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The Bohm criterion for a dusty plasma sheath

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Abstract. The formation of the sheath in a dusty plasma is investigated. The Bohm criterion is derived for two different cases: (a) when electrons are in thermodynamic equilibrium and dust grains provide the immobile, stationary background and (b) when both electrons and ions are in thermodynamic equilibrium and dust grains are moving. In the first case, Bohm criterion gets modified due to the fluctuation of the charge on the grain surface. In the second case, the collisional and Coulombic drag play important role in determining the Bohm criterion.

Keywords. Dusty plasma; Bohm criteria; plasma sheath.

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The dynamics of dusty plasma have been a subject of intense investigation in the last decade [1]. The presence of the dust grain in the plasma introduces new spatial and temporal scales. As a result, not only the collective behaviour of the plasma is altered, but also, new plasma modes are excited. For example, due to the large difference in masses (grain mass $\sim 10^{-15}$–10^{-5} g, $m_{\rm e} \sim 10^{-27}$ g, $m_{\rm i} \sim 10^{-24}$ g), the dusty plasma dynamics can be studied in two different limits: (i) the dust particle is so heavy that it provides a stationary background for the perturbations propagating in the much lighter electron–ion plasma and (ii) the perturbations are of the order of or less than the typical plasma frequencies of the dusty fluid (of the order of Hz) and wavelengths are visible to the bare eyes. Although large dust grains undergo temperature fluctuations due to collision, the mean square fluctuation in their temperature is much less than the equilibrium temperature.

The problem of sheath dynamics with the plasma–wall interactions is of great importance in a number of areas, viz., plasma ion implantation, high-density computer chip development, diamond-like film deposition, nuclear fusion etc. In plasma processing, where a target object is immersed in the plasma and pulsed repeatedly to a large negative voltage, a sheath is formed that expands in the ambient plasma. If the ion impact energy is sufficiently large, the impact ions may cause severe sputtering of the target which is an undesirable side effect. The wall may develop non-uniformities due to sputtering, re-deposition, cracking, etc. Further, sputtered material may contaminate the plasma. The emission of the secondary electron is also an important issue. Since the ions and electrons have opposite charge, the emitted secondary electrons are accelerated away from the target in the same electric field that accelerates ions toward the target. This leads to considerable power loss as part of the power goes to the secondary electrons. Therefore, secondary electron emission reduces the efficiency of the ion implantation.

The understanding of the charged boundary layer formation in a dusty plasma may have wide-ranging industrial applications. The charged boundary layer near the wall is due to the difference in the mobility of the electrons, ions and dust grains. In bounded plasmas, the Debye length gives the approximate thickness of the sheath which develops over the inverse electron plasma frequency time-scale, whenever plasma is in contact with the material wall. In the presence of a stationary sheath, in an electron–ion plasma, most of the electrons reaching the sheath boundary will be reflected back into the plasma, and number of electrons striking the wall will be equal to the number of positive ions reaching the wall. A stationary sheath exists only if the ion flow velocity satisfies the Bohm criterion at the plasma–sheath boundary, or if the electric field at the plasma–sheath interface exceeds some critical value [2–9]. In the presence of negatively charged dust grains, the plasma sheath will get modified as the sheath electric field does work to remove the negatively charged grains from its vicinity. The charge on the grain can fluctuate due to the orbit-limited plasma current fluctuation when the dust grains collect plasma particles at the random interval. The negatively charged sheath potential will affect the efficiency of negatively charged grains to collect the particles. Thus, the number of particles on the grain surface will be a function of sheath potential.

The plasma–sheath boundary, which defines the transition between the quasi-neutral bulk plasma and the charged boundary layer, is defined using Bohm criterion. The Bohm criterion for a plasma consisting of electrons, ions and charged dust grains have been a subject of past investigations [5–7]. The collisional charging of a dust grain can be regarded as the most fundamental interaction it undergoes with the ambient plasma, since the acquired charge determines the grain cross-section for all collisions with the ionized gas. In hot plasma ($\sim$10 eV), the efficiency of grain charging is limited by secondary electron emission, and, at higher temperature ($\sim$100 eV), by transparency of the dust to the impinging electrons. The variation of the dust charge introduces additional temporal scale in a dusty plasma. The effect of charge fluctuation on the positively charged grain and the modification to the Bohm criterion have been investigated recently. The derivation of the Bohm criterion for a dusty plasma in the presence of electrons, ions and negatively charged dust grains, is the aim of the present work. The role of the neutral particle is assumed to be insignificant. Such an assumption is valid in a low-pressure ($\sim$ few mT) plasma. For example, at 1 mT, ion–neutral particle collision, $\nu_{\text{in}} \sim 100$ Hz, whereas, the slowest discharge time-scale is of the order of the ion–plasma frequency. Thus, the neglect of the ion–neutral collisional effect is a plausible assumption for a low-pressure plasma. We consider two different cases: (a) when the dust is immobile and electrons are Boltzmannian and (b) when both electrons and ions are in thermodynamic equilibrium. Whereas first case will correspond to the initial stage of sheath dynamics, the latter case will be valid at the 'slower' dust plasma time-scale when the charged dust particle will respond to the electric field of the sheath.

A three-component dusty plasma consisting of electrons, ions and negatively charged dust particles is considered in the present work. The dust grains are assumed to be spherical with radius a and carry $-Ze$ charge, where Z is the number of electronic charge on the grain and e is the electric charge. The fluid description is employed to study the problem at hand. Due to the formation of the sheath near the plasma boundary, there exists two regions in a bounded dusty plasma: (a) The quasi-neutral bulk plasma, where electron, ion and dust number densities are related: $n_{\mathrm{i}0} = n_{\mathrm{e}0} + Z_0 n_{\mathrm{d}0}$, $n_{\alpha 0}$ is the number density of the αth particle ($\alpha =$ electron, ion or dust) and Z_0 is the number of the electron charge on the grain surface. (b) The sheath at the boundary where the electron and the dust number densities will be much less than the ion number density.

A stationary unmagnetized planar plasma–sheath boundary is located at $z = 0$ with the plasma filling the half space $z > 0$. The typical sheath width is a few Debye length (a spatial scale of the local electric field) that could be very small in practical applications, while the quasi-neutrality scale corresponds to the typical size of the system. This circumstances lead to the non-universality of the plasma distribution function for the whole region and allows the near-wall sheath layer to be modeled separately from the bulk plasma region.

First, we consider the case of thermalized electrons, cold ions and immobile dust particles. This is due to the fact that the large mass difference implies that the dust particles are so heavy that they can be taken as stationary for frequencies far exceeding the characteristic frequency of the dusty plasma component, forming only a stationary neutralizing background for the perturbations propagating in the plasma. The basic set of equations for the stationary cold ions is the continuity equation

$$\frac{\mathrm{d}}{\mathrm{d}z} n_{\mathrm{i}} v_{\mathrm{i}} = 0. \tag{1}$$

The ion may transfer part of its momentum due to direct ion impact [1],

$$F_{\mathrm{di}} = \pi a^2 n_{\mathrm{i}} m_{\mathrm{i}} v_{\mathrm{i}}^2 \left(1 + \frac{2Ze^2}{m_{\mathrm{i}} a v_{\mathrm{i}}^2} \right), \tag{2}$$

where $\phi_{\mathrm{d}} = -Ze/a$ has been used for the negatively charged dust potential. The Coulomb drag experienced by the ion is [1]

$$F_{\mathrm{Coul}} = 2\pi b_0^2 n_{\mathrm{i}} m_{\mathrm{i}} v_{\mathrm{i}}^2 L, \tag{3}$$

where L is the Coulomb logarithm and has a typical value between 10 and 20 and b_0 is the impact parameter corresponding to 90° deflection. One notes that the Coulomb drag is $\sim v_{\mathrm{i}} v_{\mathrm{T}}$ if v_{i} is small. Here $v_{\mathrm{T}} = \sqrt{2T_{\mathrm{i}}/m_{\mathrm{i}}}$ is the ion thermal velocity. For non-thermal ions, Coulmb drag is always proportional to v_{i}^2. Defining, $C = \pi(2Lb_0^2 + a^2)$ and $B = 2\pi a e^2/m_{\mathrm{i}}$, the ion momentum equation, in the presence of Coulomb drag and collision, can be written as

$$v_{\mathrm{i}} \frac{\mathrm{d}v_{\mathrm{i}}}{\mathrm{d}z} = -\frac{e}{m_{\mathrm{i}}} \frac{\mathrm{d}\phi}{\mathrm{d}z} - C n_{\mathrm{d}} v_{\mathrm{i}}^2 - BZ(\phi) n_{\mathrm{d}}. \tag{4}$$

It is assumed that the number of grain charge Z is a function of plasma potential, ϕ. The electron density is given by

$$n_{\rm e}(z) = n_{\rm e0} \exp\left[\frac{e\phi}{T}\right]. \tag{5}$$

The electric potential ϕ is determined by the Poisson's equation

$$\nabla^2\phi = -4\pi\left[e(n_{\rm i} - n_{\rm e}) - Zen_{\rm d}\right]. \tag{6}$$

All the dependent variables in the vicinity of the sheath can be expanded in the Taylor series as the space charge density and the potential need to be calculated in the vicinity of the sheath [8]. The integration of eq. (1) gives

$$n_{\rm i}v_{\rm i} = n_0 v_0. \tag{7}$$

Here $n_{\rm i0}, v_0$ are the ion number density and velocity respectively in the quasi-neutral plasma region where $\phi = 0$. From eq. (4) one gets

$$v_{\rm i} = v_0\left[(1-2Cz) - \frac{2BZ_0 z}{v_0^2} - \frac{2e\phi}{m_{\rm i}v_0^2}\right]^{1/2}. \tag{8}$$

Introducing, $v_{\rm B} = (T/m_{\rm i})^{1/2}$, $\lambda_{\rm D} = v_{\rm B}/\omega_{\rm pi}$, $\omega_{\rm pi} = (4\pi n_{\rm i}e^2/m_{\rm i})^{1/2}$ and defining, $s = z/\lambda_{\rm D}$, $\eta = -e\phi/T$, $C_1 = 2Cn_{\rm d0}\lambda_{\rm D}$, $B_1 = (2B/v_{\rm B}^2)n_{\rm d0}\lambda_{\rm D}$, eq. (8) can be written as

$$v = v_0\left[(1 - C_1 s) - B_1 s\left(\frac{v_{\rm B}}{v_{\rm i0}}\right)^2 + 2\eta\left(\frac{v_{\rm B}}{v_{\rm i0}}\right)^2\right]^{1/2}. \tag{9}$$

After combining eqs (7) and (9), one may write

$$\frac{n_{\rm i}}{n_{\rm i0}} = 1 - \left(\frac{v_{\rm B}}{v_{\rm i0}}\right)^2\eta + \left(\frac{v_{\rm B}}{v_{\rm i0}}\right)^2\frac{B_1 s}{2} + \frac{C_1 s}{2}. \tag{10}$$

Equation (5) becomes

$$\frac{n_{\rm e}}{n_{\rm e0}} = (1-\eta). \tag{11}$$

In order to derive the dependence of the grain charge Z on the plasma potential η, we note that the electron and ion fluxes on the grain surface will balance each other in the equilibrium, i.e. $J_{\rm e} \approx J_{\rm i}$. The electron and ion fluxes to the grain surface are given by [1]

$$J_{\rm e} = 2\sqrt{2\pi}a^2 n_{\rm e0}v_{T_{\rm e}}\exp(-\eta)\exp(-\gamma),$$
$$J_{\rm i} \approx \pi a^2 n_{\rm i0}v_{\rm i0}\left(1 + 2\gamma\frac{v_{\rm B}^2}{v_{\rm i0}^2}\right), \tag{12}$$

where $\gamma = Ze^2/aT$. Here $\phi_{\rm d} = Ze^2/a$ has been assumed while writing eq. (12). From the balance of the electron and ion fluxes on the grain surface, one may write

$$\sqrt{\frac{8}{\pi}} n_{\mathrm{e0}} v_{T_{\mathrm{e}}} \exp(-\eta - \gamma) = n_{\mathrm{i0}} v_{\mathrm{i0}} \left(1 + 2\gamma \frac{v_{\mathrm{B}}^2}{v_{\mathrm{i0}}^2}\right). \tag{13}$$

Considering that $2\gamma v_{\mathrm{B}}^2/v_{\mathrm{i0}}^2 \geq 1$, and employing quasi-neutrality condition, after assuming $v_{\mathrm{i0}} = v_{\mathrm{B}}$, and noting that in the sheath region, electron and dust number densities will be much smaller than the ion number density, eq. (13) may be written as

$$\exp(-\eta - \gamma) \approx \left(\frac{v_{\mathrm{B}}}{v_{T_{\mathrm{e}}}}\right) \left[1 + \frac{\gamma_0}{\gamma}\right]. \tag{14}$$

In general the dependence of Z on η is non-trivial and one needs to solve the above transcendental equation numerically. Furthermore, Z is also the function of ion velocity in general. The exact dependence of Z on η is not crucial for the present work and thus, we shall approximate the transcendental eq. (14) (after approximating $\ln(1 + x) \approx x$) as

$$\frac{Z(\eta)}{Z_0} \approx A\eta, \tag{15}$$

where

$$A = \frac{\gamma_0}{\mathrm{Ln}(v_{T_{\mathrm{e}}}/v_{\mathrm{B}}) - \gamma_0}. \tag{16}$$

In the steady state,

$$\nu_{\mathrm{id}} n_{\mathrm{i0}} = \nu_{\mathrm{ed}} n_{\mathrm{e0}} \tag{17}$$

and thus, one may define $\delta = \nu_{\mathrm{id}}/\nu_{\mathrm{ed}}$ and Poisson's equation (6) can be written as

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}s^2} = -P\eta + Rs. \tag{18}$$

Here $P = (v_{\mathrm{B}}/v_{\mathrm{i0}})^2 - A(1 - \delta) - \delta$, $R = \pi n_{\mathrm{d0}} \lambda_{\mathrm{D}} \left[(2Lb_0^2 + a^2) + \frac{2ae^2}{m_{\mathrm{i}} v_{\mathrm{i0}}^2}\right]$. One notes that the necessary condition for the sheath existence is,

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}s^2} > 0 \quad \text{if } s > 0, \tag{19}$$

i.e. curvature of η has a maximum at the sheath beginning. For $A = 1$, i.e., when grain charge is fixed and does not depend on the plasma potential, the above condition, in the absence of collisional and Coulombic drag, reduces to the usual Bohm criterion, $v_{\mathrm{i}} > v_{\mathrm{B}}$. However, even when $R = 0$, the dependence of grain charge on the plasma potential modifies the Bohm criterion. The critical velocity at the plasma sheath boundary becomes,

$$v_{\mathrm{c}} = \frac{v_{\mathrm{B}}}{\sqrt{A + \delta(1 - A)}}. \tag{20}$$

Thus, the modified Bohm velocity in a dusty plasma becomes a function of dust charge's dependence on the plasma potential, and depending on how strong or weak this dependence is, the critical velocity will be less than what could be the case for a two-component electron–ion plasma. Since the sheath field is modified due to the presence of the grain, the ions will acquire 'smaller' Bohm velocity than is otherwise possible. As a result, critical ion velocity at the plasma–sheath boundary will decrease. Similar result has been obtained in the past for the negative ion plasma [3,4]. However, unlike negative ion plasmas, in a dusty plasma, the charge on the grain fluctuates $Z \equiv Z(\eta)$ and the Bohm criterion is dependent on the grain charge fluctuation $A = A(Z)$ in an implicit manner. Therefore, it is not surprising that when $A = 1$, the result converges to the case of a three-component, electron, negative and positive ion plasmas [8,9].

Equation (19) will be satisfied even when P is slightly negative, since R is always positive. The solution of eq. (18) can be written as [9]

$$\eta(s) = \frac{R}{\sqrt{P}} \sin\left(\sqrt{P}s\right) + \frac{R}{P}s. \tag{21}$$

Equation (18) holds in a domain only where $\sqrt{P}|s| \ll 1$. Thus, an approximation

$$\eta(s) = \frac{1}{6}\left(R - P\right)s^3 \tag{22}$$

and

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}s^2} = (R - P)\,. \tag{23}$$

Thus, a sheath can be formed if $(R - P) > 0$.

We next consider the other limit, valid for very low frequencies (of the order of, or less than, typical plasma frequencies of the dusty fluid $\sim$ of the order of Hertz), that includes the dynamics of dust particles. This description is relevant in the aftermath of the formation of the sheath when over the dust time-scale, electrons and ions are thermalized. The electron distribution will be given by eq. (5) and ion distribution will be given by the following equation:

$$n_\mathrm{i}(z) = n_{\mathrm{i}0} \exp\left[-\frac{e\phi}{T}\right]. \tag{24}$$

The moving dust particle will face the Coulombic as well as collision drag. For a charged dust grain, moving with the velocity v_d, such a drag is [10]

$$F_\mathrm{drag} \approx \frac{8\sqrt{\pi}}{3} a^2 T n_\mathrm{i} \left(\frac{v_\mathrm{d}}{C_\mathrm{i}}\right)(2 + \gamma_0 L)\,. \tag{25}$$

Here $C_\mathrm{i} = (T/m_\mathrm{i})^{1/2}$. The dust continuity equation is

$$\frac{\mathrm{d}}{\mathrm{d}z} n_\mathrm{d} v_\mathrm{d} = 0. \tag{26}$$

The momentum equation is

$$v_{\rm d}\frac{{\rm d}v_{\rm d}}{{\rm d}z}=\frac{Z\left(\phi\right)e}{m_{\rm d}}\frac{{\rm d}\phi}{{\rm d}z}-\left[\frac{8\sqrt{\pi}}{3C_{\rm i}}a^2T\left(2+\gamma_0L\right)\right]n_{\rm i}v_{\rm d}. \tag{27}$$

Repeating the similar procedure as above, for a normalized dust velocity $v=v_{\rm d}/C_{\rm d}$, where $C_{\rm d}=\left(T/m_{\rm d}\right)^{1/2}$ one may get the following expression for v:

$$v^2=v_0^2-2\left[\phi(z)+E\phi^2\right]-2Dv_0s, \tag{28}$$

where $D=\sqrt{\frac{m_{\rm i}}{2m_{\rm d}}}\left[\frac{8\sqrt{\pi}}{3}a^2\left(2+\gamma_0L\right)\right]n_{\rm i0}\lambda_{\rm Di}, E=1/2\left(1+\gamma_0\right)$. To derive eq. (28), the fact that $Z(\phi)\approx Z_0\left(1+E\phi\right)$ has been used. Further, the integration over $n_{\rm i}v_{\rm d}$ has been approximated by $n_{\rm i0}v_0$ in the vicinity of the plasma–sheath boundary. Integrating eq. (26), $n_{\rm d}v_{\rm d}=n_{\rm d0}v_{\rm d0}$ and using eq. (28), one gets

$$\left(\frac{n_{\rm d}}{n_{\rm d0}}\right)=\left[1+\frac{2\left(\eta-E\eta^2\right)}{v_0^2}-\frac{2Ds}{v_0}\right], \tag{29}$$

where $v_0=v_{\rm d0}/C_{\rm d}$ is the normalized velocity. Integrating (24), one may write the Poisson's equation as

$$\frac{{\rm d}\eta}{{\rm d}s^2}=X\eta-Ys, \tag{30}$$

where $X=Z_0(1+1/(1+\gamma))+Z_0/v_0^2+Z_0Ds/(v_0(1+\gamma))$ and $Y=Z_0D/v_0$. In the absence of Coulombic drag and collision, when $D=0$, the necessary condition for the sheath criterion will require $X>0$. At the Bohm velocity of the dust, $v_{\rm d0}=C_{\rm d}$, i.e., $v_0=1$, $X>0$ is always satisfied. Thus, at the plasma–sheath boundary, dust particle will move with the dust acoustic speed. The effect of the drag is important. The effect of collision on sheath formation is well-known [8,9]. The Bohm criterion $v_0=1$ is only a sufficient and not a necessary condition for the sheath formation [8]. In fact, numerical result of Valentini [9] suggests that the sheath may form in the whole discharge although close to the wall $v_0>1$ is satisfied. Thus, the sheath criterion for a collisional plasma is somewhat arbitrary. In a dusty plasma, one can arrive at similar conclusion. For $\gamma_0\sim O(1)$, depending on what value D takes, necessary condition of the existence of sheath may or may not exist. For example, if $D>2(1+1/(1+\gamma_0))$, the positive sheath near the wall may not exist. Thus, the collision of the dust by the plasma particles plays an equally important role in the evolution of the sheath.

In conclusion, Bohm criterion for a dusty plasma has been derived for two cases. In the presence of Boltzmannian electrons when the dust is immobile with its charge depending upon the plasma potential, the critical ion velocity at the plasma sheath boundary may become smaller than the Bohm velocity depending upon the strength of the Coulomb and collisional drag of the dust particles. Bohm's sheath criterion is valid in the absence of the drag. Once the sheath has formed, the dust particle will respond to such an arrangement at the dust-acoustic time. Over DAW scale, electrons and ions are thermalized. The dust particle will feel the effect of the ion drag. The effect of the drag is significant. In the presence of ion drag on the grain, the Bohm criteria become somewhat arbitrary.

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