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Relativistic theory of inverse beta-decay of polarized neutron in strong magnetic field

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Abstract. The relativistic theory of the inverse beta-decay of polarized neutron, $\nu_e + n \rightarrow p + e^-$, in strong magnetic field is developed. For the proton wave function we use the exact solution of the Dirac equation in the magnetic filed that enables us to account exactly for effects of the proton momentum quantization in the magnetic field and also for the proton recoil motion. The effect of nucleons anomalous magnetic moments in strong magnetic fields is also discussed. We examine the cross-section for different energies and directions of propagation of the initial neutrino accounting for neutron polarization. It is shown that in the super-strong magnetic field the totally polarized neutron matter is transparent for neutrinos propagating antiparallel to the direction of polarization. The developed relativistic approach can be used for calculations of cross-sections of the other URCA processes in strong magnetic fields.

Keywords. Inverse beta-decay; strong magnetic fields; neutron stars.

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1. Introduction

It is by now widely recognized that strong magnetic fields can be a significant factor relevant to diverse astrophysical and cosmological environments. The presence of strong magnetic fields in proto-neutron stars and pulsars is well established. The surface magnetic fields of many radio-pulsars, that can be estimated by the observed synchrotron radiation, are of the order of $B \sim 10^{12}$–10^{14} G. There are also the so-called magnetars [1,2] whose surface magnetic fields are two or three orders of magnitude higher.

Although the internal structure of the magnetic field of a neutron star is controversial, its strength can be estimated [3] as a limit imposed by the requirement that the total energy of the star, including gravitational, electromagnetic, and thermal components, must be negative (the scalar virial theorem), so that the star is a bounded system. The scalar virial theorem sets the upper limit on the internal neutron star magnetic field on the level of $B \sim 10^{18}$ G [4]. One obtains the same estimation for the internal field if the magnetic field flux is supposed to be the trapped primordial flux.

Very strong magnetic fields are also supposed to exist in the early Universe [5]. Such fields can influence the primordial nucleosynthesis [6–8] and affect the rate of ^{4}He production.

Under the influence of strong magnetic fields the direct URCA processes like

$$n \to p + e + \bar{\nu}_e, \tag{1}$$

$$\nu_e + n \leftrightharpoons e + p, \tag{2}$$

$$p + \bar{\nu}_e \leftrightharpoons n + e^+, \tag{3}$$

can be modified. These reactions play important roles in the neutron star evolution so that the presence of strong magnetic fields significantly change the star cooling rate [9–13]. It is worth mentioning here a recent study of neutrino processes (2) and (3) in strong magnetic fields of the order 10^{16} G and implication for supernova dynamics [14].

The direct URCA processes have gained a lot of attention because of the asymmetry in the neutrino emission, which can arise in the presence of strong magnetic fields. Various authors have argued that asymmetric neutrino emission during the first few seconds after the massive star collapse could provide explanations for the observed pulsar velocities. Different mechanisms for the asymmetry in the neutrino emission from a pulsar has been studied previously [15–23]. For more complete references on the neutrino mechanisms of the pulsar kicks, see the review papers [24,25].

It is worth mentioning here that the angular dependence of the neutrino emission in URCA processes was first considered for the neutron beta-decay neutrinos in [26,27]. In these papers the probability of the polarized neutron beta-decay in the presence of a magnetic field was derived, as well as the asymmetry in the neutrino emission was studied for the first time. In the two well-known papers, [28,29], the results of [26,27] for the neutron decay rate in a magnetic field were re-derived. However, there was no discussion on the asymmetry in neutrino emission in refs [28,29].

The neutron beta-decay have been studied in different electromagnetic field configurations. The first attempts to consider the beta-decay in the field of an electromagnetic wave have been undertaken in [30] and [31]. However, the final result of [30] is very complicated and is not accessible for any numerical analysis, whereas the result of [31] for the field influence on the decay rate is far overestimated. In [32] we have considered the probability of the polarized neutron beta-decay in the superposition of a magnetic field and a field of an electromagnetic wave (the so-called Redmond field configuration) and have confirmed the results of [26,27] for the decay probability in the magnetic field and also got the probability in the presence of an electromagnetic wave field. The relativistic theory of the beta-decay of the neutron (accounting for the proton recoil motion) in the strong magnetic field has been developed in [33]. Many important technical details of the calculations, also useful for the studies performed in the present paper, can be found in [32]. The rates of the two inverse processes in eqs (3) and (4) in the presence of a magnetic field have been derived in [16].

The presence of strong magnetic fields can also stimulate the proton decay

$$p \to n + e^{+} + \nu_{\mathrm{e}}. \tag{4}$$

The decay rate of this process in the strong magnetic field and the corresponding astrophysical consequences have been discussed in [34] (see also [5]). The proton decay induced by different configurations of strong electromagnetic fields has been also considered in [30,35,36].

The present paper is devoted to a detailed study of the inverse beta-decay of neutron in a magnetic field

$$\nu_{\mathrm{e}} + n \to p + e^{-}. \tag{5}$$

The process $\nu n \to pe$ in a magnetic field has been discussed previously by several authors. The contribution of this process to the conditions for beta-equilibrium in the presence of magnetic fields has been considered in [37]. The dependence of the cross-section on the magnetic field has also been discussed [19] in the context of the pulsar kick in the case when the asymmetric magnetic field arises just after the star collapse.

A reasonable interest in the inverse beta-decay of neutron in magnetic fields has been stimulated by a belief that it can be relevant for the neutrino opacity in the proto-neutron star stage after supernova collapse. The first detailed evaluation of the magnetic field effect on the neutrino opacity can be found in [22]. In [22], as well as in [23], the calculations for the cross-section have been performed under the assumption that the magnetic field gives contribution to the phase space integrals only, whereas the process matrix elements have been considered unaffected by the magnetic field.

The first attempt to calculate modification of the neutrino sphere in pulsar due to the asymmetry in the $\nu n \to pe$ cross-section accounting for the magnetic field modifications of the matrix element has been undertaken in [9]. However, in this paper, as well as in [10], the transition to the electron lowest Landau level has been discussed. In [11] the angular asymmetry of the cross-section has been calculated only to the first order in the magnetic field.

An important effect of anisotropy in the cross-section of the inverse beta-decay has been recently considered in a series of papers [38–40] where the process $\nu n \to pe$ has been studied in the presence of a background magnetic field and the initial neutron polarization has been also accounted for. However, some of the final results of refs [38,39] for the cross-section do not coincide with corresponding results of ref. [40]. There are also some discrepancies in the figures of ref. [40]. It was also claimed in [38,40] that the developed approach is valid if the strength of magnetic field is much smaller than $B_p = (m_p^2/e) \approx 1.5 \times 10^{20}$ G and, therefore, the proton momentum quantization and proton recoil motion can be neglected. However, this is not an exact estimation for the upper limit of the magnetic field for which the calculations performed in [40] can be applied. The magnetic field limit depends on the neutrino energy and, as we show in the present paper, the expressions for the cross-section of the inverse beta-decay obtained in [38,40] are valid if the strength of the magnetic field is much smaller than the proton critical field $B'_{\mathrm{cr}} \sim 10^{18}$ G (for the range of the neutrino energies $\varkappa \sim 10$ MeV). Note that if the strength of the background field is of the order or exceeds the critical value B'_{cr} then in the calculation of the cross-section one must certainly account for the influence of the magnetic field on the proton and consider the Landau quantization of the proton momentum. Moreover, for the electrically neutral neutrino–proton–electron matter in beta-equilibrium the magnetic effects on protons are as important as those on electrons [4]. Thus, in such systems, because of charge neutrality the proton critical field may be forced to reduce to the level of much smaller electron critical field $B_{\rm cr}$.

The present paper is devoted to a detailed evaluation of the inverse beta-decay of polarized neutron cross-section in a magnetic field. For both charged particles' (e and p) wave functions we use the exact solutions of the Dirac equation in the presence of a magnetic field so that we also exactly account for the magnetic field influence on the proton. The incoming neutrino is supposed to be relativistic and effects of neutrino non-zero mass are neglected. We do not set any special limit on the neutrino energies, but it is supposed that the four-fermion weak interaction theory is relevant in our case. For astrophysical applications, and for supernovas in particular, it is of interest to consider the neutrino energies in the range of $\varkappa \sim 1$–30 MeV.

In our consideration we account for the proton momentum quantization in the magnetic field and for the proton recoil motion so that we develop here the relativistic theory of the inverse beta-decay. We also suppose that Z and W bosons are not affected by the magnetic field. The contribution of nucleons anomalous magnetic moments in strong magnetic fields is discussed. The former effect can be easily incorporated into our calculations by the corresponding shift of the masses of the nucleons [5,11,12,41]. We also show that in the case of very strong magnetic fields the process, due to the anomalous magnetic moments, can be forbidden.

2. Cross-section of inverse beta-decay

We start with the well-known four-Fermion Lagrangian,

$$\mathcal{L} = \frac{G}{\sqrt{2}} \left[\bar{\psi}_p \gamma_\mu (1 + \alpha\gamma_5)\psi_n\right] \left[\bar{\psi}_e \gamma^\mu (1 + \gamma_5)\psi_\nu\right], \tag{6}$$

where $G = G_{\rm F} \cos\theta_{\rm c}$, $\theta_{\rm c}$ is the Cabibbo angle and $\alpha = 1.26$ is the ratio of the axial and vector constants. The total cross-section of the process can be written as

$$\sigma = \frac{L^3}{T} \sum_{\text{phase space}} |M|^2, \tag{7}$$

where summation is performed over the phase space of the final particles. The matrix element of the process is given by

$$M = \frac{G}{\sqrt{2}} \int \left[\bar{\psi}_p \gamma_\mu (1 + \alpha\gamma_5)\psi_n\right] \left[\bar{\psi}_e \gamma^\mu (1 + \gamma_5)\psi_\nu\right] \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}t. \tag{8}$$

We account for the influence of the background magnetic field on the matrix element (8). The corresponding calculations are performed using the exact solutions of the Dirac equation in the magnetic field for the relativistic electron and proton.

Without loss of generality, a constant magnetic field $\vec{B}$ is taken along the z-direction. We use the notations of our previous study [33] of the beta-decay of the polarized neutron in a magnetic field with the proton recoil effects have been accounted for. We also choose the longitudinal in respect to the magnetic field vector $\vec{B}$ component of the polarization tensor

$$\mu_3 = m\sigma_3 + \rho_2[\vec{\sigma}\times\vec{P}]_z, \quad \vec{P} = \vec{p} + e\vec{A}, \quad \rho_2 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}, \tag{9}$$

for classifying the spin states of charged particles. Here σ are the Pauli matrixes, $\vec{p}$ is the momentum of the particle, $\vec{A}$ is the vector potential of the magnetic field, and $0,\ I$ are the 2×2-matrixes. A detailed discussion on the derivation of the solution of the Dirac equation in a magnetic field, and also on different spin operators that can be used for charged particles in this case, can be found in [42]. The electron wave function $\psi_e(m, n, s, p_0, p_2, p_3)$ can be written in the form

$$\psi_e = \frac{1}{L}\begin{pmatrix} C_1 U_{n-1}(\eta) \\ iC_2 U_n(\eta) \\ C_3 U_{n-1}(\eta) \\ iC_4 U_n(\eta) \end{pmatrix} \mathrm{e}^{-i(p_0 t - p_2 y - p_3 z)}, \quad \eta = x\sqrt{\gamma} + \frac{p_2}{\sqrt{\gamma}}, \quad \gamma = eB, \tag{10}$$

where $U_n(\eta)$ are Hermite functions of order n, e is the absolute value of the electron charge, p_0, p_2 and p_3 are the electron energy and momentum components, respectively. The energy spectrum

$$p_0 = \sqrt{m^2 + 2\gamma n + p_3^2}, \tag{11}$$

depends on the discreet number $n = 0, 1, 2, \ldots$ denoting the Landau levels (m is the electron mass). If one uses the spin operator (9) then the spin coefficients C_i are

$$C_{1,3} = \frac{1}{2}\sqrt{1 + s\frac{m}{\tilde{p}_\perp}}\ \sqrt{1 \pm s\frac{\tilde{p}_\perp}{p_0}}, \quad C_{2,4} = \mp\frac{1}{2}s\sqrt{1 - s\frac{m}{\tilde{p}_\perp}}\ \sqrt{1 \mp s\frac{\tilde{p}_\perp}{p_0}}, \tag{12}$$

and $\tilde{p}_\perp = \sqrt{m^2 + 2\gamma n}$. The spin number can have only the values ± 1, $s = +1$ when the electron spin is directed along the magnetic field $\vec{B}$, and $s = -1$ in the opposite case. The electrons on all Landau levels with $n \geq 1$ can have two different spin polarizations. However, in the lowest Landau state ($n = 0$) the electron spin can have the only orientation given by $s = -1$, so that the electrons moving along the direction of the magnetic field are left-handed polarized, whereas the electrons moving in the opposite direction are right-handed polarized.

The proton wave function $\psi_p(m', n', s', p_0', p_2', p_3')$ can be expressed in a similar form

$$\psi_p = \frac{1}{L}\begin{pmatrix} C_1' U_{n'}(\eta') \\ -iC_2' U_{n'-1}(\eta') \\ C_3' U_{n'}(\eta') \\ -iC_4' U_{n'-1}(\eta') \end{pmatrix} \mathrm{e}^{-i(p_0' t - p_2' y - p_3' z)}. \tag{13}$$

The dashed quantities correspond to the proton mass, number of the Landau state, energy and momentum components. Note that the positions of n' and $n'-1$ are interchanged compared to the positions of n and $n-1$ in the cases of electron. Again the proton spin values are $s' = \pm 1$. However, now at the lowest Landau level the spin orientation is along the magnetic field $\vec{B}$.

The initial neutron and neutrino are supposed to be not affected by the magnetic field, and we use the plane waves for their wave functions. The polarized neutron wave function can be chosen in the form

$$\psi_n = \frac{1}{2L^{3/2}} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix} \mathrm{e}^{-i(p_0^n t - \vec{p}_n \vec{r})}, \tag{14}$$

where the neutron spin coefficients are

$$\begin{aligned} N_{1,3} &= s_n \sqrt{1 \pm \frac{m_n}{p_0^n}} \cdot \sqrt{1 \pm s_n \cos\theta_n} \cdot \mathrm{e}^{\mp i\varphi_n/2}, \\ N_{2,4} &= \sqrt{1 \mp \frac{m_n}{p_0^n}} \cdot \sqrt{1 \mp s_n \cos\theta_n} \cdot \mathrm{e}^{\pm i\varphi_n/2}. \end{aligned} \tag{15}$$

Here m_n, p_0^n and $\vec{p}_n$ are the neutron mass, energy, momentum, and θ_n, φ_n are the polar and azimuthal neutron momentum angles. The neutron spin value s_n ($s_n = \pm 1$) classifies the neutron states with respect to the spin projection to z-direction ($s_n = 1$ corresponds to the spin orientation parallel to the magnetic field $\vec{B}$). We perform our calculations in the rest frame of the neutron, so that we shall take below $p_0^n = m_n$ and $N_3 = N_4 = 0$.

The neutrino wave function

$$\psi_\nu = \frac{1}{2L^{3/2}} \begin{pmatrix} f_1 \\ f_2 \\ -f_1 \\ -f_2 \end{pmatrix} \mathrm{e}^{-i(\varkappa t - \vec{\varkappa}\vec{r})}, \tag{16}$$

where

$$f_1 = -\mathrm{e}^{-i\varphi_\nu}\sqrt{1 - \cos\theta_\nu}, \quad f_2 = \sqrt{1 + \cos\theta_\nu}, \tag{17}$$

and $\varkappa, \vec{\varkappa}$ are the neutrino energy and momentum, respectively. We neglect effects of the neutrino mass so that $\varkappa = |\vec{\varkappa}|$. The neutrino polar and azimuthal angles are denoted as φ_ν and θ_ν.

Putting these wave functions to the matrix element of the process (8), we can perform the integrations over time t and spatial coordinates y and z and obtain three δ-functions

$$\begin{aligned} &\int \mathrm{e}^{-it(m_n + \varkappa - p_0 - p_0') + iy(\varkappa_2 - p_2 - p_2') + iz(\varkappa_3 - p_3 - p_3')} \mathrm{d}t\, \mathrm{d}y\, \mathrm{d}z \\ &= (2\pi)^3 \delta(p_0' + p_0 - m_n - \varkappa)\delta(p_2' + p_2 - \varkappa_2)\delta(p_3' + p_3 - \varkappa_3). \end{aligned} \tag{18}$$

To integrate over the coordinate x in the matrix element we use the properties of the Hermite functions (see [42,43]) and the result

$$\int_{\infty}^{\infty} U_n(\eta)U_{n'}(\eta')\mathrm{e}^{-i\varkappa_1 x}\mathrm{d}x = I_{n',n}(\rho)\mathrm{e}^{i\mu+i(n-n')\lambda}, \tag{19}$$

$$\mu = \frac{\varkappa_1(p_2+p_2')}{2\gamma}, \quad \lambda = \arctan\frac{\varkappa_1}{p_2'+p_2}, \quad \rho = \frac{\varkappa_1^2+(p_2'+p_2)^2}{2\gamma}. \tag{20}$$

The Lagguere function $I_{n',n}(\rho)$ is connected with the Lagguere polynomials $Q_n^l(\rho)$:

$$\begin{aligned} I_{n',n}(\rho) &= \frac{1}{\sqrt{n'!n!}}\mathrm{e}^{-\rho/2}\rho^{(n'-n)/2}Q_n^{n'-n}(\rho), \\ Q_n^l(\rho) &= \mathrm{e}^{\rho}\rho^{-l}\frac{\mathrm{d}^n}{\mathrm{d}\rho^n}\left(\rho^{n+l}\mathrm{e}^{-\rho}\right). \end{aligned} \tag{21}$$

Finally for the matrix element of the inverse beta-decay of the neutron we get

$$\begin{aligned} M = {} & i\sqrt{2}(2\pi)^3 G\,\mathrm{e}^{i\mu+i(n-n')\lambda} \\ & \times\Big[\Big\{2(\alpha C_1' - C_3')f_1 N_2 - (\alpha+1)(C_1'-C_3')f_2N_1\Big\}(C_2-C_4)I_{n',n}(\rho) \\ & -\Big\{2(\alpha C_2' - C_4')f_2N_1 - (\alpha+1)(C_2'-C_4')f_1N_2\Big\} \\ & \times(C_1-C_3)I_{n'-1,n-1}(\rho) \\ & +i(\alpha-1)(C_1-C_3)(C_1'+C_3')f_1N_1I_{n',n-1}(\rho)\mathrm{e}^{-i\lambda} \\ & +i(\alpha-1)(C_2-C_4)(C_2'+C_4')f_2N_2I_{n'-1,n}(\rho)\mathrm{e}^{i\lambda}\Big] \\ & \times\delta(p_0'+p_0-m_n-\varkappa)\delta(p_2'+p_2-\varkappa_2)\delta(p_3'+p_3-\varkappa_3), \end{aligned} \tag{22}$$

(here we do not include the overall term $1/4L^5$ which shall be accounted for below). Note that this expression for the matrix element follows from the relativistic matrix element of the direct beta-decay in the magnetic field calculated in [33].

Using the usual rules like

$$|\delta(p_0'+p_0-m_n-\varkappa)|^2 = \frac{T}{2\pi}\delta(p_0'+p_0-m_n-\varkappa), \tag{23}$$

$$|\delta(p_2'+p_2-p_2^n-\varkappa_2)|^2 = \frac{L}{2\pi}\delta(p_2'+p_2-p_2^n-\varkappa_2), \tag{24}$$

$$|\delta(p_3'+p_3-p_3^n-\varkappa_3)|^2 = \frac{L}{2\pi}\delta(p_3'+p_3-p_3^n-\varkappa_3), \tag{25}$$

where T and L are the quantization large time and regions in the y and z directions, we get the following relation for the squared norm of the matrix element:

$$\begin{aligned} |M|^2 = {} & (2\pi)^3 TL^2|\tilde{M}|^2\delta(p_0'+p_0-m_n-\varkappa) \\ & \times\delta(p_2'+p_2-\varkappa_2)\delta(p_3'+p_3-\varkappa_3), \end{aligned} \tag{26}$$

where

$$|\tilde{M}|^2 = 2G^2\Big[(\alpha-1)^2 f_1^2 N_1^2 (C_1-C_3)^2 (C_1'+C_3')^2 I^2_{n',n-1}(\rho) \\ +(C_2-C_4)^2\Big\{4f_1^2N_2^2(\alpha C_1'-C_3')^2 \\ +(\alpha+1)^2 f_2^2 N_1^2 (C_1'-C_3')^2\Big\} I^2_{n',n}(\rho) \\ +(\alpha-1)^2 f_2^2 N_2^2 (C_2-C_4)^2 (C_2'+C_4')^2 I^2_{n'-1,n}(\rho) + (C_1-C_3)^2 \\ \times\Big\{4f_2^2N_1^2(\alpha C_2'-C_4')^2 + (\alpha+1)^2 f_1^2 N_2^2 (C_2'-C_4')^2\Big\} I^2_{n'-1,n-1}(\rho) \\ +4(\alpha+1)(C_1-C_3)(C_2-C_4)\Big\{f_2^2N_1^2(C_1'-C_3')(\alpha C_2'-C_4') \\ +f_1^2N_2^2(C_2'-C_4')(\alpha C_1'-C_3')\Big\} I_{n',n}(\rho) I_{n'-1,n-1}(\rho)\Big]. \tag{27}$$

Let us now return back to the general expression (7) for the cross-section of the process and perform the integration and summation over the phase space of the final particles. The phase space factor for the electron and proton in the presence of a magnetic field is

$$\sum_{\text{phase space}} = \int \frac{L}{2\pi}\mathrm{d}p_2 \frac{L}{2\pi}\mathrm{d}p_3 \frac{L}{2\pi}\mathrm{d}p_2' \frac{L}{2\pi}\mathrm{d}p_3' \sum_{n=0,n'=0} \sum_{s=\pm1,s'=\pm1} g_n g_{n'}, \tag{28}$$

where $g_0 = 1$, and $g_k = 2$ for $k \geq 1$ are the degeneracies of the Landau energy levels for the electron and proton. The integrations over the proton momentum component p_2' and the electron momentum component p_3 are performed by using the two delta-functions $\delta(p_2'+p_2-\varkappa_2)$ and $\delta(p_3'+p_3-\varkappa_3)$, respectively. After these integrations we get the laws of conservation for the two momentum components, $p_3 = \varkappa_3 - p_3',\ p_2' = \varkappa_2 - p_2$.

The integration over the electron momentum component p_2 is performed by taking into account the specific for the motion in a magnetic field degeneracy of the electron energy. The corresponding phase space factor is

$$\int_{\infty}^{\infty} \mathrm{d}p_2 \to \frac{2\pi}{L}\sum_{p_2} \to eBL. \tag{29}$$

Finally we obtain the cross-section of the inverse beta-decay of the polarized neutron in a magnetic field, with the proton recoil motion effect being accounted for,

$$\sigma = \frac{eB}{32\pi}\sum_{s,s'}\sum_{n,n'}\int_{\infty}^{\infty} |\tilde{M}|^2 \delta_0(p_0'+p_0-m_n-\varkappa)\big|_{p_3=\varkappa_3-p_3'} \mathrm{d}p_3', \tag{30}$$

where $|\tilde{M}|^2$ is given by (27) with p_3 being substituted by $\varkappa_3 - p_3'$ because at this stage of calculations we have already performed the integration over the component of the electron momentum p_3 by using the corresponding δ-function.

The remaining integration over the component p'_3 of the proton momentum is performed using the $\delta_0\big(\varphi(p'_3)\big)$-function. The argument $\varphi(p'_3)$, being equated with zero, gives the law of energy conservation for the particles in the process,

$$m_n + \varkappa = \sqrt{m^2 + 2\gamma n + (\varkappa_3 - p'_3)^2} + \sqrt{m'^2 + 2\gamma n' + p'^2_3}. \tag{31}$$

Thus, the argument of the δ_0-function in (30) is a complicated function of p'_3. That is why in order to perform the integration over p'_3 we have to use the relation

$$\delta(\varphi(p'_3)) = \sum_i \frac{\delta(p'_3 - p'^{(i)}_3)}{|\varphi'(p'^{(i)}_3)|}, \tag{32}$$

where

$$\varphi'(p'^{(i)}_3) \equiv \frac{\mathrm{d}\varphi(p'_3)}{\mathrm{d}p'_3}\Big|_{p'_3 = p'^{(i)}_3}, \tag{33}$$

and $p'^{(i)}_3$ are the simple roots of the equation

$$\varphi(p'^{(i)}_3) = 0. \tag{34}$$

For the argument of the $\delta_0\big(\varphi(p'_3)\big)$-function in (30) we get

$$\varphi(p'_3) = \sqrt{\tilde{p}'^2_\perp + p'^2_3} + \sqrt{\tilde{p}^2_\perp + (\varkappa_3 - p'_3)^2} - m_n - \varkappa, \tag{35}$$

then the derivative is

$$\varphi'(p'_3) = \frac{p'_3}{\sqrt{\tilde{p}'^2_\perp + p'^2_3}} - \frac{\varkappa_3 - p'_3}{\sqrt{\tilde{p}^2_\perp + (\varkappa_3 - p'_3)^2}}, \tag{36}$$

where

$$\tilde{p}'_\perp = \sqrt{m'^2 + 2\gamma n'}, \quad \tilde{p}_\perp = \sqrt{m^2 + 2\gamma n}. \tag{37}$$

There are the two roots of eq. (34),

$$p'^{(1,2)}_3 = \frac{1}{2[(m_n + \varkappa)^2 - \varkappa^2_3]} \Bigg\{ \varkappa_3[(m_n + \varkappa)^2 + \tilde{p}'^2_\perp - \tilde{p}^2_\perp - \varkappa^2_3] \pm (m_n + \varkappa)\sqrt{[(m_n + \varkappa)^2 - \tilde{p}'^2_\perp - \tilde{p}^2_\perp - \varkappa^2_3]^2 - 4\tilde{p}'^2_\perp \tilde{p}^2_\perp} \Bigg\}. \tag{38}$$

Finally we obtain the cross-section of the inverse beta-decay of the polarized neutron in a magnetic field, with the effects of the Landau quantization of the proton momentum and of the proton recoil motion being accounted for exactly,

$$\sigma = \frac{eB}{32\pi} \sum_{s,s'} \sum_{n,n'} \sum_{i=1,2} \frac{|\tilde{M}^{(i)}|^2}{\left| \frac{p^{(i)}_3}{p^{(i)}_0} - \frac{p'^{(i)}_3}{p'^{(i)}_0} \right|}, \tag{39}$$

where one of the sums is performed over the roots $p_3'^{(i)}$ of eq. (34) given by (38), and

$$p_3^{(i)} = \varkappa_3 - p_3'^{(i)}, \quad p_0'^{(i)} = \sqrt{\tilde{p}_\perp'^2 + p_3'^{(i)2}}, \quad p_0^{(i)} = \sqrt{\tilde{p}_\perp^2 + (\varkappa_3 - p_3'^{(i)})^2}. \tag{40}$$

The squared matrix element $|\tilde{M}^{(i)}|^2$ is given by eq. (27) where the substitution $p_3' \to p_3'^{(i)}$ must be done,

$$|\tilde{M}^{(i)}|^2 = |\tilde{M}|^2_{p_3'=p_3'^{(i)}}. \tag{41}$$

Now let us consider eq. (31) in detail that gives the energy conservation law by accounting for the presence of a magnetic field. Due to the particular properties of the energy spectra of the electron and proton in a magnetic field, we can introduce two critical values of the magnetic field strength. First let us determine the critical electron magnetic field, $B_{\rm cr}$, from the condition that in the external field $B \geq B_{\rm cr}$ the electron can occupy only the lowest Landau level with the number $n = 0$. From (31) we get that for a fixed maximal neutrino energy $\varkappa_{\max}$ and for a fixed strength of the magnetic field, the maximum number of the available electron Landau level is

$$n_{\max} = \text{int}\left[\frac{(\Delta + \varkappa_{\max})^2 - m^2}{2eB}\right], \tag{42}$$

where $\Delta = m_n - m'$ is the difference in masses of the neutron and proton. From the condition $n_{\max} < 1$ (it means that the electron can occupy only the lowest Landau level with $n = 0$) we get

$$B_{\rm cr} = \frac{(\Delta + \varkappa_{\max})^2 - m^2}{2e}. \tag{43}$$

Thus, $B_{\rm cr}$ depends on the maximum available neutrino energy. For example, for different neutrino energies we have the following values of the electron critical magnetic field:

$$B_{\rm cr} \approx 8.3 \times 10^{16}\ \text{G}, \quad \varkappa_{\max} = 30\ \text{MeV}, \tag{44}$$

$$B_{\rm cr} \approx 1.1 \times 10^{16}\ \text{G}, \quad \varkappa_{\max} = 10\ \text{MeV}, \tag{45}$$

$$B_{\rm cr} \approx 1.2 \times 10^{14}\ \text{G}, \quad \varkappa_{\max} \ll m. \tag{46}$$

The critical proton magnetic field, $B'_{\rm cr}$, was determined from the condition that in the external field $B \geq B'_{\rm cr}$ the proton can occupy only the lowest Landau level with the number $n' = 0$. Again, from (31) we get that for a fixed maximum neutrino energy $\varkappa_{\max}$ and for a fixed strength of the magnetic field, the maximum number of the available proton Landau level is

$$n'_{\max} = \text{int}\left[\frac{(\varkappa_{\max} + m_n - m)^2 - m'^2}{2eB}\right]. \tag{47}$$

The proton can occupy only the Landau level with $n' = 0$ if the magnetic field strength exceeds the proton critical field

$$B'_{\rm cr} = \frac{(\varkappa_{\rm max} + m_n - m)^2 - m'^2}{2e}. \tag{48}$$

For different energies of the incoming neutrino we get

$$B'_{\rm cr} \approx 5 \times 10^{18}\ {\rm G}, \quad \varkappa_{\rm max} = 30\ {\rm MeV} \tag{49}$$

$$B'_{\rm cr} \approx 1.7 \times 10^{18}\ {\rm G}, \quad \varkappa_{\rm max} = 10\ {\rm MeV} \tag{50}$$

$$B'_{\rm cr} \approx 1.3 \times 10^{17}\ {\rm G}, \quad \varkappa_{\rm max} \ll\ m. \tag{51}$$

In figure 1 we plot the values of the critical fields $B_{\rm cr}$ (dashed line) and $B'_{\rm cr}$ (solid line) as functions of the initial neutrino energy $\varkappa$.

From the above we conclude that there are three ranges of the magnetic field strength which we call: (1) the weak field ($B \le B_{\rm cr}$), (2) the strong field ($B_{\rm cr} < B < B'_{\rm cr}$), and (3) the super-strong field ($B'_{\rm cr} \le B$). For most of the weak field range ($B \ll B_{\rm cr}$) the electron and proton Landau numbers n and n' can have very large values. Inside the strong field range ($B_{\rm cr} < B \ll B'_{\rm cr}$) only the proton number n' can have very large values, whereas the electron number is always zero. In the super-strong fields both the Landau numbers are zero, i.e., $n = n' = 0$.

The electron and proton spin properties are very different for each of the three ranges of the magnetic field strengths. In the weak fields the electron and proton can have two spin polarizations, in the strong (and the super-strong) fields the electron is always polarized against the direction of the magnetic field, and in the super-strong fields the electron and proton spin polarizations are opposite. So it is reasonable to expect that the expressions for the differential cross-sections for the inverse beta-decay in these three ranges of magnetic fields are very different. This, in particular, have to be reflected in the dependence of the cross-section on the

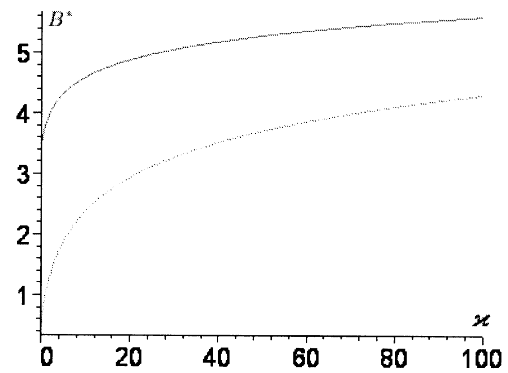

Figure 1. Dependence of the electron critical magnetic field $B_{\rm cr}$ (dashed line) and the proton critical magnetic field $B'_{\rm cr}$ (solid line) on the initial neutrino energy $\varkappa$ (MeV). The logarithmic scale is used: $B^* = \log B/B_0$, where $B_0 = m^2/e$.

polarizations of the particles and also on asymmetries in respect to the neutrino angle θ. Also it is reasonable to expect that the expression for the cross-section calculated in the presence of the strong field cannot be applied to the case of the super-strong magnetic field.

3. Cross-section in super-strong, strong and weak magnetic fields

We consider the cross-section of the inverse beta-decay of the polarized neutron, accounting also for the proton recoil motion, in the three ranges of the background field: (1) $B \leq B_{\rm cr}$, (2) $B_{\rm cr} < B < B'_{\rm cr}$, (3) $B'_{\rm cr} \leq B$. Here we would like to emphasize that in the previous section we have derived the general expression (39) for the cross-section, accounting for the proton recoil motion, that can be used for arbitrary magnetic fields. As we have already discussed in §1, the energy spectra of the electron and the proton are quantized into the Landau levels in the presence of a magnetic field. These specific properties of the energy spectra of the charged particles set the three rather different methods of further analytical calculations of the cross-section using the general expression (39).

3.1 *Cross-section in super-strong magnetic field*

Let us start with consideration of the super-strong magnetic field $B \geq B'_{\rm cr}$. Obviously, in this case the calculations are reasonably simplified because, as has been already discussed above, both the numbers of the Landau levels for the electron and proton are zero. The Laguerre functions (21) for $n = n' = 0$ are

$$I_{0,0}(\rho) = {\rm e}^{-\rho/2}, \tag{52}$$

where the argument is

$$\rho = \frac{\varkappa_1^2 + \varkappa_2^2}{2\gamma} = \frac{\varkappa_\perp^2}{2\gamma}, \quad \varkappa_\perp = \sqrt{\varkappa^2 - \varkappa_3^2} = \varkappa \sin\theta, \tag{53}$$

and the squared matrix element $|\tilde{M}|^2$ in eq. (27) is reduced to

$$\begin{aligned} |\tilde{M}_{n=n'=0}|^2 = 2G^2(C_2 - C_4)^2\{4f_1^2 N_2^2 \alpha C'_1 - C'_3)^2 \\ +(\alpha+1)^2 f_2^2 N_1^2 (C'_1 - C'_3)^2\}{\rm e}^{-\rho}. \end{aligned} \tag{54}$$

Putting back in (39), we obtain the cross-section of the process in the presence of the super-strong magnetic field

$$\begin{aligned} \sigma_{n=n'=0} = \frac{eBG^2}{8\pi} {\rm e}^{-\varkappa_\perp^2/2\gamma} \sum_{i=1,2} \frac{\left(1 + \frac{p_3^{(i)}}{p_0^{(i)}}\right)}{\left|\frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{p_0'^{(i)}}\right|} \\ \times\{a^{(i)} + b^{(i)}\cos\theta + s_n(b^{(i)} + a^{(i)}\cos\theta)\}, \end{aligned} \tag{55}$$

where

$$a^{(i)} = 3 + 2\alpha + 3\alpha^2 - 2(1-\alpha^2)\frac{m'}{p_0'^{(i)}} - (1 + 6\alpha + \alpha^2)\frac{p_3'^{(i)}}{p_0'^{(i)}},$$
$$b^{(i)} = -1 + 2\alpha - \alpha^2 + 2(1-\alpha^2)\frac{m'}{p_0'^{(i)}} - (1-\alpha)^2\frac{p_3'^{(i)}}{p_0'^{(i)}}. \quad (56)$$

The effect of the proton motion, which in this case appears exceptional due to the proton recoil in z-direction, is accounted exactly in eqs (55) and (56). It is worth mentioning that the derived expression for the cross-section in the super-strong magnetic field $B \geq B'_{\rm cr}$ can be applied for neutrinos with arbitrary (also ultra-high) energies (note that following eq. (48) the value of $B'_{\rm cr}$ is increasing with the neutrino energy increase).

If we neglect the proton momentum parallel or antiparallel to the magnetic field, we get

$$\sigma_{n=n'=0}\big|_{p_0'=m'} = \frac{eBG^2}{4\pi}{\rm e}^{-\varkappa_\perp^2/2\gamma}\{a + b\cos\theta + s_n(b + a\cos\theta)\} \times\frac{\Delta+\varkappa}{\sqrt{(\Delta+\varkappa)^2 - m^2}}, \quad (57)$$

where

$$a = 1 + 2\alpha + 5\alpha^2, \quad b = 1 + 2\alpha - 3\alpha^2. \quad (58)$$

For $\alpha = 1.26$ one can get $a = 11.5$ and $b = -1.24$. Note that the same coefficients a and b determine the neutrino asymmetry in the probability of the direct neutron beta-decay in the super-strong magnetic field [18,33].

In the case of moderate neutrino energies $\varkappa^2 \ll eB$ (the last inequality is valid in the super-strong magnetic field $B \geq B'_{\rm cr}$ for the range of the neutrino energies $\varkappa \leq 30$ MeV) the exponential term in (55) must be substituted for unit. The coefficients a_0 and b_0 for the cross-section of the inverse beta-decay of the neutron in the case when $n = n' = 0$ and for the neutrino energies $\varkappa^2 \ll eB$ have also been obtained in [9].

As it follows from the used neutron wave function, eqs (14) and (15), the neutron spin quantization axis is parallel to the magnetic field vector $\vec{B}$. Therefore, the above derived expressions for the cross-section, which contains the value $s_n = \pm 1$, describe the neutrino interaction with neutrons totally polarized along ($s_n = +1$) or against ($s_n = -1$) the magnetic field. In the case of non-polarized neutrons we have to overage the cross-section over the neutron spin

$$\sigma_{\rm unpol.} = \frac{1}{2}\sum_{s_n=\pm 1}\sigma(s_n). \quad (59)$$

We also can use the obtained expressions for the cross-section in the analysis of the neutrino interaction with partially polarized neutron matter when the number of neutrons (per unit volume) with the two different spin polarizations are N_+ and

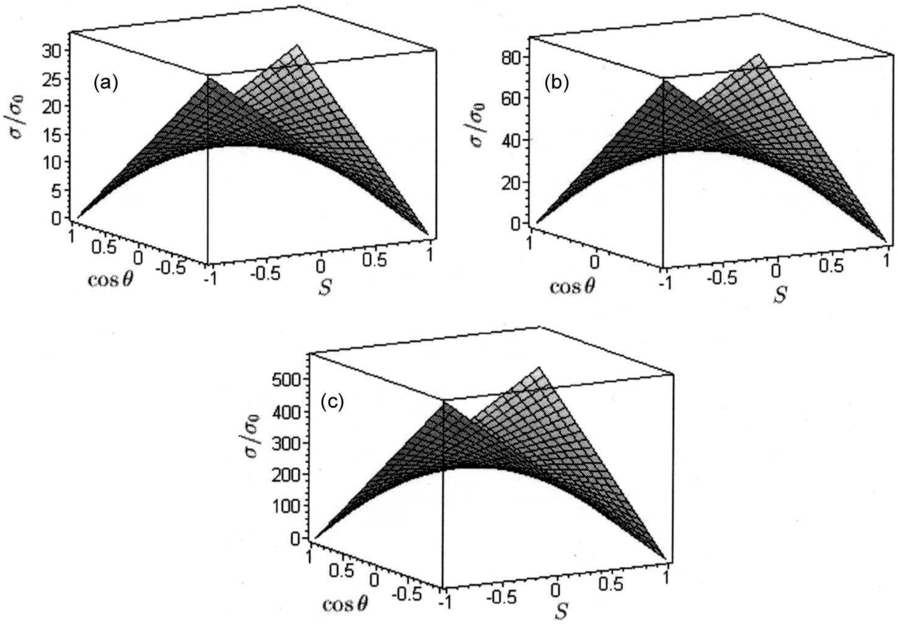

Figure 2. The cross-section σ in super-strong magnetic field $B = B'_{\rm cr}$, normalized to the cross-section σ_0 in the field-free case, for neutrinos with energy of $\varkappa = 30$ MeV (**a**), 10 MeV (**b**) and $\varkappa \ll m$ (**c**) as functions of the direction of the neutrino momentum $\cos\theta$ and polarization of neutrons S.

N_-, respectively. The partially polarized neutron matter can be characterized by the neutron polarization S determined as

$$S = \frac{N_+ - N_-}{N_+ + N_-}. \tag{60}$$

All the above obtained formulas for the cross-section can be used for the case of partially polarized neutron matter if one substitutes s_n for S.

In figures 2a,b and c, we have plotted, for different neutrino energies $\varkappa = 30$ MeV, 10 MeV and $\varkappa \ll m$, the cross-section in the magnetic field $B = B'_{\rm cr}$, normalized to the cross-section in the field-free case, as a function of neutron polarization S and $\cos\theta$ (θ is the angle the neutrino momentum makes with the magnetic field). It is clearly seen that the cross-section depends on the direction of the neutrino momentum and the neutron polarization. The most considerable increase (by the factors from a few tenth up to hundreds depending on the initial neutrino energy) of the cross-section in $B = B'_{\rm cr}$ appears in two cases: (1) nearly total neutron polarization parallel to the magnetic field ($S \approx 1$) and neutrino propagation parallel to the magnetic field $\cos\theta \approx 1$), and (2) nearly total neutron polarization antiparallel to the magnetic field ($S \approx -1$) and neutrino propagation antiparallel to the magnetic field ($\cos\theta \approx -1$).

On the opposite, the cross-section (57) vanishes to zero for the cases when the direction of the neutrons' total polarization is antiparallel to the direction of the

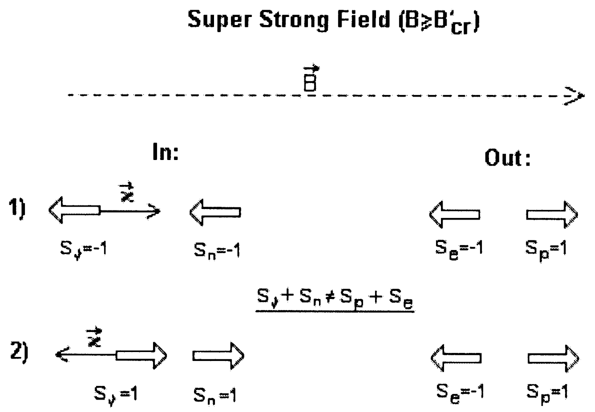

Figure 3. Initial and final particles spin orientations for the two directions of the neutrino propagation ($\cos\theta = \pm 1$) in the super-strong magnetic field $B \geq B'_{\rm cr}$. The broad arrows represent the particles spin orientations, the solid arrows show directions of the neutrino propagation, and the dashed arrow shows the direction of the magnetic field vector. The cross-section is zero when the sum of the spin numbers of the initial particles ($s_\nu + s_n = \pm 2$) is not equal to the sum of the spin numbers of the final particles ($s + s' = 0$).

neutrino momentum, $S\cos\theta = -1$. For $\cos\theta = +1$, from (57) we get

$$\sigma_{n=n'=0}\big|_{p'_0=m',\cos\theta=1} = \frac{eBG^2}{2\pi}\mathrm{e}^{-\varkappa_\perp^2/2\gamma}(1+\alpha^2)(1+S) \times \frac{\Delta+\varkappa}{\sqrt{(\Delta+\varkappa)^2-m^2}}. \tag{61}$$

Therefore, the cross-section is zero if $S = -1$. For $\cos\theta = -1$, from (57) we get

$$\sigma_{n=n'=0}\big|_{p'_0=m',\cos\theta=-1} = \frac{eBG^2}{\pi}\mathrm{e}^{-\varkappa_\perp^2/2\gamma}2\alpha^2(1-S)\frac{\Delta+\varkappa}{\sqrt{(\Delta+\varkappa)^2-m^2}}. \tag{62}$$

Therefore, the cross-section is again zero if $S = +1$.

Thus, for these two cases the neutron matter is transparent for neutrinos. This phenomenon appears due to the Landau quantization of the momentum and the spin properties of the charged particles in the strong and super-strong magnetic fields. In the field $B \geq B'_{\rm cr}$ the final electron and proton can move only parallel to a fixed line that is given by the magnetic field vector. For the neutrino also moving along this line and the neutron being at rest, the law of angular momentum conservation reduces to the law of 'spin number conservation'. Since the sum of spin numbers of the initial particles is equal to ± 2, whereas the sum of spin numbers of the final particles is zero in the two considered cases, the cross-sections must vanish. We present an illustration of the law of the 'spin number conservation' in figure 3.

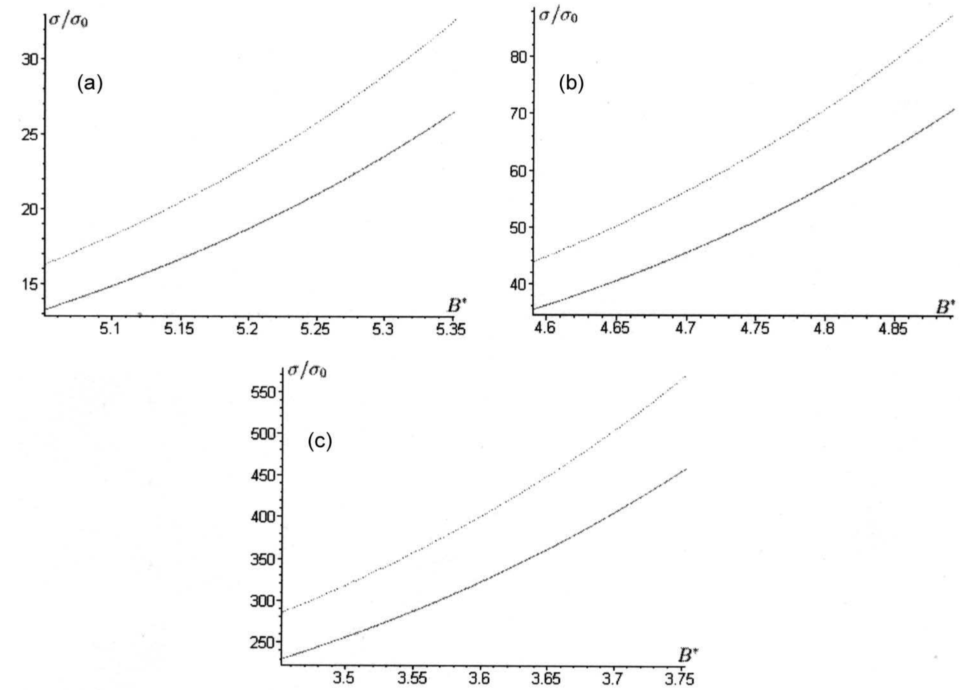

Figure 4. The cross-section in the super-strong magnetic field $B \geq B'_{\rm cr}$, normalized to the field-free case, for different neutrino energies ($\varkappa = 30$ MeV (**a**), 10 MeV (**b**) and $\varkappa \ll m$ (**c**)) in the case of unpolarized neutrons, $S = 0$. The solid and dashed lines correspond to the initial neutrino propagation parallel ($\cos\theta = 1$) and antiparallel ($\cos\theta = -1$) to the magnetic field vector. The logarithmic scale is used: $B^* = \log B/B_0$ where $B_0 = m^2/e$.

The dependence of the cross-section on the magnetic field strength $B \geq B'_{\rm cr}$ for different neutrino energies ($\varkappa = 30$ MeV, 10 MeV and $\varkappa \ll m$) and the two directions of the neutrino momentum ($\cos\theta = \pm 1$) in the case of unpolarized neutrons ($S = 0$) is shown in figures 4a, b and c. In figures 5a,b and c and figures 6a, b and c, we have plotted the cross-sections for the cases of the totally polarized neutrons ($S = \pm 1$). As we have already discussed above, neutrinos freely escape from the neutron matter when they move antiparallel to the neutron polarization, i.e., $S\cos\theta = -1$.

3.2 *Cross-section in strong magnetic field*

In the case of strong magnetic fields $B_{\rm cr} \leq B < B'_{\rm cr}$, the electron can only occupy the lowest Landau level with $n = 0$, whereas there could be many Landau levels available for the proton. The maximum number of the proton Landau level is estimated as

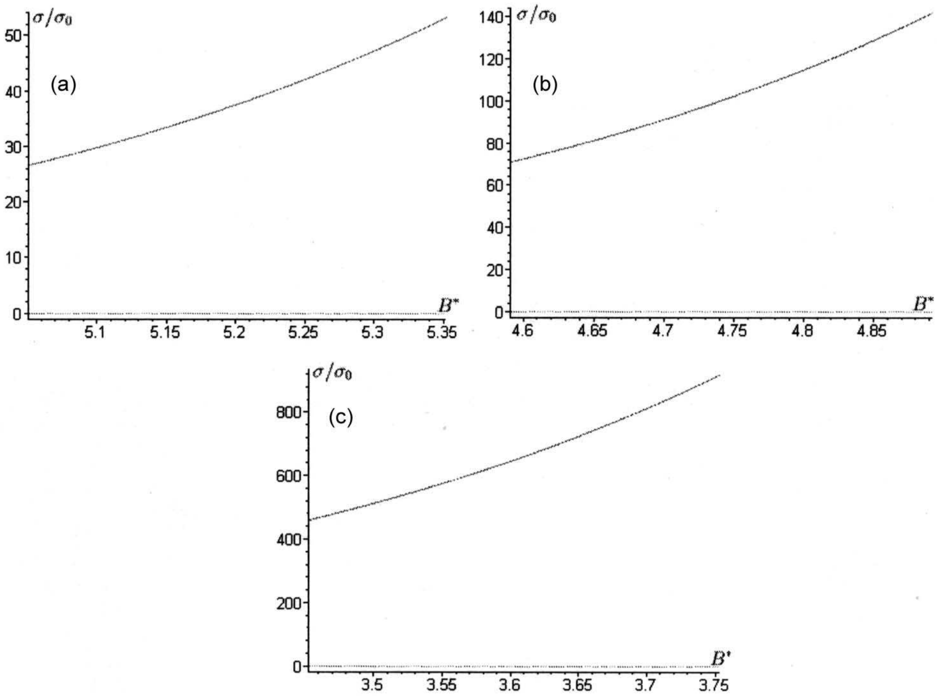

Figure 5. The cross-section in the super-strong magnetic field $B \geq B'_{\rm cr}$, normalized to the cross-section in the field-free case, for different neutrino energies ($\varkappa = 30$ MeV (**a**), 10 MeV (**b**) and $\varkappa \ll m$ (**c**)) for neutrons totally polarized parallel to the magnetic field vector ($S = 1$). The solid and dashed lines correspond to the initial neutrino propagation along ($\cos\theta = 1$) and against ($\cos\theta = -1$) the magnetic field vector. The cross-section for $\cos\theta = -1$ is exactly zero. The logarithmic scale is used: $B^* = \log B/B_0$, where $B_0 = m^2/e$.

$$n'_{\rm max} = {\rm int}\left[\frac{(m_n + \varkappa - m)^2 - m'^2}{2eB}\right] \approx {\rm int}\left[\frac{m'(\Delta + \varkappa - m)}{eB}\right]. \tag{63}$$

For the squared matrix element of the process we get from (27)

$$|\tilde{M}_{n=0}|^2 = 2G^2(C_2 - C_4)^2[(\alpha - 1)^2 f_2^2 N_2^2 (C'_2 + C'_4)^2 I^2_{n'-1,0}(\rho) + \{4f_1^2 N_2^2(\alpha C'_1 - C'_3)^2 + (\alpha + 1)^2 f_2^2 N_1^2 (C'_1 - C'_3)^2\} I^2_{n',0}(\rho)], \tag{64}$$

where the Laguerre functions with $n = 0$ are

$$I_{n',0}(\rho) = \frac{1}{\sqrt{n'!}} x^{n'/2} {\rm e}^{-\rho/2}. \tag{65}$$

Putting the squared matrix element (64) to the general formula for the cross-section, eq. (39), we get the expression for the cross-section,

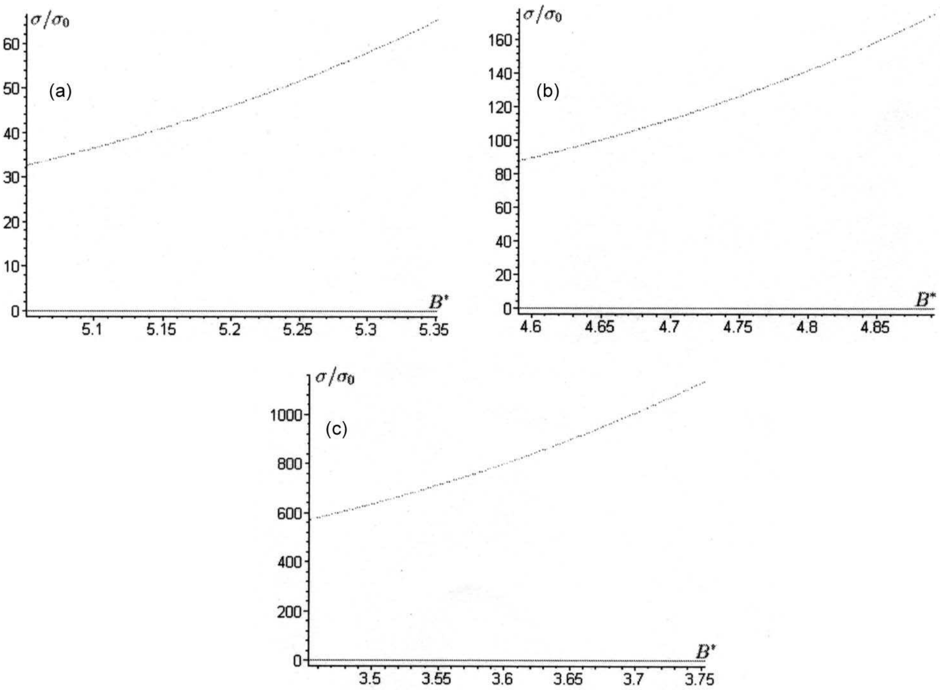

Figure 6. The cross-section in the super-strong magnetic field $B \geq B'_{\rm cr}$, normalized to the cross-section in the field-free case, for different neutrino energies ($\varkappa = 30$ MeV (**a**), 10 MeV (**b**) and $\varkappa \ll m$ (**c**)) for neutrons totally polarized antiparallel to the magnetic field vector ($S = -1$). The solid and dashed lines correspond to the initial neutrino propagation along ($\cos\theta = 1$) and against ($\cos\theta = -1$) the magnetic field vector. The cross-section for $\cos\theta = 1$ is exactly zero. The logarithmic scale is used: $B^* = \log B/B_0$, where $B_0 = m^2/e$.

$$\sigma_{n=0} = \frac{eBG^2}{8\pi} \sum_{n'=0}^{n'_{\max}} \sum_{i=1,2} \frac{\left(1 + \frac{p_3^{(i)}}{p_0^{(i)}}\right)}{\left|\frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{p_0'^{(i)}}\right|} \times \Bigg\{ \Bigg[(1+\alpha)^2 \left(1 - \frac{p_3'^{(i)}}{p_0'^{(i)}}\right)(1+S)(1+\cos\theta) + 2\left[1 + \alpha^2 - (1-\alpha^2)\frac{m'}{p_0'^{(i)}} - 2\alpha\frac{p_3'^{(i)}}{p_0'^{(i)}}\right](1-S)(1-\cos\theta)\Bigg] I^2_{n',0}(\rho) + (1-\alpha)^2\left(1 - \frac{p_3'^{(i)}}{p_0'^{(i)}}\right)(1-S)(1+\cos\theta)(1-\delta_{n',0}) I^2_{n'-1,0}(\rho)\Bigg\}, \quad (66)$$

where $\delta_{n',0}$ is the Kronecker delta ($1 - \delta_{n',0} = 0$ for $n' = 0$).

Together with the expression (38) for $p_3'^{(i)}$, eq. (66) gives the cross-section for the process in the strong magnetic field exactly accounting for the proton momentum quantization and the proton recoil motion. It should be noted that to derive (66) we have not used any constraints on the neutrino energy. Thus, eq. (66) can be used for the case of high-energy neutrino.

Alternatively, if the initial neutrino energy is much less than the proton mass, $\varkappa \ll m'$, it is possible to get an approximate analytical expression for the cross-section accounting for the proton recoil motion using the prescription that we have developed in the study [33] of the proton recoil motion effect in the beta-decay of the neutron in a magnetic field. The proton recoil motion can be characterized by the two parameters, $\alpha'_{\parallel} = p'_3/m'$ and $\alpha'_{\perp} = p'_{\perp}/m'$. The maximum values of these parameters are determined by the initial neutrino energy, and in the above-mentioned neutrino energy range $\alpha'_{\parallel}$, $\alpha'_{\perp} \ll 1$. Therefore, in order to account for the proton recoil motion one has to expand in (30), prior to integration over p'_3, the δ-function over the parameter $\alpha'_{\perp}$,

$$\delta(m_n + \varkappa - p'_0 - p_0) \approx \delta(m_n + \varkappa - p'_0 - p_0) + \frac{\gamma n'}{\tilde{p}'_3}\delta'(m_n + \varkappa - p'_0 - p_0) + O\left(\frac{p_{\parallel}'^4}{\tilde{p}_3'^5}\right), \tag{67}$$

where $\tilde{p}'_3 = \sqrt{m'^2 + p_3'^2}$. For the case of magnetic fields $B \ll B'_{\rm cr}$ the maximum number of the proton Landau level $n'_{\max} > 10$. Thus, it is possible to shift the upper limit $n'_{\max}$ in the summation over n' to infinity,

$$\sum_{n'=0}^{n'_{\max}} \to \sum_{n'=0}^{\infty}. \tag{68}$$

In addition, if one also performs expansion over the parameter $\alpha'_{\parallel}$, then it will be possible to calculate the sum over the proton Landau number n' (for details see, [33]) and get the cross-section that accounts for the transversal and longitudinal proton motions in the linear approximation. The final expression, however, is rather complicated in this case and we do not present it in this paper.

A reasonable simplification can be achieved if we neglect the proton motion in the plain orthogonal to the magnetic field vector and account only for the proton recoil in z-direction. In this case we extract the zeroth-order term in the expansion of the cross-section (30) over the parameter $\alpha'_{\perp}$ and get

$$\sigma_{n=0}\big|_{p'_{\perp}=0, p'_3\neq 0} = \frac{eBG^2}{4\pi}\sum_{i=1,2}\frac{\left(1+\frac{p_3^{(i)}}{p_0^{(i)}}\right)}{\left|\frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{\tilde{p}_3'^{(i)}}\right|}\Bigg\{\left(1+3\alpha^2-(1+\alpha)^2\frac{p_3'^{(i)}}{m'}\right) + \left(1-\alpha^2-(1-\alpha)^2\frac{p_3'^{(i)}}{m'}\right)\cos\theta + S\left[2\alpha(1-\alpha) + \left(2\alpha(1+\alpha) - 4\alpha\frac{p_3'^{(i)}}{m'}\right)\cos\theta\right]\Bigg\}. \tag{69}$$

Note that the last expression does not reproduce the cross-section $\sigma_{n=n'=0}$ given by (55) because in eq. (69), contrary to eq. (55), contributions from infinitely many proton Landau levels are included. Since both spin states $s' = \pm 1$ are not excluded now, there is only one set of values $(\cos\theta, S)$, that determines the direction of the neutrino momentum $(\cos\theta)$ and polarization of the neutrons (S), for which the cross-section vanishes. The final particles' total spin number $s + s'$ can be equal to 0 or -2, whereas for $\cos\theta = \pm 1$ and $S = \pm 1$ the initial particles' total spin number can be equal to 0 or ± 2. Therefore, the violation of the angular momentum conservation can appear only if $\cos\theta = -1$. The cross-section in this case is

$$\sigma_{n=0}\big|_{p_3'\neq 0,\ \cos\theta=-1} = \frac{eBG^2}{\pi}\sum_{i=1,2}\frac{\left(1+\frac{p_3^{(i)}}{p_0^{(i)}}\right)}{\left|\frac{p_3^{(i)}}{p_0^{(i)}}-\frac{p_3'^{(i)}}{\tilde{p}_3'^{(i)}}\right|}(1-S)\left(\alpha^2-\alpha\frac{p'^{(i)}}{m'}\right), \tag{70}$$

and vanishes, as a consequence of the law of 'spin number conservation', if neutrons are polarized in $+z$-direction, i.e. $S = 1$.

If we also neglect the effect of the proton motion in z-direction, then for the cross-section in the strong field $B_{\rm cr} < B \ll B'_{\rm cr}$ we get

$$\sigma_{n=0}\big|_{p_0'=m'} = \frac{eBG^2}{2\pi}\{1+3\alpha^2+(1-\alpha^2)\cos\theta + 2\alpha S[1-\alpha+(1+\alpha)\cos\theta]\}\frac{\Delta+\varkappa}{\sqrt{(\Delta+\varkappa)^2-m^2}}. \tag{71}$$

This result for the cross-section reproduces the one of ref. [40].

From (71) it follows that for the fixed direction of the initial neutrino propagation, i.e. for $\cos\theta = -1$, the cross-section is

$$\sigma_{n=0}\big|_{p_0'=m',\cos\theta=-1} = \frac{eBG^2}{\pi}2\alpha^2(1-S)\frac{\Delta+\varkappa}{\sqrt{(\Delta+\varkappa)^2-m^2}}. \tag{72}$$

For the neutron matter totally polarized parallel to the magnetic field vector, $S = 1$, the cross-section vanishes. The result of eq. (72) coincides with the one of [40].

The cross-section in the strong magnetic field $B = B_{\rm cr}$, normalized to the cross-section in the field-free case, calculated by using the exact eq. (66) is shown in figure 7. Note also that, as can be seen from figure 7, the cross-section for $\cos\theta = 1$ and $S = -1$ is also rather small. This is a consequence of the smallness of the value $(\alpha - 1)$ because the cross-section in this case is proportional to $(1-\alpha)^2 < 0.1$. The neutrino energy is chosen to be $\varkappa = 10$ MeV, and so the effects of the proton recoil motion cannot be screened.

In figures 8a, b and c we plot the dependence of the cross-section on the strength of strong magnetic fields for different intervals within $B_{\rm cr} \le B < B'_{\rm cr}$ in the case of unpolarized $(S = 0)$ neutrons for the initial neutrino energy $\varkappa = 10$ MeV. The super-strong magnetic field $B \ge B'_{\rm cr}$ $(B^* \sim 4.6)$ $(B^* = \log(B/B_0)$, where $B_0 = (m^2/e) = 4.41 \times 10^{13}$ G) is also included in panel (c). The solid curves correspond to $\cos\theta = 1$, the dashed curves correspond to $\cos\theta = -1$. There is a

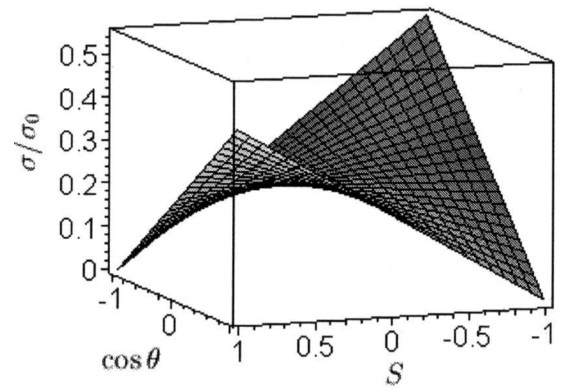

Figure 7. The cross-section in the strong magnetic field $B = B_{\rm cr}$, normalized to the cross-section in the field-free case, for neutrinos with energy of 10 MeV as functions of the direction of the neutrino momentum $\cos\theta$ and polarization S of neutrons. The cross-section in the magnetic field in the case $\cos\theta = -1,\ S = 1$ is exactly zero, whereas the cross-section in the case $\cos\theta = 1,\ S = -1$ is not zero, however it is rather small because it is proportional to $(1-\alpha)^2 < 0.1$.

fall in the cross-section for $\cos\theta = 1$ in the magnetic field $B = B'_{\rm cr}$ because in the strong magnetic field $B_{\rm cr} \le B < B'_{\rm cr}$ the cross-section is zero only for one set of the neutron spin number s_n and $\cos\theta$, i.e. for the case when $s_n = 1,\ \cos\theta = -1$, whereas in the super-strong magnetic field $B \ge B'_{\rm cr}$ the cross-section is zero for two cases $s_n = 1,\ \cos\theta = -1$ and $s_n = -1,\ \cos\theta = 1$.

The analogous dependence of the cross-sections on the strength of the magnetic field for the totally polarized neutrons with $S = -1$ and $S = 1$ are plotted in figures 9a, b and c and figures 10a, b and c, respectively. The super-strong magnetic field $B \ge B'_{\rm cr}$ ($B^* \sim 4.6$) is also included in panels (c) of figures 9 and 10.

For $S = -1$ the cross-section is small if the neutrino propagates parallel to the magnetic field ($\cos\theta = 1$) within the interval $B_{\rm cr} \le B < B'_{\rm cr}$ (see figures 9a and b) because the cross-section is proportional to $(1-\alpha)^2 < 0.1$. In the super-strong magnetic field $B \ge B'_{\rm cr}$ ($B^* \sim 4.6$) (figure 9c) the cross-section is exactly zero as we have already discussed above. For $S = 1$ in the case of $\cos\theta = -1$ for the whole range $B \ge B_{\rm cr}$ (figures 10a, b and c) the cross-section is also zero.

The plots shown in figures 7–10 for the cross-section in the magnetic field disagree with the corresponding plots for the cross-section in the strong field range ($B \ge B_{\rm cr}$) shown in figure 1 of [40]. The contradictions disappear if the solid and dashed curves in figure 1 of [40] are replaced. There is also no rapid increase of the cross-section in the field $B \ge B_{\rm cr}$ for the case of $\cos\theta = 1$ and $S = -1$, contrary to what is shown in the first panel of figure 1 of [40].

3.3 *Cross-section in weak magnetic field*

In the case of weak magnetic fields, $B < B_{\rm cr}$, many Landau levels become available for the electron so that the electron can have non-zero momentum $p_\perp = \sqrt{2\gamma n}$ in

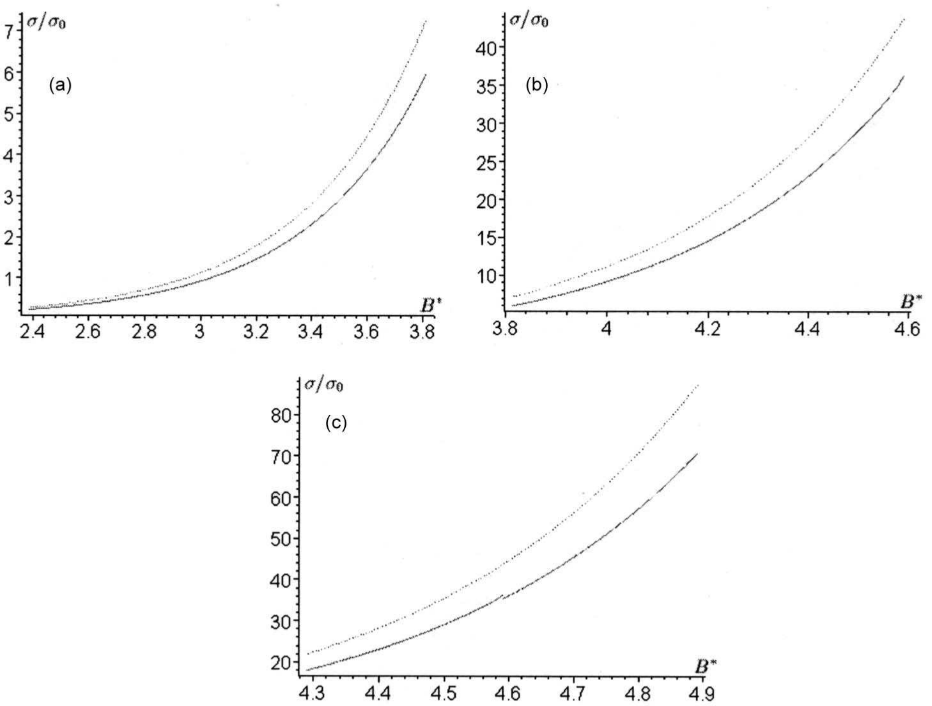

Figure 8. The cross-section in the strong magnetic field, normalized to the cross-section in the field-free case, for different intervals within $B_{\rm cr} \leq B < B'_{\rm cr}$ in the case of unpolarized ($S = 0$) neutrons. The neutrino energy is equal to $\varkappa = 10$ MeV. The super-strong magnetic field $B \geq B'_{\rm cr}$ ($B^* \sim 4.6$) is also included in panel (c). The solid and dashed lines correspond to the initial neutrino propagation along ($\cos\theta = 1$) and against ($\cos\theta = -1$) the magnetic field vector, respectively. The logarithmic scale is used: $B^* = \log B/B_0$, where $B_0 = m^2/e$.

the transverse plane. The maximum allowed value for n is given by (42). In the calculations of the cross-section in the presence of a weak magnetic field we also expand the δ-function (see eq. (67)) and perform the summation over the proton Landau number n' up to infinity. The particular contribution to the cross-section from the partial process with the electron at the lowest Landau level ($n = 0$) has been already discussed in §2.3. Therefore, we derive now the fraction $\sigma_{n\geq 1}$ of the total cross-section that is the sum of the corresponding contributions from the excited electron Landau levels with $n \geq 1$. The final result for the cross-section can be expressed as

$$\sigma_{\rm tot} = \sigma_{n=0} + \sigma_{n\geq 1}. \tag{73}$$

Putting the general expression for the squared matrix element (27) to (39), then expanding over p'_3/m' and performing summation over n' (see the previous subsection), we obtain to the first order in the proton recoil motion

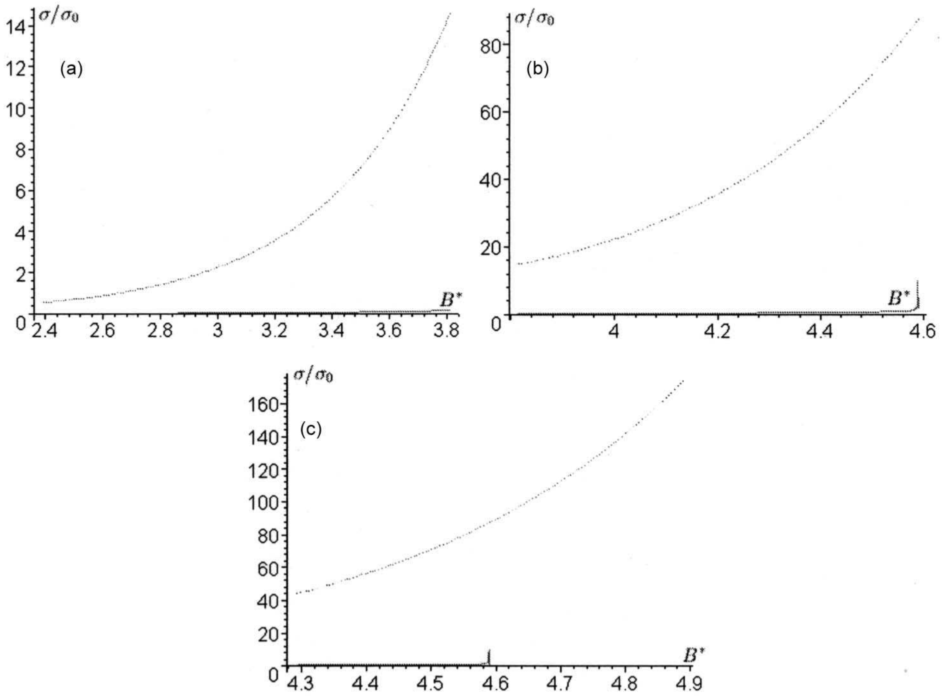

Figure 9. The cross-section in the strong magnetic field, normalized to the cross-section in the field-free case, for different intervals within $B_{\rm cr} \le B < B'_{\rm cr}$ in the case of polarized neutrons with $S = -1$. The neutrino energy is equal to $\varkappa = 10$ MeV. The super-strong magnetic field $B \ge B'_{\rm cr}$ ($B^* \sim 4.6$) is also included in panel (c). The solid and dashed lines correspond to the initial neutrino propagation along ($\cos\theta = 1$) and against ($\cos\theta = -1$) the magnetic field vector, respectively. The cross-section in the case $\cos\theta = 1$ is small for $B_{\rm cr} \le B < B'_{\rm cr}$ and is zero for $B \ge B'_{\rm cr}$. The logarithmic scale is used: $B^* = \log B/B_0$, where $B_0 = m^2/e$.

$$\sigma_{n\ge 1} = \frac{eBG^2}{2\pi} \sum_{n=1}^{n_{\max}} \sum_{i=1,2} \frac{1}{\left|\frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{p_0'^{(i)}}\right|} \times \left[1 + 3\alpha^2 + 2\alpha(1-\alpha)\frac{p_3'^{(i)}}{m'}(1+\cos\theta) + 2S\alpha(1+\alpha)\cos\theta + 2(1+\alpha)^2 \frac{\gamma n}{p_0^{(i)} m'}(1 + S\cos\theta)\right]. \tag{74}$$

In the limit of non-moving proton ($p'_0 = m'$) the contribution to the cross-section for $n \ge 1$ is

$$\sigma_{n\ge 1}\big|_{p'_0 = m'} = \frac{eBG^2}{\pi}[1 + 3\alpha^2 + 2S\alpha(1+\alpha)\cos\theta]$$

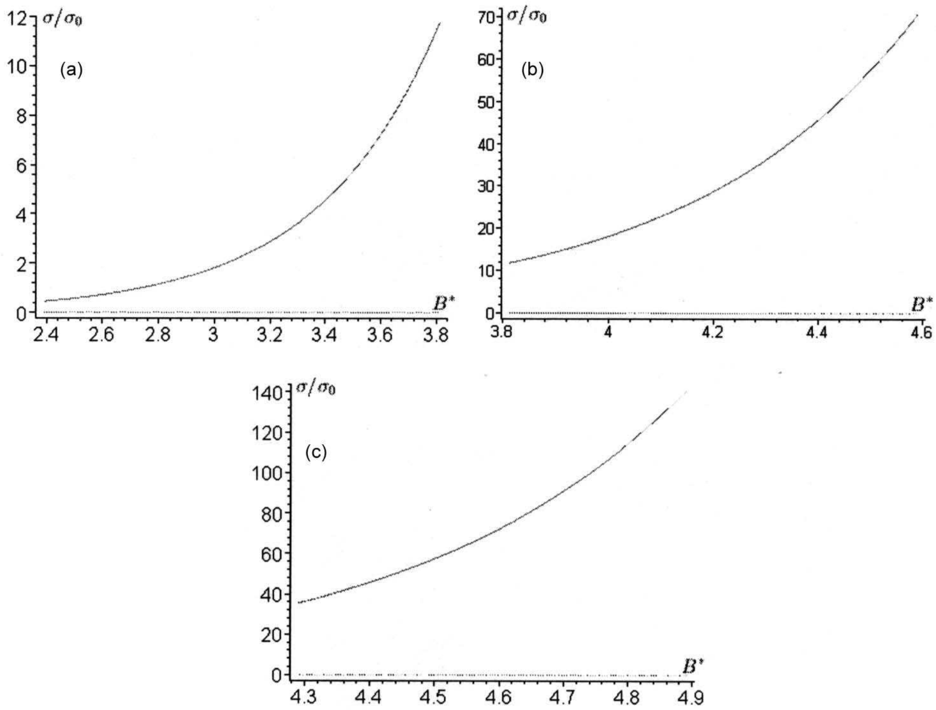

Figure 10. The cross-section in the strong magnetic field, normalized to the cross-section in the field-free case, for different intervals within $B_{\mathrm{cr}} \leq B < B'_{\mathrm{cr}}$ in the case of polarized neutrons with $S = +1$. The neutrino energy is equal to $\varkappa = 10$ MeV. The super-strong magnetic field $B \geq B'_{\mathrm{cr}}$ ($B^* \sim 4.6$) is also included in panel (c). The solid and dashed lines correspond to the initial neutrino propagation along ($\cos\theta = 1$) and against ($\cos\theta = -1$) the magnetic field vector, respectively. The cross-section for case $\theta = -1$ is equal to zero. The logarithmic scale is used: $B^* = \log B/B_0$, where $B_0 = m^2/e$.

$$\times \sum_{n=1}^{n_{\max}} \frac{\Delta + \varkappa}{\sqrt{(\Delta+\varkappa)^2 - m^2 - 2\gamma n}}. \tag{75}$$

Summing this result with the one of eq. (71), we get the result of ref. [40] for the total cross-section in the case of weak magnetic field (the proton recoil motion is neglected here)

$$\sigma_{\mathrm{tot}}\big|_{p'_0=m'} = \frac{eBG^2}{2\pi} \sum_{n=0}^{n_{\max}} \{g_n[1 + 3\alpha^2 + 2S\alpha(1+\alpha)\cos\theta] + \delta_{n,0}[(1-\alpha^2)\cos\theta + 2S\alpha(1-\alpha)]\} \times \frac{\Delta + \varkappa}{\sqrt{(\Delta+\varkappa)^2 - m^2 - 2\gamma n}}. \tag{76}$$

As it follows from (75) and (76), the cross-section has several resonances (see also [22,40]). Similar resonance in the probability of the direct beta-decay of the neutron in the magnetic field was first discovered in [26,27]. In our case the resonance appears, for the given neutrino energy $\varkappa$ and magnetic field strength B, each time when the final electron energy p_0 is exactly equal to one of the allowed ($n \le n_{\max}$) 'Landau energies' $\tilde{p}_\perp = \sqrt{m^2 + 2\gamma n}$,

$$p_0 = \varkappa + \Delta = \sqrt{m^2 + 2\gamma n}. \tag{77}$$

In figures 11a, b and c we plot the cross-section as a function of B (in the range of not very strong magnetic fields, $B \le B_{\rm cr}$) for the three different neutrino energies $\varkappa = 30$ MeV, 10 MeV and $\varkappa \ll m$. Obviously, similar resonance behavior appears in the cross-section as a function of the neutrino energy in a given fixed magnetic field. The number of resonances, which is equal to the number of terms in the sum of eq. (75), increases with the increase of the neutrino energy for a given B. The cross-section, calculated without effects of the proton recoil motion, goes to infinity in the resonance points. However, if we plot the cross-section by using eqs (66) and (74), which accounts for the proton motion, then the infinitely high spikes smooth out.

3.4 *Cross-section in the absence of magnetic field*

The inverse beta-decay in the absence of a magnetic field was considered before by many authors (see, for instance, [44,45]). To the best of our knowledge, the correlation between the neutron polarization and the direction of the neutrino propagation for the scattering (V-A)-interaction process was derived for the first time in ref. [46]. The result for the cross-section $\nu_e + n \to e + p$ in the absence of the magnetic field is

$$\sigma_0 = \frac{G^2}{\pi}\left[1 + 3\alpha^2 + 2\alpha S_n(1+\alpha)\cos\theta\right](\Delta + \varkappa)\sqrt{(\Delta+\varkappa)^2 - m^2}. \tag{78}$$

We now demonstrate, following the similar procedure described in [26,27], how in the limit of vanishing magnetic field $B \to 0$ the result in eq. (76) reduces to the one of (78).

When the field is switching off, the maximum number of Landau level $n_{\max}$ is increasing to infinity, however the product eBn remains constant,

$$\lim_{\gamma\to 0, n\to\infty} \gamma n = \frac{(\Delta+\varkappa)^2 - m^2}{2}. \tag{79}$$

In this limit we can replace the summation over n by integration using the relation (see, for example, [26])

$$\sum_{n=0}^{N} f(n) = \int_0^N f(x)\mathrm{d}x + \frac{f(N) + f(0)}{2} + \int_0^N Q(x) f''(x)\mathrm{d}x, \tag{80}$$

where the value of the last term can be estimated as

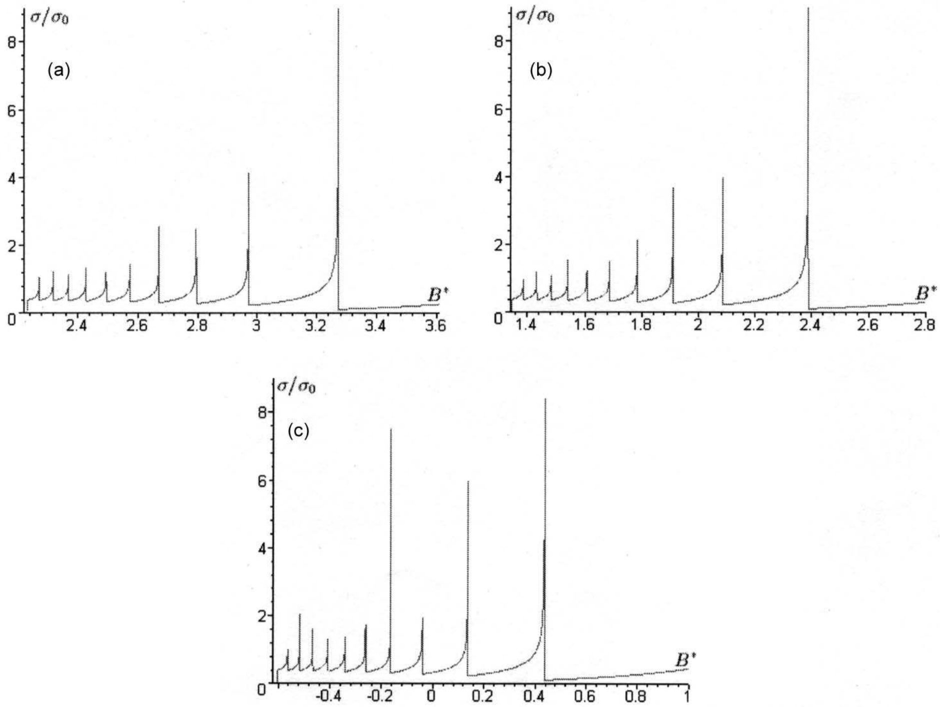

Figure 11. The resonance behavior of the cross-section in the magnetic field, normalized to the field-free case, for given neutrino energies: $\varkappa = 30$ MeV (**a**), 10 MeV (**b**) and $\varkappa \ll m$ (**c**). The logarithmic scale is used: $B^* = \log B/B_0$, where $B_0 = m^2/e$.

$$\int_0^n Q(x) f''(x) \mathrm{d}x \leqslant \frac{f'(n) - f'(0)}{8}. \tag{81}$$

In the sum over n in (76) the contribution of the lowest Landau level is diminishing in comparison with the contributions of the excited Landau levels $n > 0$. For the estimation of the former we use

$$\begin{aligned} &\lim_{\gamma \to 0} \gamma \sum_{n=0}^{n_{\max}} \frac{1}{\sqrt{(\Delta + \varkappa)^2 - m^2 - 2\gamma n}} \\ &= \lim_{\gamma \to 0} \gamma \left[\int_0^{n_{\max}} \frac{\mathrm{d}x}{\sqrt{(\Delta + \varkappa)^2 - m^2 - 2\gamma x}} + C \right] \\ &= \lim_{\gamma \to 0} \gamma \left[\frac{1}{\gamma} \sqrt{(\Delta + \varkappa)^2 - m^2} + C \right] = \sqrt{(\Delta + \varkappa)^2 - m^2}, \end{aligned} \tag{82}$$

where C is a function proportional to $\gamma^{-1/2}$. Thus, in the limit $B \to 0$ from (76) we get the cross-section of the process in the absence of a magnetic field.

3.5 *Effects of anomalous magnetic moments of nucleons*

When considering the influence of very strong magnetic fields on the inverse beta-decay of a neutron one should be careful about the effect of magnetic field on anomalous magnetic moments of a neutron and proton. In particular, it is known [5,34,41] that the interplay between anomalous magnetic moments of the neutron and proton shifts the masses of these particles. These effects are important only for the super-strong magnetic fields, when the corresponding shift of the electron energy due to the electron anomalous magnetic moment is vanishing [47,48] (see also [5]). As a result, the neutron becomes stable in the presence of magnetic fields with the strength $B \geq 1.5 \times 10^{18}$ G. On the other hand, the proton becomes unstable with respect to the the inverse beta-decay $p \rightarrow n + e^{+} + \nu_e$ if the magnetic field is increased past the strength $B \geq 2.7 \times 10^{18}$ G. Therefore, in this section, in order to complete the relativistic theory of the neutron inverse beta-decay in the super-strong magnetic field, we discuss in some detail the possible effect of the nucleons anomalous magnetic moment interactions with a magnetic field.

The energy of the moving proton and the neutron at rest in a magnetic field, with the contributions from the anomalous magnetic moments interaction being accounted for, are given respectively by

$$p'_0 = \sqrt{\left(\sqrt{m'^2 + 2eBn'} - s'k_pB\right)^2 + {p'_3}^2}, \tag{83}$$

$$p_0^n = m_n - s_n k_n B, \tag{84}$$

where the values of the proton and neutron anomalous magnetic moments

$$k_p = \frac{e}{2m'}\left(\frac{g_p}{2} - 1\right) \tag{85}$$

$$k_n = \frac{e}{2m_n}\frac{g_n}{2}, \tag{86}$$

are determined by the Lande's g-factors: $g_p = 5.58$, $g_n = -3.82$.

Taking into account these modified expressions for the proton and neutron energies, we can repeat all the above-described calculations of §2.1 applying the substitutions

$$m' \rightarrow m'^{*} = m' - k_p B, \tag{87}$$

$$m_n \rightarrow m_n^{*} = m_n - s_n k_n B. \tag{88}$$

Note that in the super-strong magnetic filed $B \geq B'_{\rm cr}$ there is only one spin state for the proton with $s' = +1$.

The law of energy conservation (31) shows that in the super-strong magnetic field there is a range of the neutron matter polarization S for which the matter becomes transparent for neutrinos. From (31) we get

$$m_n - s_n k_n B + \varkappa \geq m + m' - k_p B. \tag{89}$$

Therefore, the process $\nu_e + n \to e + p$ is forbidden if $(Sk_n - k_p) > 0$ and the magnetic field exceeds the value of B_{forb}:

$$B_{\text{forb}} = \frac{\Delta + \varkappa - m}{Sk_n - k_p}. \tag{90}$$

Note that this forbidding effect appears for nearly maximum neutron matter polarizations against the magnetic field, $-1 \leq S < k_p/k_n \approx -0.94$. The values of B_{forb} for different neutrino energies in case of maximum neutron spin polarization $S = -1$ are

$$B_{\text{forb}} \approx 8.5 \times 10^{19}\ \text{G}, \quad \varkappa_{\max} = 30\ \text{MeV}, \tag{91}$$

$$B_{\text{forb}} \approx 3.0 \times 10^{19}\ \text{G}, \quad \varkappa_{\max} = 10\ \text{MeV}, \tag{92}$$

$$B_{\text{forb}} \approx 2.2 \times 10^{18}\ \text{G}, \quad \varkappa_{\max} \ll m. \tag{93}$$

4. Conclusions

We have developed the relativistic theory of the inverse beta-decay of the polarized neutron in a magnetic field. Effects of the proton momentum quantization in the magnetic field have been included. The closed expression obtained for the cross-section in the magnetic field exactly accounts for the longitudinal and transversal motions of the proton. For the three ranges of the magnetic field (which we call the super-strong magnetic field $B \geq B'_{\text{cr}}$, the strong field $B_{\text{cr}} \leq B < B'_{\text{cr}}$, and the weak field $B < B_{\text{cr}}$) we have calculated the cross-section and discussed its dependence on the neutrino energy and angle θ, as well as on the neutron polarization S.

To describe the proton we have used the exact solution of the Dirac equation in a magnetic filed. This enables us to get the exact cross-section in the case of the super-strong magnetic field $B \geq B'_{\text{cr}}$ when the proton can occupy only the lowest Landau level $n' = 0$. We have shown that it is not correct to use the cross-section, derived under the assumption that the proton wave function is not modified by the magnetic field, in the case when only one, not many Landau levels are opened for the proton even if the proton motion is neglected. From the obtained expressions for the cross-section in the strong and super-strong magnetic filed it is clearly seen that

$$\sum_{n'=0}^{\infty} \sigma(n, n')\big|_{p'_0 = m'} \neq \sigma(n, n')\big|_{n'=0}, \tag{94}$$

and even

$$\sum_{n'=0}^{\infty} \sigma(n, n')\big|_{p'_0 = m'} \neq \sum_{n'=0}^{n'_{\max}} \sigma(n, n')\big|_{n'=0}, \tag{95}$$

if many Landau levels n' are not available. Thus we conclude that the Landau quantization of the proton momentum has to be accounted for not only the super-strong magnetic field, but even for lower magnetic fields when too many Landau levels are not opened for the proton.

We would also like to point out here that it is not possible to use the expression of the cross-section, derived for the strong magnetic field ($B_{\rm cr} < B \leq B'_{\rm cr}$), in the case of the super-strong magnetic field ($B \geq B'_{\rm cr}$) and also for lower magnetic field ($B \leq B'_{\rm cr}$) when only a few Landau levels for the proton are available.

We have shown that in the case of the total neutron polarization ($S = \pm 1$) the cross-section is exactly zero in the super-strong magnetic filed if $S\cos\theta = -1$, i.e. in the two cases: (1) $S = 1,\ \cos\theta = -1$ and (2) $S = -1,\ \cos\theta = 1$. Thus, in the super-strong magnetic field the totally polarized neutron matter is transparent for the neutrino propagating in the direction opposite to the direction of the neutron polarization. In the case of the strong magnetic filed the cross-section is exactly zero if $S = 1$ and $\cos\theta = -1$, that confirms the result of ref. [40]. These asymmetries in the cross-section appear as a consequence of the angular momentum conservation and the spin polarization properties of the electron and proton being at the lowest Landau levels in the magnetic field.

It should be noted that the developed relativistic treatment of the cross-section can be applied to the other URCA processes with two particles in the initial and final states. For instance, similar calculations can be performed for the anti-neutrino absorption process on the proton in the presence of a magnetic field,

$$\bar{\nu}_e + p \rightarrow n + e^+. \tag{96}$$

A recent study of this process in the strong-magnetic field without account for the neutron recoil can be found in [14]. The crossing symmetry makes it possible, by using the matrix element of the neutron inverse beta-decay, to write the matrix element of the former process immediately. What remains to be done is to change appropriately the phase volume of the process. With a minor modification, the above-obtained expressions for the cross-section of the neutron inverse beta-decay can be transformed for the process $\bar{\nu}_e p \rightarrow n e^+$ in a magnetic field. For example, the above-obtained expressions for the cross-section give the cross-section of the process $\bar{\nu}_e p \rightarrow n e^+$ if the signs of the values Δ and α are changed to the opposite and also the substitution $s_n \rightarrow s'$ is made.

References

[1] C Thompson and A Harding, *Astrophys. J.* **408**, 194 (1993); **473**, 332 (1996)
[2] M Baring and A Harding, *Astrophys. J.* **507**, L55 (1998)
[3] J Landstreet, *Phys. Rev.* **153**, 1372 (1967)
[4] D Lai and S Shapiro, *Astrophys. J.* **383**, 754 (1991)
[5] D Grasso and H Rubinstein, *Phys. Rep.* **348**, 163 (2001)
[6] G Greenstein, *Nature* **223**, 938 (1969)
[7] J Matese and R O'Connell, *Astrophys. J.* **160**, 451 (1970)
[8] D Grasso and H Rubinstein, *Phys. Lett.* **B379**, 73 (1996); *Astropart. Phys.* **3**, 95 (1995)
[9] A Goyl, *Phys. Rev.* **D59**, 101301 (1999), hep-ph/9812473

[10] A Gvozdev and I Ognev, *JETP Lett.* **69**, 365 (1999), astro-ph/9909154
[11] P Arras and D Lai, *Phys. Rev.* **D60**, 043001 (1999), astro-ph/9811371
[12] L B Leinson and A Perez, *J. High Energy Phys.* **9809**, 020 (1998), astro-ph/9711216
[13] D Lai and S Shapiro, *Astrophys. J.* **383**, 745 (1991)
[14] H Duan and Y-Zh Qian, *Phys. Rev.* **D69**, 123004 (2004), astro-ph/0401634
[15] N Chugai, *Sov. Astron. Lett.* **10**, 87 (1984) (*Pis'ma Astron. Zh.* **10**, 210 (1984))
[16] O Dorofeev, V Rodionov and I Ternov, *JETP Lett.* **40**, 917 (1984) (*Pisma Zh. Eksp. Teor. Fiz.* **40**, 159 (1984)); *Sov. Astron. Lett.* **11**, 123 (1985) (*Pis'ma Astron. Zh.* **11**, 302 (1985))
[17] A Kusenko and G Segre, *Phys. Rev. Lett.* **77**, 4872 (1996)
[18] A Studenikin, *Sov. J. Astrophys.* **28**, 639 (1988)
[19] G Bisnovatyi-Kogan, *Astron. Astrophys. Trans.* **3**, 287 (1993), astro-ph/9707120
[20] E Akhmedov, A Lanza and D Sciama, *Phys. Rev.* **D56**, 6117 (1997), hep-ph/9702436
[21] H Nunakawa, V Semiloz, A Smirnov and J W F Valle, *Nucl. Phys.* **B501**, 17 (1997), hep-ph/9701420
[22] E Roulet, *J. High Energy Phys.* **9801**, 013 (1998), hep-ph/9711206
[23] D Lai and Y Z Qian, *Astrophys. J.* **505**, 844 (1998), astro-ph/9802345
[24] K Bhattacharya and P Pal, hep-ph/0212118
[25] A Kusenko, astro-ph/0409521
[26] L Korovina, *Izvestija Vuzov. Fizika, Astron.* **6**, 86 (1964)
[27] I Ternov, B Lysov and L Korovina, *Mosc. Univ. Phys. Bull.* (Vest. MSU, Fiz. Astr.) No. 5, 58 (1965)
[28] J Matese and R O'Connell, *Phys. Rev.* **180**, 1289 (1969)
[29] L Fassio-Canuto, *Phys. Rev.* **187**, 2141 (1969)
[30] A Gazazian, *Izvestija Acedemy Nauk Armenii* **18**, 126 (1965)
[31] I Baranov, *Izvestija Vuzov. Fizika, Astron.* **4**, 115 (1974)
[32] I Ternov, V Rodionov, V Zhulego and A Studenikin, *Sov. J. Nucl. Phys.* **28**, 747 (1978) (Yadernaja Fizika **28**, 1454 (1978)); *Ann. d. Phys.* **37**, 406 (1980)
[33] A Studenikin, *Sov. J. Nucl. Phys.* **49**, 1031 (1989) (Yadernaja Fizika **49**, 1665 (1989))
[34] M Bander and H Rubinstein, *Phys. Lett.* **B289**, 385 (1992); Preprint Uppsala Univ. PT 11, 1991
[35] G Zharkov, *Sov. Phys. JETP* **89**, 1489 (1985)
[36] A Studenikin, *Weak processes in external electromagnetic fields*, PhD Thesis (Moscow State University, 1983) p. 127
[37] B Cheng, D Schramm and J Truran, *Phys. Lett.* **B316**, 521 (1993)
[38] K Bhattacharya and P Pal, hep-ph/9911498
[39] K Bhattacharya and P Pal, hep-ph/0001077
[40] K Bhattacharya and P Pal, *Pramana – J. Phys.* **62**, 1041 (2004), hep-ph/0209053
[41] I Mamsourov and H Goudarzi, hep-ph/0404086
[42] A A Sokolov and I M Ternov, *Synchrotron radiation* (Pergamon Press, New York, 1968)
[43] I Gradstein and I Ryzhik, *Table of integrals, series, and products*, 4th edition (Academic Press, 1980)
[44] D Tubbs and D Schramm, *Astrophys. J.* **201**, 467 (1975)
[45] S Bruenn, *Astrophys. J. Suppl.* **58**, 771 (1985)
[46] B Kerimov, *Izv. Ak. Nauk USSR* **15**, 1 (1961)
[47] I Ternov, V Bagrov, V Bordovitsyn and O Dorofeev, *JETP* **28**, 1206 (1969)
[48] J Schwinger, *Particles, sources and fields* (Addison-Wesley, 1988) Vol. 3