PRAMANA — journal of physics

Vol. 65, No. 1
July 2005
pp. 85–93

Quasi-binary incident electron–centre of mass collision in (e, 3e) process on He and He-like ions

R CHOUBISA and K K SUD
Department of Physics, College of Science Campus, M.L.S. University, Udaipur 313 002, India
E-mail: kksud@yahoo.com

MS received 10 August 2004; revised 9 November 2004; accepted 12 November 2004

Abstract. We present in this communication the results of our first Born calculation in the three-Coulomb (3C) wave approach for the $(e, 3e)$ process on He and He-like ions at an incident electron energy 5599 eV in the coplanar constant θ_{12} as well as out-of-plane constant ϕ_{12} modes. These two geometrical modes are such that the quasi-binary collision between the incident electron and centre of mass of the ejected electrons is in the scattering plane. The theoretical formalism has been developed using plane waves, Le Sech wave function and approximated BBK-type wave function respectively for the incident and scattered, bound and ejected electrons to calculate five-fold differential cross-section (FDCS) of the $(e, 3e)$ process. We emphasize on the similarities and dissimilarities (asymmetries) in the angular profile of the FDCS in two modes as well as the effects of post-collision interaction (between the ejected electrons) and nuclear charge Z on the angular profile of the FDCS. We observe that with the increment of nuclear charge the two quasi-binary collisions approach towards identical behaviour at larger mutual angles and thus bringing less asymmetry in FDCS for higher Z target.

Keywords. $(e, 3e)$ process; three-Coulomb wave approach; five-fold differential cross-section; constant θ_{12} mode; constant ϕ_{12} mode.

PACS Nos 34.80.Dp; 34.50.Fa

1. Introduction

A pioneer experiment in the field of electron impact double ionization performed by Lahmam-Bennani *et al* [1] on Ar atom proved to be an important step to understand the complicated Coulomb four-body problem in which an incident electron on a target ejects two bound electrons and these two electrons as well as the scattered electron are detected coincidentally. Such type of coincident experiments are referred as a kinematically complete $(e, 3e)$ experiment in which information about the collision dynamics, target electron momentum density and the correlated motion of the electrons can be obtained by measuring the five-fold differential cross-section (FDCS), which is differential in solid angles of all the outgoing elec-

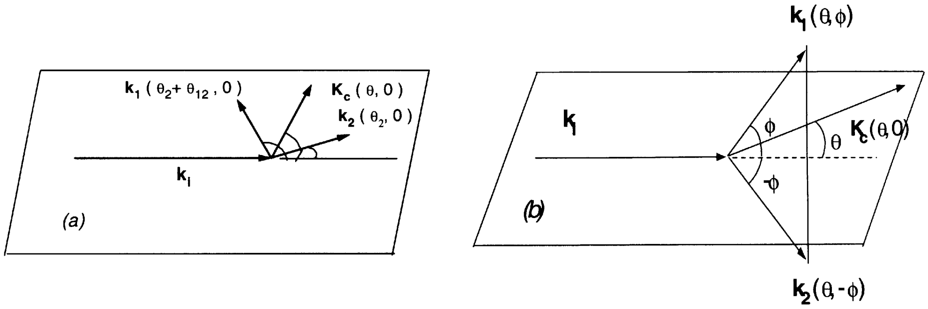

Figure 1. Schematic diagram of the $(e, 3e)$ process on He atom showing the geometrical arrangements. (**a**) Coplanar constant θ_{12} mode and (**b**) out--of-plane constant ϕ_{12} mode. The momenta of the incident electron, ejected electrons 1, 2 and centre of mass of the ejected electrons are depicted by the symbols $k_{\rm i}, \mathbf{k}_1, \mathbf{k}_2$ and $\mathbf{K}_{\rm c}$ respectively. The polar and azimuthal angles of the ejected electrons 1, 2 and the centre of mass of the ejected electrons are depicted in each frame in the small parentheses. The length of the arrows of the momentum vectors are not scaled with the magnitude of the vectors.

trons and energies of the two ejected electrons. Further, the study of the $(e, 3e)$ process gained a great momentum by the first $(e, 3e)$ experiment on He atom [2] as such type of experiment leads to a pure four-body problem consisting of three unbound electrons and He^{2+} ion, having no relevant internal structure, in the continuum state which can be solved more effectively and easily by the existing theoretical models (three-Coulomb wave function approach (3C and C4FS) or convergent close-coupling (CCC) approach) as compared to outer shell double ionization on more heavy inert atoms (Ne, Ar, Kr) in which the target as well as the residual ion has a complicated internal structure. Most of the existing investigations have been done at higher electron impact energies and low momentum transfer to utilize the fact that under the condition of low momentum transfer, $(e, 3e)$ process should converge to photo double ionization, i.e. $(\gamma, 2e)$ process, and as expected some similarities have been observed between the two processes [3–5]. However, in these investigations it was not clear what would be the analogue structure in the $(e, 3e)$ process corresponding to binary and recoil peaks in the $(e, 2e)$ process. A number of workers [6–9] have attempted to give this answer and showed that the centre of mass $\mathbf{K}_{\rm c} = \mathbf{k}_1 + \mathbf{k}_2$ of the pair of the ejected electrons, where $\mathbf{k}_1, \mathbf{k}_2$ are the momenta of the ejected electrons, acts as an 'effective single particle' during the collision and as in the $(e, 2e)$ process, binary and recoil peaks are observed in the FDCS versus $\mathbf{K}_{\rm c}$ direction curve in constant θ_{12} mode due to quasi-binary incident electron–centre of mass collision. Further, this type of distribution is found to be symmetric about the direction of the momentum transfer $\mathbf{K}$ for any first-order theory (see Choubisa *et al* [7]) and any deviation from the symmetry in the FDCS indicates the importance of higher-order projectile target interaction in the $(e, 3e)$ process (see Choubisa *et al* [8]).

Most of the investigations of the $(e, 3e)$ process on atoms have been done in coplanar geometry in which all particles are in the scattering plane defined by incident and scattered electrons. Ford *et al* [10] and Dorn *et al* [11] have measured

FDCS in out-of-scattering plane on Mg ($3s^2$) and He ($1s^2$) targets respectively by keeping the ejected electrons at $+45°$ up and $-45°$ down the scattering plane thus keeping the incident electron–centre of mass collision (i.e., quasi-binary collision) in the scattering plane. We depict in figure 1, the geometrical arrangements for the $(e, 3e)$ process on atom in the coplanar constant θ_{12} and out-of-plane constant ϕ_{12} geometrical modes and also various symbols used in the present communication. In the constant θ_{12} mode, the mutual angle between the ejected electrons is kept constant ($\theta_{12} = \theta_1 - \theta_2$ is constant) and the variation of FDCS is investigated as a function of $\theta(\theta = (\theta_1 + \theta_2)/2)$, i.e. the direction of centre of mass of the ejected electrons while in the constant ϕ_{12} mode, the electrons are ejected at equal but opposite azimuthal angles (i.e., $\phi_1 = -\phi_2$) and FDCS is investigated as a function of θ ($\theta_1 = \theta_2 = \theta$), i.e. with the direction of the momentum of the centre of mass of the ejected electrons in the scattering plane. We present in this paper a comparison of the angular profile of the FDCS in constant θ_{12} mode and constant ϕ_{12} mode in the first Born approximation using 3C wave function approach. The purpose of presenting FDCS in these geometrical modes is that in both cases the quasi-binary collision between the incident electron and centre of mass is in the scattering plane. However, the difference is in the ejection directions of the ejected electrons and thus the angular profiles of FDCS in these two modes for equal θ_{12} and ϕ_{12} angles provide an insight into the $(e, 3e)$ process on atom. We also investigate the effect of variation of the post-collision interaction (i.e., constant mutual angle) between the ejected electrons and momentum transfer as well as effect of nuclear charge Z on the angular profile of the FDCS in both modes to reveal some interesting investigation about the quasi-binary collision.

We calculate FDCS at high incident, lower excess ejected electron energies and at a small momentum transfer using our well-described first Born theoretical model in 3C wave approach [7–9]. The FDCS for the $(e, 3e)$ process on atom in the first Born approximation is given as

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}\Omega_\mathrm{s}\mathrm{d}\Omega_1\mathrm{d}\Omega_2\mathrm{d}E_1\mathrm{d}E_2} = (2\pi)^4 \frac{k_\mathrm{s}k_1k_2}{k_\mathrm{i}} |T_{fi}^{B1}|^2, \tag{1}$$

where $\mathrm{d}\Omega_\mathrm{s}, \mathrm{d}\Omega_1$ and $\mathrm{d}\Omega_2$ are solid angle elements of scattered, first and second ejected electrons respectively and $\mathrm{d}E_1, \mathrm{d}E_2$ are the energy band pass of the respective ejected electrons. The momenta of the incident and scattered electrons are depicted as k_i and k_s respectively. We describe the matrix element T_{fi}^{B1} in the following form:

$$\begin{aligned} T_{fi}^{B1} = &-\frac{1}{2\pi^2K^2} \\ &\times \left[\langle\psi_\mathrm{f}(\mathbf{r}_1, \mathbf{r}_2)| - Z + \exp(i\mathbf{K}\cdot\mathbf{r}_1) + \exp(i\mathbf{K}\cdot\mathbf{r}_2)|\psi_\mathrm{i}(\mathbf{r}_1, \mathbf{r}_2)\rangle\right], \end{aligned} \tag{2}$$

where Z is the atomic number of He-like ions. The wave function $\psi_\mathrm{f}(\mathbf{r}_1, \mathbf{r}_2)$ is the correlated approximate BBK-type wave function [12] for the ejected electrons and $\psi_\mathrm{i}(\mathbf{r}_1, \mathbf{r}_2)$ is the Le Sech-type wave function [13] for the bound electrons. For brevity, we do not describe the detailed description of the theoretical formalism here and refer the reader to our earlier papers [7–9] for more details.

2. Results and discussion

We present the results of our calculation of the FDCS in constant θ_{12} and constant ϕ_{12} modes by varying the angle of the centre of mass of the ejected electrons (θ) from 0° to 360° (here angles are measured counter-clockwise with respect to the direction of the incident electron). The result of the present calculation using the first Born approximation for He atom and He-like ions are presented in §§2.1 and 2.2 respectively. We depict the angular profile of the FDCS in constant θ_{12} mode by dashed curve and in constant ϕ_{12} mode by solid curve in all the figures.

2.1 *(e, 3e) process on He*

In the present kinematics, the scattering angle is kept fixed and so it fixes the direction of the momentum transfer. Further, the angle of momentum of the centre of mass is varied in such a manner that the escape angles $\theta_{1,2} = \theta \pm (\theta_{12}/2)$ are in coplanar constant θ_{12} mode whereas in the constant ϕ_{12} mode the polar angles are equal, i.e. $\theta_1 = \theta_2 = \theta$, and the azimuthal angles are equal but opposite, i.e. $\phi_1 = -\phi_2$. As expected, the angular profile of the FDCS in constant θ_{12} mode exhibits binary and recoil peak structure and is observed for constant mutual angles up to 120° and then changes in the binary peak region begins to appear in the angular profile of the FDCS for larger mutual angles (see dashed curve in figures 2d–2f). The formation of the binary peak may be attributed to quasi-binary incident electron–centre of mass collision whereas the recoil peak to the recoil of the ion (see Choubisa *et al* [7,8] and Lahmam-Bennani *et al* [6]). We also present the result of our calculation of FDCS in the constant ϕ_{12} mode by keeping same mutual angle between the ejected electrons as in the constant θ_{12} mode and plotting FDCS in both the geometrical modes with the angle θ (see solid and dashed curves of figure 2). The geometrical arrangement of constant ϕ_{12} mode is simply a rotation of 90° of the plane formed by $\mathbf{k}_1, \mathbf{k}_2$ and $\mathbf{K}_c$ from the coplanar constant θ_{12} mode about the direction of centre of mass $\mathbf{K}_c$ which brings again the direction of the momentum of the centre of mass in the scattering plane and hence as far as collision between the incident electron and centre of mass is concerned it again occurs in the scattering plane as in the constant θ_{12} mode. As expected, we also observe binary and recoil peak structure in the constant ϕ_{12} mode (see solid curve) as in the constant θ_{12} mode. Further, the angular profile of FDCS in both modes are almost similar for mutual angles up to 120° due to unchanged mutual angle between the ejected electrons and hence unchanged momentum $\mathbf{K}_c$ of the centre of mass in the rotation of the plane formed by $\mathbf{k}_1, \mathbf{k}_2$ and $\mathbf{K}_c$. However, for higher mutual angles the differences are clearly noticeable and are as follows:

(i) We observe that the binary peak splits and two new peaks are formed in the constant θ_{12} mode (see dashed curves in figures 2e and 2f). The location of the peaks is an indication of collinear Wannier escape of the ejected electrons (see Dorn *et al* [11]) as it corresponds to a situation in which one of the ejected electrons is near to the direction of momentum transfer ($\theta = \theta_K$) while the other one is ejected roughly opposite to it.

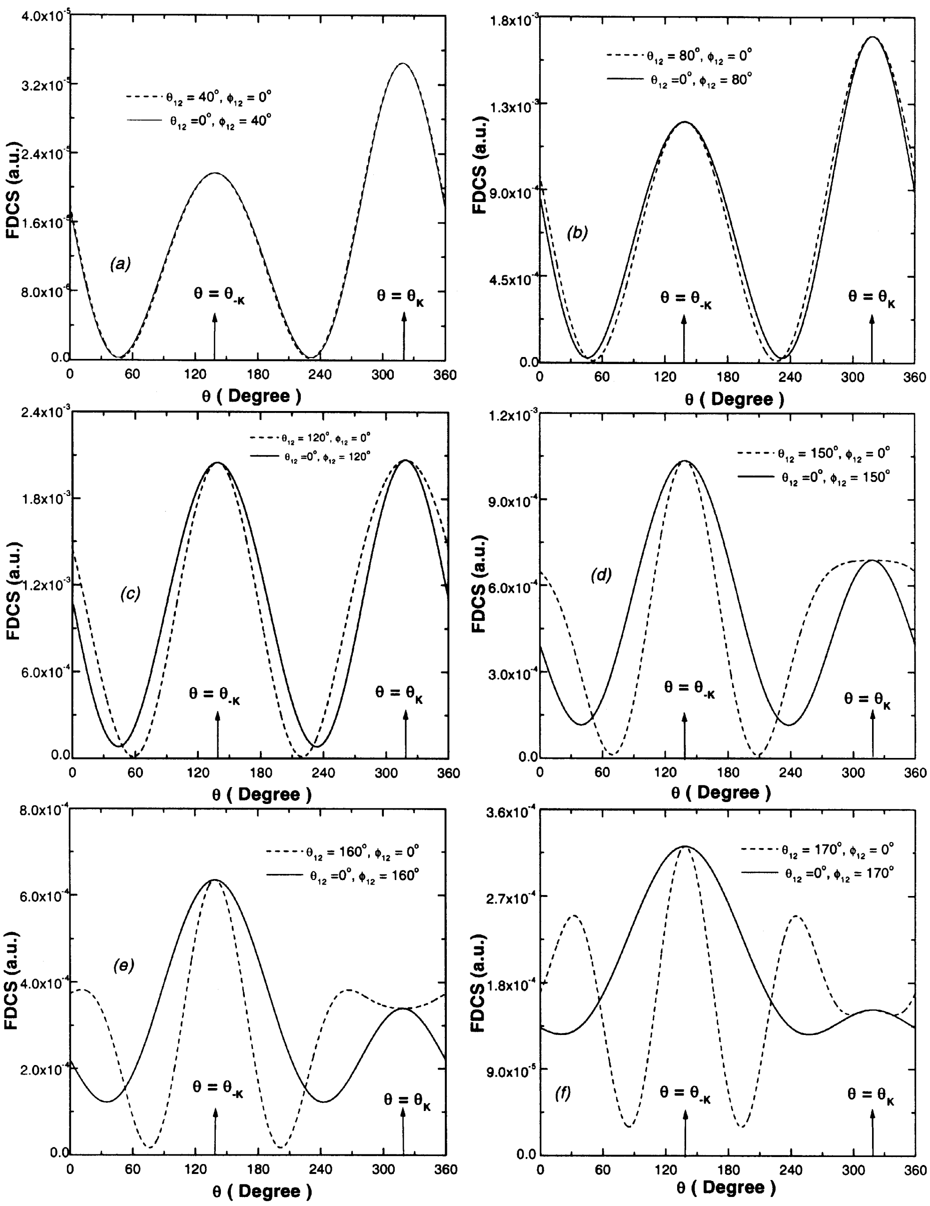

Figure 2. FDCS is plotted as a function of θ (i.e. the angle of momentum of the centre of mass of the ejected electrons) in the constant θ_{12} mode and out-of-plane constant ϕ_{12} mode. The angular profiles of FDCS in the constant θ_{12} mode and constant ϕ_{12} mode are depicted by dashed and solid curve respectively in each frame. The kinematical parameters are $E_{\mathrm{i}} = 5599$ eV, $E_{\mathrm{s}} = 5500$ eV, $E_1 = E_2 = 10$ eV and $\theta_{\mathrm{s}} = 0.45°$. The values of θ_{12} and ϕ_{12} for the two geometrical modes are presented in each frame. The arrows at $\theta = \theta_{\mathrm{K}}$ and $\theta = \theta_{-\mathrm{K}}$ indicate the direction of momentum transfer and the opposite direction of it respectively.

(ii) The splitting in the binary peak is not observed in the constant ϕ_{12} mode (see solid curve in figures 2e and 2f). We further observe that the recoil peak region is broadened while the binary peak is contracted in constant ϕ_{12} mode because the width of half maximum is larger in the recoil region while it is smaller in the binary peak region (see solid curve of figure 2 and compare it with the dashed curve in the corresponding frame). The broadness of the recoil peak and narrowness of the binary peak reveal that more effective role is played by the ion and a lesser role by free quasi-binary collision of projectile and the centre of mass at higher mutual angles in the out-of-plane ejection of the ejected electrons.

2.2 *(e, 3e) process on He-like ions*

In the previous section, we have investigated the effect of in- and out-of-plane ejection of electrons on the quasi-binary collision between the incident electron and centre of mass on He atom. In this section, we present the results of our investigations of the effect of atomic number Z as well as geometrical effects on the quasi-binary collision in the $(e, 3e)$ process on He-like ions and in particular for H^- to Be^{2+} targets. We have also performed calculation for He and Li^+ targets but for the sake of brevity we have not presented our results here. It may be mentioned that very few studies of $(e, 3e)$ process on He-like ions are reported in the literature [9,14–16]. Further, these studies have been limited to either in θ-variable mode or constant θ_{12} mode whereas the present calculation is in the constant ϕ_{12} mode. Furthermore, comparison with the FDCS in the constant θ_{12} and constant ϕ_{12} modes has not been reported earlier in the literature and so we hope that the results of the present study on He-like ions will provide insight on the quasi-binary collision in $(e, 3e)$ process on He-like ions. We present the results of FDCS in the first Born approximation in the constant θ_{12} mode (dashed curves) and constant ϕ_{12} mode (solid curves) for H^- in the frames (a), (c), (e) and (g) (given in the left frames) as well as for Be^{2+} in frames (b), (d), (f) and (h) (given in the right frames) of figure 3. The calculation has been performed at an incident energy 5599 eV and at momentum transfer 0.75 a.u. for both the targets. The results are summarized here:

- The FDCS evaluated for any mutual ejection angle on He-like ions in both the geometrical modes is the same when the centre of mass of the ejected electrons is either along the direction of momentum transfer or opposite to it (see, arrow at $\theta = \theta_K$ and $\theta = \theta_{-K}$). It shows that within the first Born approximation, ejection of the centre of mass of the ejected electrons along the axis of the momentum transfer is equally probable in both geometrical modes regardless of the ejection directions of the ejected electrons and hence reveals an important symmetrical criterion for the first Born calculation. However, the second-order projectile target interaction is expected to break the symmetry present in the FDCS in the first Born calculation.
- For H^- target (the lightest target), we observe that asymmetry in FDCS starts at very low mutual angle $\theta_{12} = \phi_{12} = 80°$ (see figures 3a, 3c, 3e and 3g) whereas for Be^{2+} (i.e., the heaviest target considered here), asymmetry in

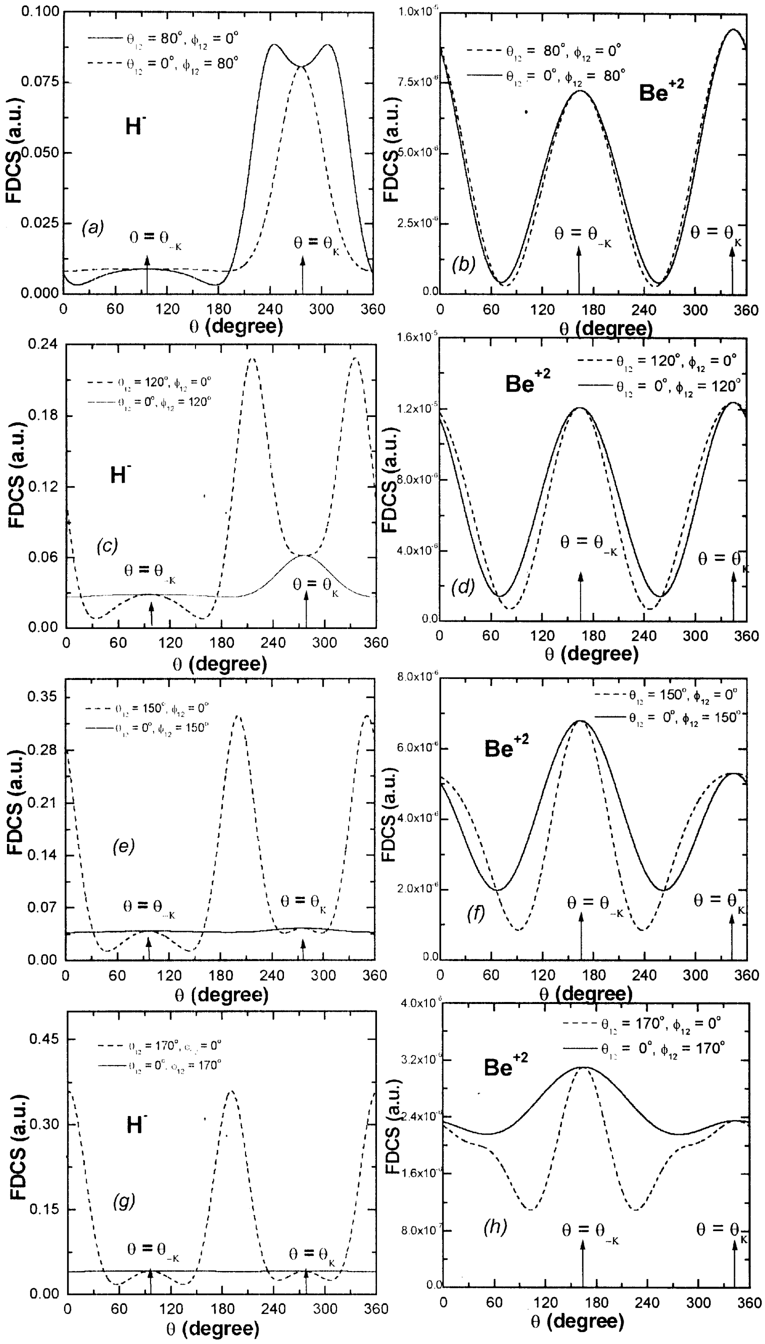

Figure 3. Same as figure 2 but for H^- (frames (a), (c), (e), (g)) and Be^{2+} (frames (b), (d), (f), (h)) targets and for kinematics: $E_\mathrm{i} = 5599$ eV, $E_1 = E_2 = 10$ eV and $K = 0.75$ a.u.

FDCS is almost negligible up to mutual angle $\theta_{12} = \phi_{12} = 120°$ (see figures 3b, 3d, 3f and 3h). This shows that with the increment of nuclear charge the two quasi-binary collisions approach towards identical behaviour to larger mutual angles and thus bringing less asymmetry in FDCS for higher Z target.

- At higher mutual angle (i.e., $\theta_{12} = \phi_{12} = 150°, 160°$ and $170°$), with the increment of Z it is observed that the isotropic behaviour of FDCS in the constant ϕ_{12} mode at higher mutual angle decreases (compare the solid curves for H^- and Be^{2+} of figure 3). For example, if we choose $\phi_{12} = \theta_{12} = 160°$ and observe the trend of FDCS in the constant ϕ_{12} mode for H^-, we have almost a parallel line to the x-axis showing that the angular profile of FDCS is highly isotropic whereas for Be^{2+} the angular profile of FDCS in constant ϕ_{12} mode varies more sharply with θ for $\phi_{12} = \theta_{12} = 160°$.
- Splitting in the binary peak is observed at a low mutual angle $\theta_{12} = 80°$ for H^- target while with the increment of Z it gets weakened (see the split peaks near $\theta = \theta_K$ in the left frames of figure 3). Furthermore, it is observed to nearly disappear for Be^{2+} target (see the right frames of figure 3). This brings an important conclusion that the splitting of the binary peak, which is due to Wannier escape, is very sensitive to nuclear charge Z. As Z increases, the bound electrons locate more closely to the nucleus thus giving less possibility of Wannier-like collision for the ejected electrons.
- For H^- target, the asymmetry in FDCS in both the geometrical modes is found to be largest, which indicates that the out-of-plane ejection of the ejected electrons is much less probable for H^-. With the increment of Z, the asymmetry in FDCS in both the modes gets weakened as the out-of-plane ejection dominates with the increment of Z (see solid and dashed curves of figure 3).
- It will be an interesting task to include the second-order effect in the present first Born calculation for He-like ions to investigate its effect on the asymmetry of FDCS. The calculation in second Born approximation is in progress and will be reported elsewhere.

3. Conclusions

In conclusion, we have reported the results of our investigation of the two possible geometrical modes in the $(e, 3e)$ process on He and He-like ions in which the projectile-centre of mass of ejected electrons is in the scattering plane and the ejected electrons are either in or out of the scattering plane. In both the modes, the angular profiles of the FDCS with the variation of the angle of the momentum of the centre of mass of the ejected electrons exhibit similarity at lower mutual angles but the differences are observed for higher mutual angles due to splitting in binary peak in the constant θ_{12} mode resulting from Wannier escape of the ejected electrons. Furthermore, with the increment of Z of He-like ion targets, the two quasi-binary collisions approach towards identical behaviour at larger mutual angles and thus bringing less asymmetry in FDCS for higher Z target. We suggest more experimentation in different geometrical modes to understand better $(e, 3e)$ process on He and He-like ions.

Acknowledgements

RC acknowledges the Council of Scientific and Industrial Research (CSIR), New Delhi for senior research fellowship. The support from the special assistance programme of the University Grants Commission (UGC), New Delhi to the Department of Physics is also gratefully acknowledged.

References

[1] A Lahmam-Bennani, C Dupre and A Duguet, *Phys. Rev. Lett.* **63**, 1582 (1989)
[2] I Taouil, A Lahmam-Bennani, A Duguet and L Avaldi, *Phys. Rev. Lett.* **81**, 4600 (1998)
[3] A Lahmam-Bennani, I Taouil, A Duguet, M Lecas, L Avaldi and J Berakdar, *Phys. Rev.* **A59**, 3548 (1999)
[4] A Kheifets, I Bray, A Lahmam-Bennani, A Duguet and I Taouil, *J. Phys.* **B32**, 5047 (1999)
[5] A Dorn, R Moshammer, C D Schröter, T J M Zouros, W Schmitt, H Kollmus, R Mann and J Ullrich, *Phys. Rev. Lett.* **82**, 2496 (1999)
[6] A Lahmam-Bennani, C C Jia, A Duguet and L Avaldi, *J. Phys.* **B35**, L215 (2002)
[7] R Choubisa, A S Bhullar and K K Sud, *Pramana – J. Phys.* **60**, 1187 (2003)
[8] R Choubisa, G Purohit and K K Sud, *J. Phys.* **B36**, 1731 (2003)
[9] G Purohit, R Choubisa, D K Sharma and K K Sud, *Indian J. Phys.* **78(10)**, 1067 (2004)
[10] M J Ford, J P Doering, J H Moore and M A Coplan, *Rev. Sci. Instrum.* **66**, 3137 (1995)
[11] A Dorn, A Kheifets, C D Schröter, B Najjari, C Hohr, R Moshammer and J Ullrich, *Phys. Rev.* **A65**, 032709 (2002)
[12] M Brauner, J S Briggs and H Klar, *J. Phys.* **B22**, 2265 (1989)
[13] C Le Sech, *J. Phys.* **B30**, L47 (1997)
[14] K Muktavat and M K Srivastava, *J. Phys.* **B34**, 2975 (2001)
[15] P Lamy, C Dal Cappello, B Joulakain and C Le Sech, *J. Phys.* **B27**, 3359 (1994)
[16] B Nath and C Sinha, *J. Phys.* **B33**, 5525 (2000)