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Models of plastic depinning of driven disordered systems

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Abstract. Two classes of models of driven disordered systems that exhibit history-dependent dynamics are discussed. The first class incorporates local inertia in the dynamics via nonmonotonic stress transfer between adjacent degrees of freedom. The second class allows for proliferation of topological defects due to the interplay of strong disorder and drive. In mean field theory both models exhibit a tricritical point as a function of disorder strength. At weak disorder depinning is continuous and the sliding state is unique. At strong disorder depinning is discontinuous and hysteretic.

Keywords. Collective transport; depinning; disorder; plasticity.

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1. Introduction

Nonequilibrium depinning transitions from static to moving states underlie the physics of a wide range of phenomena, from fracture propagation in heterogeneous solids to flux flow in type-II superconductors [1]. One class of models, overdamped elastic media pulled by a uniform force, has been studied extensively. These exhibit a nonequilibrium phase transition from a pinned to a sliding state at a critical value F_T of the driving force. This nonequilibrium transition displays universal critical behavior as in equilibrium continuous transitions, with the medium's mean velocity v acting as an order parameter [2,3]. In overdamped elastic media, the sliding state is unique and no hysteresis can occur [4].

The elastic medium model is often inadequate to describe real physical systems that exhibit plastic response on various scales. Plasticity, used here in a loose sense, may arise in systems with an underlying periodic structure, such as vortex lattices or charge density waves, for strong disorder. This yields large deformations of the driven medium with the proliferation of topological defects, which are continuously generated and healed by the interplay of drive, disorder and interactions [5–7]. In other situations, such as crack propagation in heterogeneous solids [8], and the motion of a helium-4 droplet contact line on a rough surface [9], the dissipative elastic medium model is made inadequate by the presence of inertial effects or other nonlocal stress propagation mechanisms. In both cases the depinning transition may become discontinuous and sometime hysteretic.

Several coarse-grained models of driven extended systems that can lead to history-dependent dynamics have been proposed in the literature [1,10–13]. Here we examine two classes of such models. In the first class of models the displacement of the driven medium from some undeformed reference configuration remains single-valued, as appropriate for systems without topological defects, but the elastic interactions are modified by assuming local underdamped dynamics. This is modeled via a linear stress–strain relation, where the stress transfer between displacements of different parts of the manifold is nonmonotonic in time. Models of this type have been used to incorporate the effect of inertia or elastic stress overshoot in crack propagation in solids [13,14]. A related model was proposed by us [12,15,16] as an effective description of topological defects in the driven system. Below we show that the presence of topological defects in a solid can be described as a viscous force that allows a moving portion of the medium to overshoot a static configuration before relaxing back to it, so that these two models are identical. In the second class of models topological defects are explicitly allowed by removing the constraint that displacements be single-valued [10,17]. We consider a simple realization of this type of models proposed some time ago in the context of charge density waves, where the scalar displacement describes the phase of the electronic condensate and the system only exhibits one-dimensional periodicity along the direction of motion. It is obtained by assuming a nonlinear coupling among neighboring displacements that incorporates the crucial feature that the displacement becomes undefined at the location where the amplitude of the order parameter collapses. This is incorporated in the model as an instantaneous jump of the displacement or phase of an amount equal to its period, known as phase slip.

The two classes of models exhibit remarkably similar behavior in the mean-field limit, where the interaction are assumed to have infinite range. As a function of disorder strength, they both have a tricritical point separating continuous depinning transitions from discontinuous depinning transitions. Depinning is continuous at weak disorder with mean-field critical exponents that are identical to those obtained for a dissipative elastic medium. Above a critical disordered strength, depinning becomes discontinuous and hysteretic. The origin of the hysteresis is, however, different in the two models, as discussed below. Preliminary numerical work also suggests that differences may arise in finite dimensions.

2. Two classes of models: Viscous and phase slip couplings

We restrict ourselves to the dynamics of a scalar field describing deformations along the direction of mean motion and discretize space, denoting by $u_i(t)$ the displacement of the i-degree of freedom from some undeformed reference configuration, at time t. We stress that all models discussed are coarse-grained, with u_i representing the displacement of a region pinned collectively by disorder. The microscopic dynamics is always assumed to be overdamped, but velocity-dependent couplings can arise upon coarse-graining. The equation of motion in the laboratory frame is written by balancing all the forces acting on each segment of the driven medium as [18]

$$\partial_t u_i = \sigma^\alpha(\{u_i\}, t) + F + f_\mathrm{p}(i, u_i) \ , \tag{1}$$

where time has been scaled so that the friction coefficient is unity, F is the external driving force, f_p is the pinning force, and σ^α represents the stress due to interactions with the neighbors. The label α will be used below to identify two ways of modeling the coupling. For driven periodic manifolds, such as vortex lattices or charge density waves, the pinning force is periodic in the displacement u_i, with $0 \le u_i \le 1$, and can be written as $f_\mathrm{p}(i, u_i) = h_i Y(u_i - \gamma_i)$, where $Y(u)$ is a periodic function specified below, h_i are random forces with distribution $\rho_i(h)$, and γ_i are random phases chosen independently and uniformly in $[0, 1)$.

First, we consider a class of models with single-valued displacements u_i and a linear stress–strain relation of the general form,

$$\sigma^\alpha(\{u_i\}, t) = \int_{-\infty}^{t} \mathrm{d}t' \ \frac{1}{Z} \sum_{\langle j \rangle} J_{ij}^\alpha(t - t') \big[u_j(t') - u_i(t')\big] \ , \tag{2}$$

where the sum is over the Z sites j that are nearest neighbors of i. A suitable choice of the stress-transfer or memory function, $J_{ij}^\alpha(t)$, yields various models described in the literature. The familiar elastic model is obtained by assuming instantaneous stress transfer, i.e., $J_{ij}^\mathrm{E}(t) = K\delta(t)$, with K an elastic constant. More generically, all monotonic models, defined as those with $J_{ij}(t) \ge 0$, for all i, j, t, exhibit a continuous depinning transition with universal critical behavior and a unique sliding state [1]. The choice $J_{ij}^\mathrm{V} = \eta\partial_t\delta(t)$ yields purely viscous stresses, i.e., $\sigma^\mathrm{V} = \eta \sum_{\langle j \rangle} \big[\dot{u}_j(t) - \dot{u}_i(t)\big]$, provided we identify $v_i = \dot{u}_i \equiv \partial_t u_i$ with the local flow velocity of the medium. In this case depinning always occurs at $F = 0$ for pinning force distributions without a finite lower bound, but there is a critical point above which the system can switch discontinuously and hysteretically between a macroscopic slow moving state and a fast moving state [12,15]. We recently considered models where the stress transfer function is taken to have the form appropriate for a viscoelastic fluid, that responds elastically on short time-scales and flows viscously at long times [12]. This viscoelastic coupling was proposed as an effective way of describing the local slip due to dislocations generated at the boundaries between coarse-grained degrees of freedom. The connection between the viscoelastic model and the presence of free dislocations in the medium was made more precise in ref. [19] where it was shown that the equations describing the dynamics of equilibrium deformations of a two-dimensional lattice with a finite concentration n_d of free annealed dislocations can be recast in the form of the phenomenological equations of a viscoelastic fluid introduced many years ago by Maxwell [20]. In a scalar version of the equations of viscoelasticity, local compressional stresses are written in the form given in eq. (2), with

$$J_{ij}^\mathrm{VE}(t) = K_\infty\delta(t) - \frac{K_\infty - K_0}{\tau} \mathrm{e}^{-t/\tau} \ , \tag{3}$$

where K_∞ and K_0 are the high and low frequency compressional moduli, respectively, and $\tau \sim (n_d)^{-1}$ is the microscopic relaxation time [19]. Shear stresses have a similar form, with shear moduli replacing compressional ones and $G_0 = 0$ as a fluid has no zero-frequency elastic restoring forces in response to shear stresses. The nonzero long-wavelength compressional elasticity ($K_0 \neq 0$) is associated with density conservation and plays a crucial role in controlling the physics of depinning in driven lattices. For this reason in our scalar model we assume a coupling of the form (3) in all directions. On time-scales t short compared to τ, the contribution to the stress coming from the second term in eq. (3) is negligible compared to the first term and σ^{VE} reduces to the stress of an elastic solid, given by

$$\sigma^{\mathrm{VE}}(\{u_i\}, t \ll \tau) \approx \frac{K_\infty}{Z} \sum_{\langle j \rangle} [u_j(t) - u_i(t)] \; . \tag{4}$$

Conversely, for $t \gg \tau$, one can expand the relative displacements for $t' \sim t$, and

$$\sigma^{\mathrm{VE}}(\{u_i\}, t \gg \tau) \approx \frac{K}{Z} \sum_{\langle j \rangle} [u_j(t) - u_i(t)] + \frac{\eta}{Z} \sum_{\langle j \rangle} [\dot{u}_j(t) - \dot{u}_i(t)] \; , \tag{5}$$

with $K = K_0$ and $\eta = (K_\infty - K_0)\tau$. The model of driven depinning studied below is obtained from eq. (1) with the simplified form given in eq. (5) for the stress–strain relation

$$\partial_t u_i = \frac{K}{Z} \sum_{\langle j \rangle} [u_j(t) - u_i(t)] + \frac{\eta}{Z} \sum_{\langle j \rangle} [\dot{u}_j(t) - \dot{u}_i(t)] + F + f_{\mathrm{p}}(i, u_i) \; , \tag{6}$$

and will be referred to as viscous/elastic model (VE). The presence of dislocations generated by disorder is incorporated in a mean-field-type approximation as a local inertial response of the driven system embodied by the viscous coupling of strength $\eta \sim 1/n_d$. This model assumes a fixed number of topological defects and does not describe the creation and annihilation of dislocations due to the interplay of drive, disorder and interactions. Furthermore, in a driven disordered solid unbound, dislocations can be pinned by disorder and do not equilibrate with the lattice. The resulting dynamics cannot be described as a locally underdamped response of the systems. The model given in eq. (6) may be used to describe some of the effects of topological defects near depinning, but becomes inapplicable at large driving forces where dislocations recombine as the lattice reorders.

Before studying the dynamics of the VE model, it is useful to discuss its relationship to other models studied in the literature. In particular, the form of the stress transfer function given in eq. (5) is the same as that used recently by Fisher and Schwarz to incorporate the effect of stress overshoot on propagation of cracks in heterogeneous solids [13,14], although the random pinning force considered there is not periodic. These authors consider an automaton model where time is discrete. It is straightforward, to define an automaton version of our VE model, where both the displacement u_i and time are discrete. The displacement takes integer values and the automaton is updated according to the rule

$$\begin{aligned} u_i(t+1) &= u_i + \Theta(F_i(u_i)) \; , \\ v_i(t+1) &= \Theta(F_i(u_i)) \; , \end{aligned} \tag{7}$$

where v_i can have values 0 and 1 and $F_i(u_i)$ is the total force at site i given by

$$F_i(u_i) = \frac{K}{Z}\sum_{\langle j\rangle}\left[u_j(t) - u_i(t)\right] + \frac{\eta}{Z(1+\eta)}\sum_{\langle j\rangle} v_j(t) + F + \tilde{f}_{\rm p}(i)\ . \qquad (8)$$

The pinning force $\tilde{f}_{\rm p}(i)$ becomes a random number chosen uniformly from an interval $[0, h_0]$ [21]. The automaton can be obtained in the limit of very deep periodic pinning wells, when the dynamics is dominated by discrete events corresponding to jumps of the displacement from one well to the next. As long as $v \ll 1$, where 1 is the artificial upper limit of the mean velocity in the discrete model, the automaton dynamics mimics the continuous time dynamics reasonably well. The automaton version of the viscoelastic model given in eqs (7) and (8) is identical to its dynamics of the model of crack propagation with stress overshoot studied by Schwarz and Fisher, provided the strength M of the stress overshoot is identified with the combination $\eta/(1+\eta)$. The two models differ in the type of pinning considered as the random force $f_{\rm p}(i, u_i)$ used in refs [13,14] which is not periodic. By establishing the connection between these two models we have shown that distinct physical mechanisms (inertia, nonlocal stress propagation, unbound topological defects) play a role in different physical systems that can be described generically by a coarse-grained model that includes a coupling to local velocities of the driven manifold.

In the second class of models topological defects are explicitly allowed by removing the constraint of single-valued displacements. At a strong pinning center, deformations of the driven medium can be large and lead to the accumulation of a large strain. When the distortion is released through a collapse of the amplitude of the order parameter (i.e., the creation of a topological defect), the displacement abruptly advances of an amount of order 1, while the amplitude quickly regenerates. This process is known as phase slippage in superconductors and superfluids. On time-scales large compared to those of the microscopic dynamics, it can be described approximately as a 'phase slip': an instantaneous (modulo 1) hop of the displacement of an integer unit, modeled as a coupling periodic in the difference in displacements between neighboring degrees of freedom

$$\sigma^{\rm PS}(\{u_i\}, t) = \frac{K}{Z}\sum_{\langle j\rangle}\sin 2\pi\left[u_j(t) - u_i(t)\right]\ . \qquad (9)$$

If the relative displacements are small, this reduces to an elastic coupling of strength K. This model has been studied before in the mean-field limit by Strogatz and collaborators for a sinusoidal pinning force [10]. In this case depinning is always hysteretic in mean field. Very recently we were able to solve the model in mean field for arbitrary pinning potential and show that more generically both continuous and discontinuous depinning transitions are obtained as the parameter K is varied [17].

Below we compare the behavior of these two classes of models: models where the displacement remains single valued and deviations from elasticity are introduced via local inertial couplings and models where the displacement ceases to be single valued and topological defects can be generated for strong disorder.

3. Mean-field solution

The mean-field approximation for the VE model is obtained in the limit of infinite-range elastic and viscous interactions. Each displacement then couples to all others only through the mean velocity, $v = N^{-1}\sum_i \dot{u}_i$, and the mean displacement, $\overline{u} = N^{-1}\sum_i u_i$. We look for solutions moving with stationary velocity, so that $\overline{u} = vt$. Since all displacements u_i are coupled, they can now be indexed by their disorder parameters γ and h, rather than the spatial index i. The mean-field dynamics is governed by the equation

$$(1+\eta)\dot{u}(h,\gamma) = K\big(vt - u\big) + F + \eta v + f_{\rm p}(u; h, \gamma). \tag{10}$$

It is useful to first review the case $\eta = 0$, where eq. (10) reduces to the mean-field theory of a driven elastic medium [3]. The mean-field velocity is determined by the self-consistency condition $\langle u(h,\gamma) - vt\rangle_{h,\gamma} = 0$, where the subscripts h, γ indicate averages over the distribution of pinning strengths, $\rho(h)$, and over the uniformly-distributed phases, γ. For piecewise harmonic pinning, $Y(u) = 1/2 - u$, for $0 \le u \le 1$, no moving solution exists for $F < F_T = \left\langle \frac{h^2}{2(K+h)} \right\rangle_h$. Just above threshold the mean velocity has a universal dependence on the driving force, with $v \sim (F - F_T)^{\beta}$. In mean field the critical exponent β depends on the shape of the pinning force: $\beta = 1$ for the discontinuous piecewise harmonic force and $\beta = 3/2$ for generic smooth forces. Using a functional RG expansion in $4-\epsilon$ dimensions, Narayan and Fisher [3] showed that the discontinuous force captures a crucial intrinsic discontinuity of the large scale, low-frequency dynamics, giving the general result $\beta = 1 - \epsilon/6 + \mathcal{O}(\epsilon^2)$, in reasonable agreement with numerical simulations in two and three dimensions [22,23]. For simplicity and to reflect the 'jerkiness' of the motion in finite-dimensional systems at low velocities, we use piecewise harmonic pinning.

When $\eta > 0$, the nature of the depinning differs qualitatively from the $\eta = 0$ case in that both unique and multi-valued solutions can exist depending on the values of the parameters. The solution for general η can be found from that for $\eta = 0$ by substituting the effective driving force $G = F + \eta v$ for F in the $v_{\eta=0}(F)$ relation and scaling the velocity down by $1+\eta$. The linear transformation $F = G - \eta v$ then gives the general $v(F)$ curve. The mean velocity in the sliding state is given by the solution

$$F - F_T = v\big[1 - M(\eta, K)\big] + \left\langle \frac{h^2}{K(K+h)[\mathrm{e}^{(K+h)/(1+\eta)v} - 1]} \right\rangle_h , \tag{11}$$

where $M(\eta, K) = (1+\eta)\left\langle \frac{h^2}{(K+h)^2} \right\rangle_h$. For $M(\eta, K) < 1$ there is a unique sliding solution with mean velocity near threshold given by

$$v \sim \frac{(F - F_T)^{\beta}}{1 - M(\eta, K)} , \tag{12}$$

with $\beta = 1$. The condition $M(\eta, K) = 1$ determines a critical line separating unique from multi-valued solutions $v(F)$. The phase diagram in the (F, h_0) plane is shown in figure 1 for $\rho(h) = \delta(h - h_0)$ (provided $K \neq 0$, the topology of the

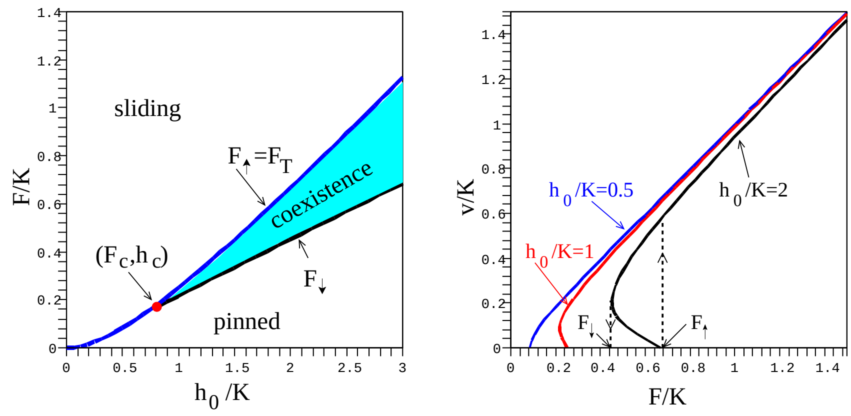

Figure 1. Mean-field solution of the VE model with a piecewise parabolic pinning potential, $\rho(h) = \delta(h - h_0)$ and $\eta = 5$. Left frame: phase diagram. Here 'coexistence' refers to multistability of the solutions to the equations of motion. Right frame: velocity versus drive for $h_0/K = 0.5$ (blue), $h_0/K = 1$ (red) and $h_0/K = 2$ (black). Also shown for $h_0/K = 2$ are the discontinuous hysteretic jumps of the velocity obtained when F is ramped up and down adiabatically.

phase diagram does not depend on the form of $\rho(h)$). There is a tricritical point at $(h_c, F_c = F_T)$, with $h_c = K/(\sqrt{1+\eta} - 1)$. For $h_0 < h_c$, a continuous depinning transition at F_T separates a stable pinned state [24] from a sliding state with unique velocity given by eq. (12). In finite dimensions, we expect this transition to remain in the same universality class as the depinning of an elastic medium ($\eta = 0$). This is corroborated by numerical studies and analysis by Schwarz and Fisher [14] of the related stress overshoot model (but with non-periodic pinning). For $h_0 > h_c$ the $v(F)$ curves are multivalued, which leads to hysteresis when F is ramped up and down adiabatically. The hysteresis is easily understood as a consequence of the global nature of the viscous coupling in mean field, where the driving force is replaced by an effective drive $F + \eta v$. Clearly this does not affect the static state where $v = 0$, so that upon ramping up the driving force from the pinned state the system always depins at $F_\uparrow = F_T$. When the force is ramped down from the sliding state where $v \neq 0$, the system sees a larger effective drive $F + \eta v$ and repins at the lower value $F_\downarrow$. It is important to appreciate the crucial role of a finite value of K in eq. (6). When $K = 0$ each degree of freedom has its own velocity v_i and for any broad $\rho(h)$ there are always some degrees of freedom that experience zero pinning force, yielding no stable pinned phase at any $F > 0$ [12]. For finite long-time elasticity, i.e., when $K \neq 0$, the elastic forces or particle conservation enforce a uniform time-averaged velocity for all degrees of freedom and one obtains a stable pinned phase for $F < F_T$. Finally, we note that the VE model is also closely related to a model of sliding CDWs that incorporates the coupling of the CDW to normal carriers by adding a global velocity coupling [11,25] to the Fukuyama–Lee–Rice model.

We now turn to the phase slip model, where the stress is given by eq. (9). In mean-field theory the nonequilibrium state can be described in terms of two order parameters. The first is the coherence of the phases, measured by the amplitude r of a complex order parameter defined by

$$re^{i\psi} = \frac{1}{N}\sum_{i=1}^{N} e^{i2\pi u_i} \ , \tag{13}$$

with ψ a mean phase. In the absence of interactions among the phases or external drive, u_i are locked to the random phases γ_i and the state is incoherent, with $r = 0$. In the opposite limit of very strong interactions we expect perfect coherence of the static state, with all u_i becoming equal to the mean phase and $r \to 1$. The second-order parameter is the average velocity of the system, given by

$$v = \frac{1}{N}\sum_{i=1}^{N} \dot{u}_i(t). \tag{14}$$

The mean velocity is the order parameter for the transition between static and moving phases. In the sliding state $\psi = vt$. The mean-field equation of motion is

$$\dot{u}(h,\gamma) = F - Kr\sin(2\pi u - \psi) + hY(u-\gamma) \ , \tag{15}$$

where the effective coupling is proportional to the coherence r. The self-consistency condition for the mean-field theory is given by

$$re^{i\psi} = \int_0^1 d\gamma \int dh\rho(h)e^{i2\pi u(h,\gamma)} \ . \tag{16}$$

In [17] we developed a general method for solving the mean-field equations (15) and (16) for arbitrary pinning potential. The resulting phase diagram for a piece-wise parabolic pinning potential is shown in figure 2. We see immediately that this is richer than the phase diagram for the VE model, as now both disorder-driven transitions between static coherent and incoherent phases as well as force-driven transitions between static and moving phases can occur. Although, as shown in [17], the detailed shape of the phase boundaries depends on the specific form of the pinning potential in mean field, the types of phases and the schematic topology of the phase diagram are general. At low driving forces both coherent (CS) and incoherent (IS) static phases are stable. The transition from the coherent phase at weak disorder to the incoherent phase at strong disorder is discontinuous for piece-wise parabolic pinning. It also becomes hysteretic for most other pinning potentials [17]. When the driving force is ramped up adiabatically from the coherent state, the system depins continuously at $F_\uparrow^c$ to a unique sliding state and the coherence grows smoothly towards unity. When the force is ramped up from the incoherent static state, where $r = 0$, the system depins discontinuously at $F_\uparrow^i = F_{\rm sp}$, with $F_{\rm sp} = h_0$ the single particle threshold, given by the maximum pinning force. This is easily understood from eq. (15) as when $r = 0$ the displacements can remain decoupled even for finite values of K. At $F_\uparrow^i$ the coherence also jumps discontinuously to a finite value, as shown in figure 3. When the force is ramped back down from the

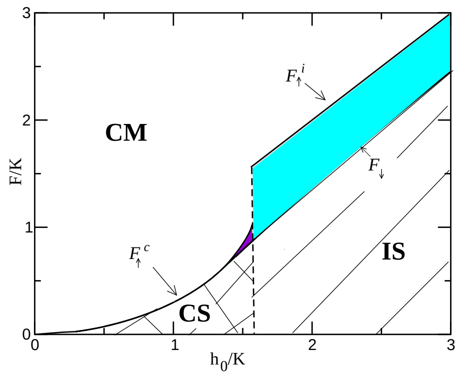

Figure 2. Mean-field phase diagram for the PS model with a piecewise parabolic pinning potential and $\rho(h) = \delta(h - h_0)$. In the cyan region both stable CM and IS solutions exist. In the small purple region near the tricritical point hysteretic depinning from the CS state is obtained numerically. The discontinuous increase of the depinning threshold at the critical point is a peculiarity of the piecewise pinning potential and is replaced by a sharp, but continuous rise for other pinning potentials.

sliding state, the system repins at the lower force $F_\downarrow$, where the coherence also jumps back to zero. The moving state is always coherent in mean field, although incoherent sliding states should be possible in finite dimensions [26]. The origin of the mean-field hysteresis observed when the system depins from a static incoherent state is easily understood. When the force is ramped up each degree of freedom depins essentially independently. Once the medium starts sliding, disorder becomes

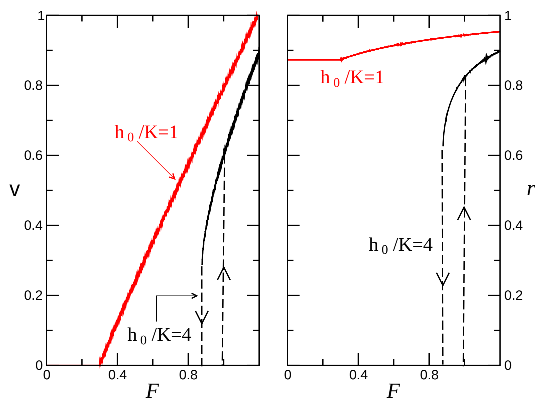

Figure 3. Coherence (right) and velocity (left) as functions of drive for $h_0/K = 1$ (red curves, showing the CS $\rightarrow$ CM continuous depinning) and $h_0/K = 4$ (black curves, showing the IS $\rightarrow$ CM hysteretic depinning). The curves are obtained by numerical integration of eq. (15) as F is ramped up and then down.

less important and the system becomes coherent and therefore much stiffer. Such a system cannot easily adjust to disorder and therefore when the force is decreased, it remains in a sliding state down to a lower force. This type of hysteresis is clearly strongly enhanced in mean-field, where only states with zero or perfect coherence are possible. Hysteresis has been observed in numerical simulations of the PS model in three dimensions [27], although this work did not establish conclusively that hysteresis survives in the limit of infinite system size. Preliminary work by us [28] suggests that there is no hysteresis in infinite systems in three dimensions.

4. Conclusion

We have discussed the depinning transition of two classes of models that allow for history-dependent response. In mean field theory both models exhibit a tricritical point as a function of disorder strength. At weak disorder depinning is continuous and the sliding state is unique. Above the tricritical point depinning is discontinuous and hysteretic. Numerical studies of these models are currently under way to establish how much of the mean field behavior survives in finite dimensions. Simulations of the PS model in three dimensions suggest that the hysteresis disappears for large system sizes [27–30]. In this model the hysteresis may indeed be an artifact of the mean-field approximation that yields a sharp distinction between coherent and incoherent static states. In finite dimensions this may be replaced by a smooth growth of local coherence over a broad range of length scales, without discontinuous jumps. The mean-field tricritical point may then become a strong crossover between continuous depinning transitions characterized by (nonuniversal) exponents $\beta < 1$ at weak disorder to $\beta > 1$ at strong disorder [30]. Hysteresis seems more robust in the VE model, or in general in models with local inertia. Preliminary numerical simulations of an anisotropic version of the VE models in two dimensions indicate a stubborn persistence of hysteresis with increasing system size even for relatively weak disorder. In this case the existence of hysteresis may not be a good test of the precise nature of the depinning transition [31].

Once local inertia or topological defects are introduced in the model, various depinning scenarios are possible. The depinning may be discontinuous with hysteresis (like an equilibrium first-order phase transition) or with hysteresis that vanishes in the infinite system limit. Another possibility is that the transition in continuous and critical, in the sense that it is possible to identify diverging correlation lengths as the depinning threshold is approached adiabatically from above or from below. Finally, as originally suggested by Ramanathan and Fisher [32] and more recently explored by Maimon and Schwarz [31], even critical behavior with hysteresis that survives in the infinite system limit is possible. Sorting out these various scenarios for the models discussed will require more extensive numerical studies in finite dimensions.

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