnan, *Found. Phys. Lett.* **15**, 1 (2002)

[10] C S Unnikrishnan, *Curr. Sci.* **79**, 195 (2000); available at http://www.ias.ac.in/currsci/backissues.htm

[11] B d'Espagnat, *On the Unnikrishnan approach to the notion of locality*, quant-ph/0302167