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Bianchi Type-IX viscous fluid cosmological model in general relativity

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Abstract. Bianchi Type-IX viscous fluid cosmological model is investigated. To get a deterministic model, we have assumed the condition $a = b^m$ (m is a constant) between metric potentials and $\eta \propto \theta$ where η is the coefficient of shear viscosity and θ the scalar of expansion in the model. The coefficient of bulk viscosity (ζ) is taken as constant. The physical and geometrical aspects of the model are also discussed.

Keywords. Bianchi Type-IX; viscous fluid.

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1. Introduction

Bianchi Type-IX cosmological models are interesting because these models allow not only expansion but also rotation and shear and in general are anisotropic. Many Relativists have taken keen interest in studying Bianchi Type-IX universes because familiar solutions like Robertson–Walker universe with positive curvature, the de-Sitter universe, the Taub-NUT solutions etc. are of Bianchi Type-IX space-times. In these models, neutrino viscosity does not guarantee isotropy at the present epoch. Viscosity is important in cosmology for a number of reasons. Misner [1,2] has studied the effect of viscosity on the evolution of cosmological models. Collins and Stewart [3] have studied the effect of viscosity on the formation of galaxies. Murphy [4] has studied the influence of viscosity on the nature of initial singularity. Weinberg [5] derived general formula for bulk and shear viscosity and used these to evaluate the rate of cosmological entropy production. Heller and Klimek [6] have investigated viscous universe without initial singularity. They have shown that introduction of bulk viscosity removes the initial singularity. Roy and Prakash [7,8] investigated viscous fluid cosmological models of Petrov Type-ID and non-degenerate Petrov Type-I in which coefficients of viscosities are constants. Banerjee *et al* [9] investigated Bianchi Type-I viscous fluid cosmological models with both viscosities. Santosh *et al* [10] obtained exact solutions for isotropic homogeneous cosmological model with bulk viscosity. Bali and Jain [11,12] have obtained some expanding and shearing Bianchi Type-I viscous fluid cosmological models in which coefficient of shear viscosity is proportional to the rate of expansion in the model and free gravitational field is Petrov Type-ID and non-degenerate. Robertson–Walker cosmological models with bulk viscosity and equation of state $p = (\gamma - 1)\rho$, $0 \leq \gamma \leq 2$, is investigated by Mohanty and Pradhan [13]. Caderni and Fabri [14] have carried out the research on homogeneous viscous universe and investigated models of Bianchi Type-V, Type-VIII and Type-IX.

In this paper, we have investigated Bianchi Type-IX viscous fluid cosmological model in General Relativity. To get a deterministic model, we have assumed the condition $a = b^m$ between metric potentials, m is a constant and $\eta \propto \theta$ where η is the coefficient of shear viscosity and θ is the scalar of expansion in the model and coefficient of bulk viscosity (ζ) is taken as constant. In earlier works, mostly Bianchi Type-I cosmological models are investigated when $\eta =$ constant (Roy and Prakash [7,8]), Banerjee *et al* [9] and $\eta \propto \theta$ (Bali and Jain [11,12]). Bianchi Type-I model is FRW model with zero curvature. In this paper, we have investigated Bianchi Type-IX viscous fluid cosmological model which is a closed FRW model having positive curvature and $\eta \propto \theta$ is used to get a deterministic model and coefficient of bulk viscosity is assumed as constant. It has been shown that the energy conditions: (i) $\varepsilon + p > 0$, (ii) $(\varepsilon + 3p) > 0$ are satisfied and remain the same in the presence of viscosity when $m < 1$. To justify these conditions, the detailed calculations are given in Appendix. The behaviour of the model in the presence and absence of viscosity is discussed. The physical and geometrical aspects of the model are also discussed.

We consider Bianchi Type-IX metric

$$\begin{aligned} \mathrm{d}s^2 = &-\mathrm{d}t^2 + a^2\mathrm{d}x^2 + b^2\mathrm{d}y^2 + (b^2\sin^2 y + a^2\cos^2 y)\mathrm{d}z^2 \\ &-2a^2\cos y\,\mathrm{d}x\mathrm{d}z, \end{aligned} \tag{1.1}$$

where a and b are functions of t alone.

The energy–momentum tensor for viscous fluid distribution is given by Landau and Lifshitz [15] as

$$\begin{aligned} T_i^j = &(\varepsilon + p)v_i v^j + p g_i^j - \eta(v_{i;}^j + v_{;i}^j + v^j v^\ell v_{i;\ell} + v_i v^\ell v_{;\ell}^j) \\ &-\left(\zeta - \frac{2}{3}\eta\right) v_{;\ell}^\ell (g_i^j + v_i v^j), \end{aligned} \tag{1.2}$$

where p is the isotropic pressure, ε the density, η and ζ the two coefficients of viscosity, v^i the flow vector satisfying

$$g_{ij}v^i v^j = -1. \tag{1.3}$$

We assume the coordinates to be co-moving, so that

$$v^1 = 0 = v^2 = v^3 \quad \text{and} \quad v^4 = 1. \tag{1.4}$$

The Einstein's field equation

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -8\pi T_i^j \quad (c = 1, G = 1 \text{ in gravitational unit}) \tag{1.5}$$

for the line-element (1.1) leads to

$$\left[\frac{2\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+\frac{1}{b^2}-\frac{3}{4}\frac{a^2}{b^4}+\Lambda\right]=-8\pi\left[p-2\eta\frac{\dot{a}}{a}-\left(\zeta-\frac{2}{3}\eta\right)\theta\right] \tag{1.6}$$

$$\left[\frac{\ddot{a}}{a}+\frac{\dot{a}\dot{b}}{ab}+\frac{\ddot{b}}{b}+\frac{a^2}{4b^4}+\Lambda\right]=-8\pi\left[p-2\eta\frac{\dot{b}}{b}-\left(\zeta-\frac{2}{3}\eta\right)\theta\right] \tag{1.7}$$

$$\left[\frac{2\dot{a}\dot{b}}{ab}+\frac{\dot{b}^2}{b^2}-\frac{a^2}{4b^4}+\frac{1}{b^2}+\Lambda\right]=8\pi\varepsilon. \tag{1.8}$$

Here dot over a and b denotes ordinary differentiation with respect to t and $\theta = v^{\ell}_{;\ell}$.

2. Solution of the field equations

Equations (1.6)–(1.8) are three equations in six unknowns a, b, ε, p, ζ and η. To get a deterministic solution, we assume that

$$a = b^m, \tag{2.1}$$

$$\eta \propto \theta, \tag{2.2a}$$

and

$$\zeta = \text{constant}, \tag{2.2b}$$

where θ is the expansion in the model, η the coefficient of shear viscosity and ζ the coefficient of bulk viscosity.

Equations (1.7) and (1.8) lead to

$$\left[\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}-\frac{\ddot{a}}{a}-\frac{\dot{a}\dot{b}}{ab}-\frac{a^2}{b^4}+\frac{1}{b^2}\right]=16\pi\eta\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right). \tag{2.3}$$

Condition (2.2) leads to

$$\eta=\ell\left(\frac{\dot{a}}{a}+\frac{2\dot{b}}{b}\right), \tag{2.4}$$

where ℓ is a constant.

Equation (2.3) together with (2.1) and (2.4) leads to

$$b\ddot{b}+\alpha\dot{b}^2=\frac{b^{2m-2}}{(1-m)}-\frac{1}{(1-m)}. \tag{2.5}$$

Here

$$\alpha = \left[(1+m) - 16\pi\ell\frac{(m^2+m-2)}{(1-m)}\right]. \tag{2.6}$$

Equation (2.5) leads to

$$\frac{\mathrm{d}}{\mathrm{d}b}(f^2) + \frac{2\alpha}{b}(f^2) = \frac{2b^{2m-3}}{(1-m)} - \frac{2}{(1-m)b}, \tag{2.7}$$

where

$$\dot{b} = f(b). \tag{2.8}$$

From eq. (2.7), we have

$$\left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)^2 = \left[\frac{b^{2(m-1)}}{(1-m)(m+\alpha-1)} + \frac{\beta}{b^{2\alpha}} - \frac{1}{\alpha(1-m)}\right], \tag{2.9}$$

where β is the constant of integration.

Thus the metric (1.1) reduces to the form

$$\mathrm{d}s^2 = -\left(\frac{\mathrm{d}t}{\mathrm{d}b}\right)^2 \mathrm{d}b^2 + b^{2m}\mathrm{d}x^2 + b^2\mathrm{d}y^2 + (b^2\sin^2 y + b^{2m}\cos^2 y)\mathrm{d}z^2 -2b^m \cos y \mathrm{d}x\mathrm{d}z, \tag{2.10}$$

where $a = b^m$. After using (2.9) into the metric (2.10), we have

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{\left[\frac{T^{2m-2}}{(1-m)(m+\alpha-1)} + \frac{\beta}{T^{2\alpha}} - \frac{1}{\alpha(1-m)}\right]} + T^{2m}\mathrm{d}X^2 + T^2\mathrm{d}Y^2 +(T^2\sin^2 Y + T^{2m}\cos^2 Y)\mathrm{d}Z^2 - 2T^{2m}\cos Y\mathrm{d}X\mathrm{d}Z, \tag{2.11}$$

where $b = T, x = X, y = Y, z = Z$.

3. Some physical and geometrical features

The pressure and density for the model (2.11) are given by

$$\begin{aligned} 8\pi p = &\left[\frac{(-m^2)(64\pi\ell+21) - m(64\pi\ell - 3\alpha + 6) + (128\pi\ell - 3\alpha + 15)}{12(m+\alpha-1)(1-m)}\right] T^{2m-4} \\ &+ \frac{\beta}{3}\left[(-m^2)(16\pi\ell+3) - m(16\pi\ell - 3\alpha) + (32\pi\ell + 3\alpha)\right]\frac{1}{T^{2\alpha+2}} \\ &+ \left[\frac{m^2(16\pi\ell+3) + m(16\pi\ell) - 32\pi\ell}{3\alpha(1-m)}\right]\frac{1}{T^2} \\ &+ \left[(8\pi\zeta)(m+2)\sqrt{\frac{T^{2m-4}}{(1-m)(m+\alpha-1)} + \frac{\beta}{T^{2\alpha+2}} - \frac{1}{\alpha(1-m)T^2}}\right] - \Lambda, \end{aligned} \tag{3.1}$$

and

$$8\pi\varepsilon = \left[\frac{m^2+m(\alpha+6)-(\alpha-5)}{4(1-m)(m+\alpha-1)}\right]T^{2m-4} + \left[\frac{\beta(2m+1)}{T^{2\alpha+2}}\right] + \left[\frac{(-m)(\alpha+2)+(\alpha-1)}{\alpha(1-m)}\right]\frac{1}{T^2} + \Lambda. \tag{3.2}$$

The energy conditions given by Ellis [16] are: (i) $(\varepsilon + p) > 0$ and (ii) $(\varepsilon + 3p) > 0$.
The condition (i) leads to

$$\left[\frac{(-m^2)(32\pi\ell+9)-m(32\pi\ell-3\alpha-6)+(64\pi\ell-3\alpha+15)}{6(m+\alpha-1)(1-m)}\right]T^{2m-4} + \left[\frac{(m^2)(16\pi\ell+3)+m(16\pi\ell-3\alpha-6)-(32\pi\ell-3\alpha+3)}{3\alpha(1-m)}\right]\frac{1}{T^2} + \frac{\beta}{3}\left[(-m^2)(16\pi\ell+3)-m(16\pi\ell-3\alpha-6)+(32\pi\ell+3\alpha+3)\right]\frac{1}{T^{2\alpha+2}} + 8\pi\zeta(m+2)\sqrt{\frac{T^{2m-4}}{(1-m)(m+\alpha-1)}+\frac{\beta}{T^{2\alpha+2}}-\frac{1}{\alpha(1-m)T^2}} > 0 \tag{3.3}$$

and the condition (ii) leads to

$$\left[\frac{(-m^2)(16\pi\ell+5)-m(16\pi\ell-\alpha)+(32\pi\ell-\alpha+5)}{(m+\alpha-1)(1-m)}T^{2m-4} + \beta\left\{(-m^2)(16\pi\ell+3)-m(16\pi\ell-3\alpha-2)+(32\pi\ell+3\alpha+1)\right\}\frac{1}{T^{2\alpha+2}} + \frac{(m^2)(16\pi\ell+3)+m(16\pi\ell-\alpha-2)-(32\pi\ell-\alpha+1)}{\alpha(1-m)T^2} + 24\pi\zeta(m+2)\sqrt{\frac{T^{2m-4}}{(1-m)(m+\alpha-1)}+\frac{\beta}{T^{2\alpha+2}}-\frac{1}{\alpha(1-m)T^2}}\right] > 2\Lambda \tag{3.4}$$

which gives condition on Λ. Thus energy conditions (i) $\varepsilon + p > 0$, (ii) $\varepsilon + 3p > 0$ are satisfied and remain the same for viscous fluid when $0 < m < 1$. To justify eqs (3.3) and (3.4), the detailed calculations are given in Appendix.

The expansion (θ) and the shear (σ) in the model (2.11) are given by

$$\theta = (m+2)\sqrt{\frac{T^{2(m-2)}}{(1-m)(m+\alpha-1)}+\frac{1}{T^{2\alpha+2}}\left(\beta-\frac{T^{2\alpha}}{\alpha(1-m)}\right)} \tag{3.5}$$

and

$$\sigma = \sqrt{\frac{2}{3}}(1-m)\sqrt{\frac{T^{2(m-2)}}{(1-m)(m+\alpha-1)}+\frac{1}{T^{2\alpha+2}}\left(\beta-\frac{T^{2\alpha}}{\alpha(1-m)}\right)}. \tag{3.6}$$

The model (2.11) starts with a big-bang at $T = 0$ where $\alpha > 0$ and $m < 2$, and the expansion in the model decreases as time increases. The expansion in the model stops at $T = \infty$. When $T \to 0$ then $\varepsilon \to \infty$, $p \to \infty$ and when $T \to \infty$ then $\varepsilon \to \Lambda$, $p \to -\Lambda$. The model (2.11) represents a realistic model. Since $\lim_{T\to\infty}(\sigma/\theta) \neq 0$, the model (2.11) does not approach isotropy for large values of T in general.

However, if $m = 1$, the model (2.11) isotropizes because

$$\lim_{T\to\infty} \frac{\sigma}{\theta} = \sqrt{\frac{2}{3}} \left(\frac{1-m}{m+2}\right) = 0 \quad \text{when } m = 1.$$

This shows that the model isotropizes for $m = 1$. Also

$$\sigma = \sqrt{\frac{2}{3}}(1-m)\sqrt{\frac{T^{2(m-2)}}{2m(1-m)} + \frac{1}{T^{2(m+2)}} - \frac{1}{(m^2-1)T^2}}.$$

This shows that the model (2.11) isotropizes at later time when $-2 < m < 2$ which agrees with 'Cosmic No-Hair Conjecture' [17].

In the absence of viscosity, i.e., when $\ell \to 0$, the metric (2.10) reduces to

$$\begin{aligned} \mathrm{d}s^2 = {} & \frac{-\mathrm{d}T^2}{\left[\frac{T^{2m-2}}{2m(1-m)} + \frac{\beta}{T^{2m+2}} + \frac{1}{(m^2-1)}\right]} + T^{2m}\mathrm{d}x^2 + T^2\mathrm{d}y^2 \\ & + (T^2 \sin^2 y + T^{2m}\cos^2 y)\mathrm{d}z^2 - 2T^{2m}\cos y\mathrm{d}x\mathrm{d}z. \end{aligned} \tag{3.7}$$

From eq. (2.6), we have

$$\alpha = \left[(1+m) - 16\pi\ell\frac{(m^2+m-2)}{(1-m)}\right].$$

From the above equation, we find that in the absence of viscosity, $\alpha = 1 + m$ as $\ell = 0$.

In the absence of viscosity, $\alpha = 1 + m$ as $\ell = 0$, the expressions for pressure, density, expansion (θ) and shear (σ) are given by

$$\begin{aligned} 8\pi p = {} & \left[\frac{(3m^2+m-2)}{4m(m-1)}\right]T^{2m-4} + \beta(2m+1)\frac{1}{T^{2m+4}} \\ & + \frac{m^2}{m^2-1}\left(\frac{1}{T^2}\right) - \Lambda \end{aligned} \tag{3.8}$$

$$8\pi\varepsilon = \frac{(1+m)(2+m)}{4m(1-m)}T^{2m-4} + \frac{\beta(2m+1)}{T^{2m+4}} + \left[\frac{m(m+2)}{m^2-1}\right]\frac{1}{T^2} + \Lambda \tag{3.9}$$

$$\theta = (m+2)\sqrt{\frac{T^{2m-4}}{2m(1-m)} + \frac{\beta}{T^{2m+4}} - \frac{1}{(m^2-1)T^2}} \tag{3.10}$$

$$\sigma = \frac{\sqrt{2}}{\sqrt{3}}(1-m)\sqrt{\frac{T^{2m-4}}{2m(1-m)} + \frac{\beta}{T^{2m+4}} - \frac{1}{(m^2-1)T^2}}. \tag{3.11}$$

The energy conditions $(\varepsilon + p) > 0$ and $(\varepsilon + 3p) > 0$ lead to

$$\left[\frac{(2m+1)}{2(1-m)}T^{2m-4} + \frac{2\beta(2m+1)}{T^{2m+4}} + \frac{2m}{1-m}\left(\frac{1}{T^2}\right)\right] > 0 \tag{3.12}$$

and

$$\left[\frac{(5m^2+3m-4)}{2m(1-m)}T^{2m-4} + \frac{4\beta(2m+1)}{T^{2m+4}} + \frac{2m(2m+1)}{(m^2-1)T^2}\right] > 2\Lambda \tag{3.13}$$

which gives condition on Λ. Thus energy conditions (i) $\varepsilon + p > 0$, (ii) $\varepsilon + 3p > 0$ in the absence of viscosity, are satisfied when $0 < m < 1$.

In the absence of viscosity, the expansion in the model starts with a big-bang at $T = 0$ and the expansion in the model decreases with time. When $T \to 0$ then $\varepsilon \to \infty$, $p \to \infty$ where $m < 2$ and when $T \to \infty$ then $\varepsilon \to \Lambda$ and $p \to -\Lambda$. Since $\lim_{T\to\infty}(\sigma/\theta) \neq 0$. Hence the model does not approach isotropy for large values of T in the absence of viscosity in general.

However, if $m = 1$ then the model (2.11) in absence of viscosity, isotropizes in general. Also

$$\sigma = \sqrt{\frac{2}{3}}(1-m)\sqrt{\frac{T^{2(m-2)}}{2m(1-m)} + \frac{\beta}{T^{2(m+2)}} - \frac{1}{(m^2-1)T^2}}.$$

Thus in the absence of viscosity $\sigma \to 0$ when $T \to \infty$ which shows that the model (3.10) isotropizes at a later time when $-2 < m < 2$ which agrees with 'Cosmic No-Hair Conjecture' [17].

Appendix

$$\alpha = (1+m) - \frac{16\pi\ell(m^2+m-2)}{1-m} = \frac{(1-m)(1+m) + 16\pi\ell(1-m)(m+2)}{(1-m)}$$
$$= 1 + m + 16\pi\ell(m+2) > 0, \quad (0 < m < 1).$$

For energy conditions, (i) $\varepsilon + p > 0$, (ii) $\varepsilon + 3p > 0$

$$\begin{aligned}\varepsilon + p = &\left[\frac{-m^2(32\pi\ell+\alpha) - m(32\pi\ell - 3\alpha - 6) + (64\pi\ell - 3\alpha + 15)}{6(m+\alpha-1)(1-m)}\right]T^{2m-4} \\ &+ \left[\frac{m^2(16\pi\ell+3) + m(16\pi\ell - 3\alpha - 6) - (32\pi\ell - 3\alpha + 3)}{3\alpha(1-m)}\right]\frac{1}{T^2} \\ &+ \frac{\beta}{3T^{2\alpha+2}}\left[-m^2(16\pi\ell+3) - m(16\pi\ell - 3\alpha - 6) + (32\pi\ell + 3\alpha + 3)\right] \\ &+ 8\pi\zeta(m+2)\sqrt{\frac{T^{2m-4}}{(1-m)(m+\alpha-1)} + \frac{\beta}{T^{2\alpha+2}} - \frac{1}{\alpha(1-m)T^2}},\end{aligned}$$

where β is the constant of integration.

$$\begin{aligned}\alpha(1-m) &= 1-m^2-16\pi\ell(m^2+m-2)\\ &= (1-m)(1+m)-16\pi\ell(m-1)(m+2)\\ &= (1-m)[1+m+16\pi\ell(m+2)]>0\end{aligned}$$

when $m<1$.

$$\begin{aligned}(m+\alpha-1)(1-m) &= \left[m-1+1+m-\frac{16\pi\ell(m^2+m-2)}{1-m}\right](1-m)\\ &= (2m-2m^2-16\pi\ell(m^2+m-2))\\ &= 2m[(1-m)-8\pi\ell(m+2)(m-1)]\\ &= 2m(1-m)[1+8\pi\ell(m+2)]>0\end{aligned}$$

when $m<1$.

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$$-32\pi\ell(m^2+m-2)-3(3m^2-2m-5)+3\alpha(m-1)>0.$$

Since

$$\left.\begin{aligned}m^2+m-2=(m-1)(m+2)<0\\ 3m^2-2m-5=(m+1)(3m-5)<0\end{aligned}\right\}\quad \text{as } m<1,$$

$$\alpha(m-1)<0.$$

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$$\begin{aligned}&= 16\pi\ell(m^2+m-2)+3(m^2-2m+1)+3\alpha(1-m)\\ &\quad 16\pi\ell(m^2+m-2)=16\pi\ell(m-1)(m+2)<0\quad \text{as } m<1\\ &\quad 3(m^2-2m+1)=3(1-m)^2>0\\ &\quad 3\alpha(1-m)>0.\end{aligned}$$

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$$-16\pi\ell(m^2+m-2)-3(m^2-2m-1)+3\alpha(m+1).$$

Now

$$\begin{aligned}&-16\pi\ell(m^2+m-2)=16\pi\ell(1-m)(m+2)>0\quad \text{as } m<1\\ &-3(m^2-2m-1)>0\quad \text{as } m<1\\ &\alpha(m+1)>0\quad \text{as } \alpha>0.\end{aligned}$$

Therefore the energy condition $\varepsilon+p>0$ is satisfied.

For energy condition

$$\varepsilon+3p>0$$

$$
\begin{aligned}
8\pi(\varepsilon+3p) = & \left\{\frac{-m^2(16\pi\ell+5)-m(16\pi\ell-\alpha)+(32\pi\ell-\alpha+5)}{(m+\alpha-1)(1-m)}\right\}T^{2m-4} \\
& +\beta\frac{1}{T^{2\alpha+2}}\left\{(-m^2)(16\pi\ell+3)-m(16\pi\ell-3\alpha-2)+(32\pi\ell+3\alpha+1)\right\} \\
& +\left\{\frac{m^2(16\pi\ell+3)+m(16\pi\ell-\alpha-2)-(32\pi\ell-\alpha+1)}{\alpha(1-m)T^2}\right\} \\
& +24\pi\zeta(m+2)\sqrt{\frac{T^{2m-4}}{(1-m)(m+\alpha-1)}+\frac{\beta}{T^{2\alpha+2}}-\frac{1}{\alpha(1-m)T^2}} \\
& -2\Lambda.
\end{aligned}
$$

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$$-16\pi\ell(m^2+m-2) = -16\pi\ell(m-1)(m+2) > 0 \quad \text{as } m<1$$

$$5(1-m^2) = 5(1-m)(1+m) > 0 \quad \text{for } m<1\alpha(m-1)<0 \quad \text{as } m<1.$$

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$$
\begin{aligned}
& -16\pi\ell(m^2+m-2) = -16\pi\ell(m-1)(m+2) > 0 \quad \text{as } m<1 \\
& -(3m^2-2m-1) = -(m-1)(3m+1) > 0 \quad \text{as } m<1.
\end{aligned}
$$

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$$
\begin{aligned}
& 16\pi\ell(m^2+m-2) = 16\pi\ell(m+2)(m-1) < 0 \quad \text{as } m<1 \\
& 3m^2-2m-1 = (m-1)(3m+1) < 0 \quad \text{as } m<1 \\
& \alpha(1-m) > 0.
\end{aligned}
$$

Thus the energy condition

$$\varepsilon+3p>0$$

is satisfied and it gives condition on Λ.

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