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A perspective on nonlinear dynamics

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Abstract. We present a brief report on the conference, a summary of the proceedings, and a discussion on the field of nonlinear science studies and its current frontiers.

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The study of nonlinear dynamical systems has been an active area of research for over twenty five years now. Well after the initial upsurge of interest caused by the seminal works in the 1960s and 70s – associated with the names of Lorenz, Gardner, Greene, Kruskal and Miura, Kolmogorov, Arnold and Moser, Ruelle and Takens, Feigenbaum, and Libchaber among others – has died down, the field continues to grow exponentially. Of the more than 16,000 articles that address nonlinear science that have been published in the *Physical Review* since 1960, almost a third have appeared in the past four years alone! And these statistics do not include the other major specialized journals that focus specifically on nonlinearity and chaos [1].

The conference on Perspectives in Nonlinear Dynamics (PNLD Chennai, July 2004) [2], the proceedings of which constitute the remaining articles in this volume, and which was nucleated by the related meeting, Statphys-22 (Bangalore, July 2004) seemed like an appropriate occasion for a discussion of the current status of the field, with some focus on the work on chaos and nonlinearity that has been going on in India. The meeting itself benefited from considerable international participation, and the several articles that follow testify to the high quality of the meeting and the discussion that it generated. Diverse topics were covered at the meeting, in areas that ranged from integrable systems to turbulence.

In this article, in addition to summarizing the meeting – covering both the papers in this collection as well as those that were only presented at the conference – it is our aim to present a selective perspective of the still evolving area of nonlinear science studies.

The study of fluid turbulence continues to be a frontier area of nonlinear science: some aspects are well-explored, while much remains uncharted. The article by Sreenivasan and Bershadskii [3] suggests the existence of logarithmic scaling for structure functions in the near-dissipation range for which classical power-law scaling does not apply. Experimental data, shown to support this suggestion, can be considered as the basis for extended self-similarity. Vinod and Govindarajan [4] review instability mechanisms leading to turbulence in boundary layers and show that spot birth is related to the pattern of secondary instability. The article by Verma [5] discusses incompressible turbulence and the issue of its non-locality in real and in Fourier space. He also contrasts the Burgers' and the Navier–Stokes equations, using a non-local field theory. In the final article in this section by Ananthakrishna [6], the intriguing cross-over phenomenon from a low dimensional chaotic to an infinite dimensional scale invariant power-law regime in the behaviour of material stress is explained in terms of a dynamical model. Other talks at the conference dealt with the transition to turbulence in pipe flow (Eckhardt) [6a] and in a rotating fluid (Bhattacharjee), as well as the effects of magnetic fields (Das), an aspect of considerable importance in plasma physics, and an experimental study of modal structures in Rayleigh–Bénard convection (Krishan). Numerical simulations of Rayleigh–Bénard convection as a system of spatio-temporal chaos was also discussed (Cross).

The applicability of statistical techniques in the study of chaotic dynamics, and the implications of nonlinearity in statistical physics form the focus of the articles on statistical methods. Murthy *et al* [7] review a variety of statistical methods employed in the study of deterministic and stochastic dynamical systems including power spectral analysis and aliasing, extreme value, order, and recurrence time statistics. In a paper of high practical utility, Chandré and Uzer [8] summarize a time–frequency analysis method that allows the analysis of the structure and geometry of high dimensional complex dynamical systems. This method uses the wavelet transform of a single trajectory to characterize dynamical properties like resonance transitions and trappings.

Several very diverse applications were presented at the meeting. Muruganandam and Francisco [9] discuss the bred vector dimension used to identify regimes where a finite time forecast is feasible and demonstrate its use for a two-dimensional coupled map lattice. The article by Parvate and Gangal [10] discusses processes where time evolution takes place on Cantor-like fractal subsets of the real time line and sets up a new calculus called the F^{α}-calculus for the analysis of such equations. Jaganathan and Sinha [11] introduce q-deformed nonlinear maps in the next article and study the q-deformed logistic map which turns out to have co-existing attractors, a phenomenon rare in one-dimensional maps. Such extensions are reminiscent of and are inspired by Tsallis' non-extensive generalization of the Boltzmann–Gibbs statistics; in a talk presented at the conference (Tsallis), the dynamical foundations of such statistics was discussed, along with a discussion of its pertinence at the edge of chaos (Robledo).

Obtaining macroscopic, thermodynamic quantities from the details of the microscopic dynamics also remains an active area of research. A version of the fluctuation–dissipation theorem can be obtained for certain non-equilibrium systems (Gaspard), though obtaining transport properties from first principles still poses a problem for all but the most elementary models. Coupled oscillator systems have been popular since the studies of Fermi *et al* [12], and here Krishna Mohan and Sen [13] discuss the formation of discrete solitary and antisolitary waves in the time evolution of such conservative, discrete, nonlinear chains, which precludes the approach to an equilibrium.

Inferring details of the dynamical state through analysis of time-series data has been an ongoing theme of research for several decades now. New applications and new experimental situations need novel analysis. Some of these were, an application of cluster-weighted modeling, a bayesian method for the extraction of Lyapunov exponents (Ghosh) and techniques for inferring the causal relatedness of different time-series signals measured simultaneously (Rangarajan, and separately, Prasad).

The two papers in this volume that are on experiments are new approaches to computation using chaotic circuits by Murali *et al* [14] and further explorations of Shil'nikov homoclinic chaos in Chua's circuit by Dana *et al* [15] which identifies interesting phenomena and routes to chaotic dynamics. The former is a new way to realize different logic gates using the same chaotic circuit in different parameter regimes, using a simple thresholding operation. This type of versatile device could be one of the future building blocks for a new concept of computation that utilizes chaotic dynamics for performing basic logic operations. The latter presents observations of bursting dynamics in Chua's circuit, a well-known paradigm system that has been the topic of study for over a decade, but which still yields new phenomena to understand and incorporate in the phenomenology of chaotic systems. Shil'nikov chaos and bursting dynamics are important in many different types of systems, ranging from fluids and optics to neurons, and a deeper understanding of these phenomena in a well-characterized Chua circuit could provide insights that are significant for a wide range of systems. Research on nonlinear dynamics with electronic circuits is a very active area for the development of new tools for computation, communication, information processing and sensor technology. It is also a promising approach to explore synchronization and collective dynamical phenomena in arrays and networks, which represent new frontiers of nonlinear dynamics in high dimensional systems. Other papers presented at the meeting dealt with coupled laser systems (Roy) and switching in electrical circuits (Banerjee).

A comprehensive review of synchronization phenomena (Kurths) provided a good focus for a number of related studies, viz., on effects of delays (Kye, and separately, Nandakumaran), controlling intermittency (Goswami), and communication (Kim). Related work on low-dimensional chaos dealt with multistability and attractor hopping (Feudel).

The six papers on extended systems are concerned with high dimensional dynamical systems. Considerations of cluster formation and synchronization on networks by Amritkar and Jalan [16] reveal interesting new phenomena that characterize the dynamics of elements coupled together in different types of networks. Sen *et al* [17] introduce time-delayed coupling between limit cycle oscillators and examine issues of phase-locking and amplitude death as the nature of the coupling (global or local) between the oscillators is changed. Dynamics and security in complex networks such as the internet or the power grid of a nation is addressed by Lai *et al* [18] in the context of possible attacks that may attempt to disrupt communication or power distribution. The focus is on the effect of attacks that disrupt links rather than nodes of the network. The basic phenomenon of lag synchronization is studied for externally driven coupled Duffing oscillators by Pisarchik and Jaimes-Reátegui [19], while hysteresis and spatial synchronization in coupled non-identical chaotic oscillators is analyzed by Prasad *et al* [20] in terms of measures such as Lyapunov exponents, with possible applications in understanding epileptic seizures. Coupled oscillators also provide the framework for an examination of the gaits of quadrupeds and their dynamical symmetry properties. The paper of Castellini *et al* [21] is motivated by applications in robotics. Finally, stochastic resonance in bimodal maps, coupled map lattices and Josephson junctions are investigated numerically by Ambika *et al* [22], with emphasis on multisignal amplification and spatial aspects.

It is a challenge for nonlinear dynamicists to understand phenomena in such high-dimensional systems. An important issue that is pervasive and needs to be addressed concerns the overwhelming amount of data that can be gathered from high dimensional and extended systems, numerically or experimentally. Pattern formation in systems ranging from reaction–diffusion models (Kapral) to leaf–venation (Couder) is a representative area of considerable experimental and theoretical interest. What should be measured, and how should the data be presented and analyzed? We are often limited by our ability to imagine and visualize high dimensional dynamics, and new ways of representing the phenomena may make a difference on how we can develop and understand them. As computational methods and data acquisition techniques have progressed, so have our ambitions with regard to the complexity of the systems we can study. Systems with time delays present a particularly challenging class of phenomena where our intuition needs to be developed and guided through the study of basic examples and the development of paradigm systems. The motivations for such studies are often provided by biology, and the possible applications, even in the near future, are limited only by our imagination.

One such area is medical diagnostics. It is evident that tools that can help in accurate diagnosis are invaluable, for both disease prevention, and appropriate medical care. In this context, it has proved advantageous to take a 'systems' view: a number of physiological functions – the heart and brain being principal among them – can be viewed as dynamical systems. Measurement of their output, as ECG or EEG signals, and subsequent analysis through techniques developed for nonlinear dynamical systems has been useful and instructive, especially with respect to the study of human heart. Zebrowski and Baranowski [23] analyse heartbeats with a view to quantifying the variability, and show that several of the features observed in intermittent dynamical systems in the presence of multiplicative noise can be seen in heartbeat variability. The paper by Breuer and Sinha [24] deals with ventricular tachycardia and its prevention by appropriate pacing, taking into account the physical inhomogeneities in the organ. At the conference there were several other contributions that dealt explicitly with the heart as dynamical system (Ditto, and separately, Pumir).

Ecological systems are inherently complex because of the diversity of ecological species and the complexity of their interactions. In the course of biological evolution many organisms have evolved complex strategies to capture prey and avoid predators. For instance, many species rely on the formation of swarms to confuse predators and reduce predation loss. Because of this complexity at the individual level, models of ecological population dynamics are generally characterized by a high degree of nonlinearity (and often incorporate stochasticity as well). For sustainability and conservation of ecological systems, the dynamics and functioning of these complex nonlinear systems need to be understood. Many of the techniques developed for the mathematical analysis of nonlinear dynamical systems find fruitful application in this context (Gross). At a more detailed level, complex and adaptive behaviour such as flocking (Chaté) can also be analyzed within the fabric of nonlinear science and represents a unique context in which adaptive behaviour leads to self-organization. Clearly, these are areas in which much activity can be expected in the next decade.

The quantum implications of classical chaos – the area of quantum chaos – have been debated at length for over 25 years, starting with ideas of regular and irregular quantum states [25] and leading to the elegant and powerful periodic orbit theory [26]. Current work in the area recognizes distinctions in the nature of quantum states of systems that have underlying classical systems that are chaotic, integrable, or mixed, in terms of the eigenstates or the eigenvalues. The two papers in this volume in this area examine, respectively, the correspondence between the evolving classical phase space density and a quantal counterpart [27], and the effect of entanglement on quantum chaotic eigenstates [28]. Current developments in quantum information suggest that the effects of chaos on entanglement and in setting the limits to computability may be significant [29], and in this context, the tunneling properties of macroscopic wave functions (Nakamura) will be important.

The article by Valsakumar [30] puts forth the point of view that the quantum mechanical treatment of discrete time evolution reveals interference between non-degenerate quantum states which decay exponentially. This decoherence can be important in certain situations. The last two articles in this volume deals with integrability, probably the earliest characteristic of nonlinear dynamical systems to be recognized as distinctive. In the penultimate article, Balakrishnan [31] shows how techniques from differential geometry can assist in the analysis of static and moving curves, and in particular, discusses anholonomy, the change in geometric phase that can be associated with such curves. In the concluding article, Lakshmanan [32] gives a brief history of this fascinating subject wherein the combination of dispersion and nonlinearity conspire to produce coherence in the form of long lived structures. Since their discovery in 1965, solitons and solitary waves have been a dominant area of research by providing a contrast, as it were, to dynamical chaos. The extensive research in this area has led to significant understanding of the nature of integrability, its consequences, its diagnostics and its potential applications [32].

We have not been able to include here a description of the more than sixty posters which were presented at the meeting. As a result, several areas of nonlinear dynamics will appear under-represented in this volume. These include the analysis of nonlinear evolution equations, their experimental realizations, the study of active and reacting systems and their mixing behaviour, as well as stochastic dynamics and resulting phenomena. The deep connections between statistical mechanics and nonlinear dynamics, as well as the mathematical underpinnings of the theory of dynamical systems, are some important areas which were not discussed in PNLD, and we look forward to future activities which will redress the balance.

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[1] The statistics we collect is from PROLA and includes *Physical Review* A–E and *Letters*, and counts articles which have the words *chaos* or *nonlinearity* in the title or abstract; some of the other journals which focus on nonlinear science, but of which we have not included statistics are *Chaos*, *Int. J. Bifurcat. Chaos*, *Physica D*, *Nonlinearity*, *J. Nonlinear Sci. Nonlinear Phenomena in Complex Systems*. In addition, numerous articles that deal with chaos and nonlinearity appear in a variety of other journals

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