PRAMANA — journal of physics

Vol. 64, No. 2
February 2005
pp. 1–7

Angular momentum transfer in incomplete fusion

B S TOMAR[1], K SURENDRA BABU[2], K SUDARSHAN[1], R TRIPATHI[1] and A GOSWAMI[1]
[1]Radiochemistry Division, Bhabha Atomic Research Centre, Mumbai 400 085, India
[2]School of Studies in Physics, Vikram University, Ujjain 456 010, India
Corresponding author. E-mail: bstomar@apsara.barc.ernet.in

MS received 12 March 2004; revised 22 September 2004; accepted January 2005

Abstract. Isomeric cross-section ratios of evaporation residues formed in ^{12}C+^{93}Nb and ^{16}O+ ^{89}Y reactions were measured by recoil catcher technique followed by off-line γ-ray spectrometry in the beam energy range of 55.7–77.5 MeV for ^{12}C and 68–81 MeV for ^{16}O. The isomeric cross-section ratios were resolved into that for complete and incomplete fusion reactions. The angular momentum of the intermediate nucleus formed in incomplete fusion was deduced from the isomeric cross-section ratio by considering the statistical de-excitation of the incompletely fused composite nucleus. The data show that incomplete fusion is associated with angular momenta slightly smaller than critical angular momentum for complete fusion, indicating the deeper interpenetration of projectile and target nuclei than that in peripheral collisions.

Keywords. Heavy-ion reactions; incomplete fusion; isomeric cross-section ratio; ^{12}C, ^{16}O beams; ^{93}Nb; ^{89}Y targets; angular momentum.

PACS Nos 25.70.Gh; 25.70.Ji

1. Introduction

During the past 10–15 years several reports have been published [1–6] which suggest that incomplete fusion (ICF) reactions start competing with complete fusion (CF) just above the Coulomb barrier. Though the cross-sections for ICF reactions at beam energies more than 10 MeV/amu have been well-explained by the sum rule model of Wilczynski [7], which envisages the localization of the different ICF channels in the angular momentum space above the critical angular momentum for CF, the data at lower beam energy could not be explained on the basis of the above prescription owing to the fact that the maximum angular momentum values ($l_{\rm max}$) are close to the critical angular momentum ($l_{\rm cr}$) thereby precluding any window for ICF above $l_{\rm cr}$. In the projectile energy range of around 10 MeV/nucleon the measurement of angular momenta in incomplete fusion reactions showed that these reactions are associated with peripheral collisions [8,9] though a few studies particularly those involving spherical targets showed the involvement of angular momenta much lower than l_{cr} in these reactions [10]. Thus, whether ICF is associated with peripheral collisions at beam energies around 5 MeV/nucleon is not still clearly understood. There are very few data in the literature on measurement of angular momentum in ICF reactions at lower incident beam energies [11–13]. Most of the studies on angular momentum in ICF reactions have been carried out by measurement of γ-ray multiplicity. Another method of measurement of angular momentum is through measurement of isomeric cross-section ratio (ICR) of evaporation residues (ERs). This method has been extensively used for the measurement of angular momenta of fission fragments [14] as well as the angular momentum distribution in sub-barrier fusion reactions [15].

Fusion of one part of the projectile with the target nucleus leads to an intermediate nucleus, which attains equilibrium, and subsequently de-excites analogous to a compound nucleus (CN). By measuring the ICR of ERs, one can deduce the angular momentum of the intermediate nucleus using the statistical model codes for the de-excitation of the intermediate nucleus by particle and gamma emission. No such studies have been carried out for the determination of angular momentum involved in ICF reactions. Our previous measurements of excitation functions and recoil range distributions [16,17] of ERs in $^{12}C + ^{93}Nb$ and $^{16}O + ^{89}Y$ systems had clearly shown significant contribution of ICF in the cross-sections of alpha emission products (rhodium products) in both the systems and alpha transfer products (technetium products) in $^{12}C + ^{93}Nb$ reaction. In this work we report the results of the analysis of isomeric cross-section ratios of the 99Rh evaporation residue formed in ICF to deduce the angular momentum involved in ICF reactions. The angular momenta of the intermediate nucleus were deduced using the statistical model code CASCADE [18]. With a view to fix the input parameters in CASCADE code, the experiments were also carried out on isomeric cross-section ratios of ^{97}Rh in ^{12}C-induced reaction on ^{89}Y wherein the rhodium isotopes are formed in complete fusion reaction.

2. Experimental

The experiments were carried out at BARC–TIFR pelletron facility at Mumbai. Recoil catcher technique followed by off-line γ-ray spectrometry was used for measurement of ICRs of ERs. Rolled metal foils of niobium and yttrium of thickness around 1.5 mg/cm^2 were bombarded with ^{12}C and ^{16}O beams respectively for a period of 45 min to 1 h. The recoiling evaporation residues were stopped in aluminium catcher foils of thickness 2 mg/cm^2. In each irradiation a stack of two target catcher foils was used. Three such irradiations were carried out to encompass the beam energy range of 55.7–77.5 MeV for ^{12}C and 68–81 MeV for ^{16}O. The details of the experiment are given in [16]. In the case of $^{12}C + ^{89}Y$ reaction the beam energy of ^{12}C was varied in the range of 52–63 MeV. After the irradiation the target and the following catcher foil were assayed for γ-ray activity of ERs using a 60CC HPGe coupled to a PC-based 4K channel analyzer. The resolution of the detector was 2 keV at 1332 keV. The counting was continued for a period of 1 month so as to follow the decay of the long-lived ERs. The peak areas of the γ lines were calculated by linear subtraction of the Compton background. The cross-section of evaporation

Table 1. Nuclear spectroscopic data used in this work.

Nuclide	Spin	Half-life	E (keV)	I (%)
$^{99}Rh^{m}$	9/2 +	4.5 h	340.8	69.1
$^{99}Rh^{g}$	$\frac{1}{2}$ −	16.0 d	527.7	40.7
^{98}Rh	2 +	9.05 m	652.4	94.2
$^{97}Rh^{m}$	$\frac{1}{2}$ −	44.3 m	188.6	51.2
$^{97}Rh^{g}$	9/2 +	31.1 m	421.5	75.0

Table 2. Experimental and calculated ICRs of ^{97}Rh in^{12}C + ^{89}Y reaction.

$E_{\rm lab}$ (MeV)	$(\sigma_{\rm h}/\sigma_{\rm l})^{97}$ Rh Expt.	$(\sigma_{\rm h}/\sigma_{\rm l})^{97}$ Rh CASCADE
52	8.8 ± 0.4	6.7
55.4	10.1 ± 0.3	8.1
63	16.0 ± 1.0	13.8

residues was calculated using the standard activation equation [16]. The nuclear spectroscopic data of radionuclides of interest are given in table 1.

3. Results and discussion

Table 2 shows the measured isomeric cross-section ratios of ^{97}Rh in ^{12}C + ^{89}Y reaction at various ^{12}C beam energies. In this system ^{97}Rh is formed as a dominant reaction product of CF process, wherein the angular momentum distribution of the de- exciting compound nucleus (CN) can be calculated quite accurately using the formula,

$$\sigma_{\rm l} = \pi \Delta^2 (2l+1) T_{\rm l}. \tag{1}$$

The $T_{\rm l}$ value can be calculated using the one-dimensional barrier penetration formula [18]. The isomeric cross-section ratio of ^{97}Rh was calculated using the CASCADE code. The calculations were carried out for a range of a. The strength functions for E1, M1 and E2 were taken from [19]. The moment of inertia of the excited nuclei was taken as 0.85 times rigid body value. Level density prescription of Reisdorf was used in the calculations. The CASCADE code gives the population along the yrast line following the emission of particles and statistical γ-rays from the compound nucleus. This yrast line population is apportioned into the high-spin and low-spin isomeric cross-sections assuming a sharp cut-off approximation, according to which spin states below $J_{\rm h}$ (spin of high-spin isomer) decay to low-spin isomer while those above $J_{\rm h}$ decay to high-spin isomer. The sharp cut-off approximation is commonly used in calculating the angular momenta of fission fragments from measured isomeric yield ratios of the fission products [14]. The calculated isomeric cross-section ratios of ^{97}Rh were compared with the experimental data. The best agreement between the calculated and experimental values of isomeric cross-sections of ^{97}Rh was obtained for $a = A/8.0$ MeV^{-1}. These set of parameters were used for subsequent calculations.

Table 3 gives the cross-sections and ICR of ^{99}Rh at various projectile energies in ^{12}C + ^{93}Nb and ^{16}O + ^{89}Y reactions. The error on the cross-sections is the propagated error due to counting statistics, detection efficiency and target thickness. The overall error on the ICR values is around 10–15%.

The measured ICR is the resultant of the cross-section for the formation of the ER by complete as well as incomplete fusion. In order to resolve the measured ICR into that for CF and ICF, the ICR for CF ($R_{\rm CF}$) was deduced using the statistical de-excitation code CASCADE. The CF cross-sections for ^{99}Rh were taken from CASCADE predictions as the code successfully reproduced ICR in the case of ^{12}C + ^{89}Y and the excitation functions for all the ERs, in ^{12}C + ^{93}Nb and ^{16}O + ^{89}Y [16], which are formed by CF alone. The measured ICR ($R_{\rm exp}$) can be expressed as

$$R_{\rm exp} = \frac{\sigma_{\rm h}({\rm CF}) + \sigma_{\rm h}({\rm ICF})}{\sigma_{\rm l}({\rm CF}) + \sigma_{\rm l}({\rm ICF})}, \tag{2}$$

where h and l refer to the high- and low-spin isomer, and

$$\sigma({\rm CF}) + \sigma_{\rm h}({\rm CF}) + \sigma_{\rm l}({\rm CF}), \tag{3}$$

$$\sigma({\rm ICF}) = \sigma_{\rm h}({\rm ICF}) + \sigma_{\rm l}({\rm ICF}) \tag{4}$$

$$R_{\rm CF} = \frac{\sigma_{\rm h}}{\sigma_{\rm l}}. \tag{5}$$

σ(CF) and $R_{\rm CF}$ were deduced from CASCADE calculations. σ(ICF) was obtained by subtracting the CF cross-section from the experimental cross-section. Thus $\sigma_{\rm h}$(ICF) and $\sigma_{\rm l}$(ICF) were obtained by solving eqs (2)–(5).

Table 3. The isomeric cross-section ratios of ^{99}Rh in ^{12}C + ^{93}Nb and ^{16}O + ^{89}Y reactions.

$E_{\rm lab}$ (MeV))	$\sigma_{\rm h}$ ^{99}Rh (mb)	$\sigma_{\rm l}$ ^{99}Rh (mb)	$\sigma_{\rm h}/\sigma_{\rm l}$
^{12}C+^{93}Nb → ^{101}Rh+^{4}He			
47.4	59.6 ± 4.6	11.6 ± 1.3	5.1 ± 0.7
55.7	137.8 ± 7.8	17.9 ± 1.6	7.7 ± 0.8
63.3	180.7 ± 13.5	16.7 ± 1.3	10.8 ± 1.2
63.9	173.1 ± 11.5	14.8 ± 1.7	11.7 ± 1.5
70.9	89.8 ± 6.6	6.7 ± 0.6	14.5 ± 1.8
77.5	89.4 ± 9.5	6.9 ± 1.1	12.9 ± 2.4
^{16}O+^{89}Y → ^{101}Rh+^{4}He			
57.6	58.3 ± 5.5	9.9 ± 0.8	5.8 ± 0.7
68.1	135.5 ± 6.6	11.1 ± 0.9	12.2 ± 1.1
71.2	150.2 ± 7.7	10.3 ± 0.9	14.5 ± 1.4
81.2	74.2 ± 7.7	2.9 ± 0.4	25.9 ± 3.7
93.3	83.2 ± 8.3	6.2 ± 0.7	13.3 ± 2.0

4. Deduction of angular momentum of the intermediate nucleus (^{101}Rh) from isomeric cross-section ratio of ^{99}Rh

Fusion of a part of the projectile with the target nucleus results in an intermediate nucleus (IN), which attains equilibrium in all degrees of freedom, and therefore its subsequent de-excitation is analogous to a compound nucleus. However, one needs to decide the initial excitation energy and angular momentum of the intermediate nucleus. The excitation energy of IN was calculated assuming the breakup fusion picture of ICF. Considering the ICF component in rhodium products arises from reactions of the type,

$$^{12}\mathrm{C}\ ^{93}\mathrm{Nb} \rightarrow\ ^{101}\mathrm{Rh} +\ ^{4}\mathrm{He} \quad \text{and} \quad ^{16}\mathrm{O} +\ ^{89}\mathrm{Y} \rightarrow\ ^{101}\mathrm{Rh} +\ ^{4}\mathrm{He}$$

the excitation energy of IN (^{101}Rh) was calculated using the formulae [20],

$$(2/3)E(^{12}\mathrm{C})(93/101) + Q_{\mathrm{gg}} \quad \text{and} \quad (3/4)E(^{16}\mathrm{O})(89/101) + Q_{\mathrm{gg}} \tag{6}$$

for the two systems respectively, where Q_{gg} is the ground state Q value for the above ICF reactions.

The intermediate nucleus (IN) was assumed to have fixed excitation energy, though a distribution of excitation energy is expected owing to the kinetic energy and angular distribution of the outgoing α particles. However, the distribution of excitation energy and angular momentum of IN will affect the second moment of the angular distribution, but not the mean value of angular momentum. Since in the present work only the mean angular momentum are deduced, this aspect has not been considered. Thus the angular momentum of IN was taken as single spin considering the binary nature of the ICF reaction. The initial spin $\langle J \rangle$ of the IN was varied in steps of $1\hbar$ from $10\hbar$ to $15\hbar$ and the statistical model analysis was carried out using CASCADE code. The final population of the residual nucleus (^{99}Rh), was apportioned into high (h) and low (l) spin states assuming sharp cut-off approximation as discussed above. Table 4 gives the mean angular momentum of ^{101}Rh formed in ICF reactions in ^{12}C + ^{93}Nb and ^{16}O + ^{89}Y systems. The error on the deduced angular momenta is around 15%, considering the uncertainty in the prediction of the isomeric cross-section ratio by CASCADE code and the errors on the experimental data.

In order to calculate the angular momentum (l_e) carried away by α particles emitted in ICF, the formalism of Inamura *et al* [12,21] was used according to which

$$l_e = \frac{R}{h}[2\mu(E_\alpha - V_c)]^{1/2}, \tag{7}$$

where E_α is the kinetic energy of the α particles emitted in the ICF reaction, μ is the reduced mass in the exit channel and V_c is the potential barrier for α particles at radius R, which was taken as sharp absorption radius, defined as

$$R = 1.07(A_a^{1/3} + A_A^{1/3}) + 3.0fm \tag{8}$$

The subscripts a and A represent α particle and the intermediate nucleus formed following fusion of the projectile fragment with the target nucleus. The calculated

Table 4. Angular momenta deduced from ICR of ^{99}Rh in ^{12}C + ^{93}Nb and ^{16}O + ^{89}Y reaction.

E_{lab} (MeV)	$(\sigma_{\mathrm{h}}/\sigma_{\mathrm{l}})$ Expt.	$(\sigma_{\mathrm{h}}/\sigma_{\mathrm{l}})$ CF	$(\sigma_{\mathrm{h}}/\sigma_{\mathrm{l}})$ ICF	$\langle J\rangle$ ^{101}Rh ($\hbar$)	L_{cr} ($\hbar$)	L_{ICF} ($\hbar$)
(a) For ^{12}C+^{93}Nb reaction						
55.7	7.7 ± 0.8	6.9	8.8	13.3	27.9	22.9
63.3	10.8 ± 1.2	20.3	8.2	13.2	33.8	24.8
63.9	11.7 ± 1.5	14.8	10.4	14.3	34.2	26.0
70.9	14.5 ± 1.8	22.3	11.7	13.8	38.7	27.2
77.5	12.9 ± 2.4	20.2	11.5	13.5	42.5	28.1
(b) For ^{16}O+^{89}Y reaction						
68.1	12.2 ± 1.1	11.8	12.9	14.0	32.5	22.2
71.2	14.5 ± 1.4	15.8	13.7	14.3	35.3	23.2
81.2	25.9 ± 3.7	15.3	72.5	17.8	42.9	28.8

l_e values were added to the mean angular momentum of IN (^{101}Rh) to obtain the angular momentum involved in ICF (L_{ICF}). The E_α value was taken as the energy corresponding to beam velocity, that is, in the mass ratio of ejectile to projectile. The kinetic energy spectrum of the outgoing α particles and its angular distribution would affect the width of the angular momentum distribution but not the mean value of angular momentum. The L_{ICF} values in ^{12}C + ^{93}Nb and ^{16}O + ^{89}Y systems are given in table 4, along with the critical angular momentum (l_{cr}) calculated using the prescription of Wilczynski [22]. The L_{ICF} values deduced from the isomeric cross-section ratios are found to be slightly lower than the critical angular momentum for CF. The lower than l_{cr} value of L_{ICF} suggests that ICF at lower beam energies may not be associated with only peripheral collisions. Instead, results of the present study indicate deeper interpenetration of the projectile and target during the ICF reactions. Similar observations have been made in the earlier reports [11,12]. Determination of angular momentum involved in ICF reaction using the γ-ray multiplicity in ^{12}C + ^{169}Tm also yielded similar results [13]. This is further corroborated by the observation of significant ICF cross-section even at beam energy as low as 5 MeV/nucleon, which cannot be well-reproduced by the sum rule model, as the maximum angular momentum is very close to the critical angular momentum for the complete fusion of projectile and target, thereby precluding a separate window for CF and ICF. This suggests that ICF competes with CF even at beam energies close to Coulomb barrier.

5. Conclusions

In the present work the angular momentum involved in ICF reaction of ^{12}C with ^{93}Nb and that of ^{16}O with ^{89}Y has been deduced from the isomeric cross-section ratio of ^{99}Rh. The results indicate that ICF is associated with collisions having deeper interpenetration of the projectile and the target than that in peripheral collisions.

Acknowledgements

The authors thank the operation crew of the Pelletron accelerator for their cooperation during the experiment. Thanks are also due to D C Ephraim for preparing the targets.

References

[1] D J Parker, J J Hogan and J Asher, *Phys. Rev.* **C35**, 161 (1987)
[2] I Tserruya, V Steiner, Z Fraenkel, P Jacobs, D G Kovar, W Henning, M F Vineyard, and B G Glagola, *Phys. Rev. Lett.* **60**, 14 (1988)
[3] B S Tomar, A Goswami, G K Gubbi, A V R Reddy, S B Manohar, Bency John and S K Kataria, *Phys. Rev.* **C58**, 3478 (1998)
[4] E Gadioli, *Acta Phys. Pol.* **B30**, 1493 (1999)
[5] B Bindu Kumar, S Mukherjee, S Chakrabarty, B S Tomar, A Goswami and S B Manohar, *Phys. Rev.* **C57**, 743 (1998)
[6] S Chakrabarty, B S Tomar, A Goswami, G K Gubbi, S B Manohar, Anil Sharma, B Bindu Kumar and S Mukherjee, *Nucl. Phys.* **A678**, 355 (2000)
[7] J Wilczynski, K Siwek-Wilczynska, J Van Driel, S Gonggrijp, D C J M Hageman, R V F Janssens, J Lukasiak and R H Siemssen, *Phys. Rev. Lett.* **45**, 606 (1980)
[8] K A Geoffroy, D G Sarantites, M L Halbert, D C Hensley, R A Dayras and J H Baker, *Phys. Rev. Lett.* **43**, 1303 (1979)
[9] W Trautmann, Ole Hansen, H Tricoire, W Hering, R Ritzka and R Trombik, *Phys. Rev. Lett.* **53**, 1630 (1984)
[10] H Tricoire, C Gerschel, N Perrin, H Sergolle, L Valentin, D Bachelier, H Droube and A Gizon, *Z. Phys.* **A343**, 223 (1982)
[11] G Duchene, P Romain, F A Beck, Ph Benet, D Disdier, B Hass, B Lott, V Rauch, F Scheibling, J P Vivien, S K Basu, E Bozek, K Zuber, D Di Gregorio and J Fernandez-Niello, *Phys. Rev.* **C47**, 2043 (1993)
[12] T Inamura, A C Kahler, D R Zolnowski, U Garg, T T Sugihara and M Wakai, *Phys. Rev.* **C32**, 1539 (1985)
[13] K Sudarshan, S Sodaye, K Surendra Babu, S Mukherjee, S K Rathi, K Mahata, A Goswami and B S Tomar, *Phys. Rev.* **C69**, 027603 (2004)
[14] B S Tomar, A Goswami, S K Das, B K Srivsatava, R Guin, S M Sahakundu, S B Manohar and Satya Prakash, *Phys. Rev.* **C38**, 1787 (1988)
[15] D E DiGregorio, Y Chan, E Chavez, A Dacal, M E Ortiz, J Suro and R G Stockstad, *Phys. Rev.* **C43**, 687 (1991)
[16] B S Tomar, A Goswami, A V R Reddy, S K Das, P P Burte, S B Manohar and Satya Prakash, *Z. Phys.* **A343**, 223 (1992)
[17] B S Tomar, A Goswami, A V R Reddy, S K Das, P P Burte, S B Manohar and Bency John, *Phys. Rev.* **C49**, 941 (1994)
[18] F Puhlhofer, *Nucl. Phys.* **A280**, 267 (1977)
[19] P M Endt, *At. Data. Nucl. Data Tables* **26**, 47 (1981)
[20] H Morgenstern, W Bohne, W Galster and K Grabisch, *Phys. Rev. Lett.* **52**, 1104 (1984)
[21] T Inamura, T Kojima, T Nomura, T Sugitate and H Utsunomiya, *Phys. Lett.* **B84**, 71 (1982)
[22] J Wilczynski, *Nucl. Phys.* **A216**, 386 (1973)