PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 64, No. 1
January 2005
pp. 31–37

Nuclear matter equation of state and σ-meson parameters

A B SANTRA[1] and U LOMBARDO[2]
[1]Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai 400 085, India
[2]Dipartimento di Fisica, Universita di Catania and INFN-LNS, Via S. Sofia 44, I-95123 Catania, Italy
E-mail: santra@apsara.barc.ernet.in

MS received 9 December 2003; revised 16 April 2004; accepted 13 September 2004

Abstract. We try to determine phenomenologically the extent of in-medium modification of σ-meson parameters so that the saturation observables of the nuclear matter equation of state (EOS) are reproduced. To calculate the EOS we have used Brueckner–Bethe–Goldstone formalism with Bonn potential as two-body interaction. We find that it is possible to understand all the saturation observables, namely, saturation density, energy per nucleon and incompressibility, by incorporating in-medium modification of σ-meson–nucleon coupling constant and σ-meson mass by a few per cent.

Keywords. Nuclear matter equation of state; medium modification of meson parameters.

PACS Nos 21.65.+f; 21.10.Dr; 21.30.Fe

The symmetric nuclear matter is characterized by its EOS, the empirical information about which are through saturation density, ρ_0, the energy per nucleon, E/A, at the saturation density and the incompressibility, K, of saturated nuclear matter. Values of these quantities have been found to be: $\rho_0 = 0.17 \pm 0.02$ fm^{-3}, $E(\rho_0)/A = -16 \pm 1$ MeV and $K = 210 \pm 30$ MeV [1].

A large number of calculations of the nuclear matter EOS have been done by various groups within non-relativistic and relativistic Brueckner and variational framework using both phenomenological and microscopic two-body nucleon–nucleon interaction potential [1–9] constrained by nucleon–nucleon scattering phase shifts and deuteron properties. It has been observed that saturation densities obtained from these EOS in non-relativistic calculations with different versions of two-body interaction potential lie on a coester line which is systematically away from the experimental saturation density. Relativistic calculations [1,10–12] produce another coester line which passes near the experimental saturation density. However, non-relativistic calculation with two-body and an additional three-body potential with a few adjustable parameters [13–15] improve the situation in reproducing the saturation observables.

In recent years ideas from non-perturbative QCD, namely, modification of hadron parameters in nuclear medium due to partial restoration of chiral symmetry, have been employed [16,17] to understand the saturation observables of nuclear matter. These calculations were done in relativistic Dirac–Brueckner [16] as well as non-relativistic Brueckner–Bethe–Goldstone [17] formalism by taking the Bonn potential as two-body interaction, in which the mesonic degrees of freedom have been treated explicitly. Extent of modification of the parameters of Bonn potential (nucleon mass, cut-off masses at the meson–nucleon vertices and masses of vector and scalar mesons) was guided by prescriptions of Brown–Rho scaling [18], estimations of quark–meson coupling model [19] and QCD sum rule [20].

By using relativistic Dirac–Brueckner–Hartree–Fock approach alone to calculate the EOS and Brown–Rho scaling for modification of hadron parameters, it was not possible to get saturation in nuclear matter EOS [16]. To improve the situation, additional many-body effect coming from the hadronic sector, namely, modification of σ-meson (two-pion correlator in 0^+ channel) spectral function due to in-medium polarization of pion to nucleon–hole and delta–hole states, was included to get the saturation observables close to experimental values. These effects were incorporated in one-boson exchange model of nucleon–nucleon interaction by suitably modifying the mass and coupling with nucleon of the zero-width σ-meson. Modification of σ-meson–nucleon coupling mocks up, as it were, the modification of σ-meson width in nuclear medium.

On the other hand, in [17] using non-relativistic Brueckner–Bethe–Goldstone formalism it was possible to get saturation in nuclear matter EOS using Brown–Rho scaling alone for modification of hadron parameters. However, energy per nucleon and incompressibility could not be matched with corresponding experimental values. It was possible to match the energy per nucleon to its experimental value, by taking a non-linear form for the modification of cut-off masses with respect to density, but the incompressibility came out very large.

Modification of hadronic (particularly mesons) parameters may originate from two sources: firstly due to interactions in the hadronic sector and secondly due to partial restoration of chiral and scale symmetry of QCD in nuclear medium. The latter effect gives rise to Brown–Rho scaling [18]. As it is not possible to reproduce the saturation observables of nuclear matter by modifying the hadron parameters according to Brown–Rho scaling alone, and the spectral function of σ-meson is most vulnerable to be affected by in-medium pion polarization [16], we aim here to determine phenomenologically the extent of in-medium modification of σ-meson parameters, which alone will be required to understand the saturation observables of nuclear matter within non-relativistic Brueckner–Bethe–Goldstone framework. It is also very much of current interest [21] to know the extent of in-medium modification of σ-meson spectral function.

We consider σ-meson to be of zero width and assume that its role in one-boson-exchange model of nucleon–nucleon interaction can be taken care of through its mass, coupling with nucleon and cut-off mass at the σ-nucleon interaction vertex. We concentrate on the modification of only two parameters, σ-meson mass and σ-nucleon coupling constant.

As it is well-known that σ-meson is responsible for generating the medium range attraction in nucleon–nucleon interaction, reduction of its mass would generate

Table 1. Meson parameters used in Bonn-B potential.

Meson	$g^2/4\pi$	f/g	Mass (MeV)	Λ (MeV)
π	14.40	–	138.03	1700
η	3.00	–	548.80	1500
ρ	0.90	6.10	769.00	1850
ω	24.50	0	782.60	1850
δ	2.488	–	983.00	2000
σ	8.9437	–	550.00	1900
σ^*	18.3773	–	720.00	2000

*Parameters of σ-meson to use for $T = 0$ nucleon–nucleon potential.

attractive effect on the EOS, thus pushing the saturation density away from its experimental value. However, reduction of σ-nucleon coupling would generate repulsive effects, which may bring the saturation density towards the experimental value. So, first we modify σ-nucleon coupling, $g_{\sigma\mathrm{NN}}$, as

$$g^*_{\sigma\mathrm{NN}} = g_{\sigma\mathrm{NN}}\,(1 - \alpha\,\tilde{\rho})\,, \tag{1}$$

where $\tilde{\rho} = \rho/\rho_0$ with ρ_0 as the nuclear matter saturation density (0.17 fm^{-3}), $g^*_{\sigma\mathrm{NN}}$ is the in-medium coupling constant at density ρ and α is a parameter which determines the extent of modification of the coupling constant at saturation density. Dependence of meson–nucleon coupling constant with density in nuclear medium has been discussed in relativistic mean field model [22] and quark meson coupling model [19].

We calculate the EOS in Brueckner–Bethe–Goldstone formalism taking Bonn potential as two-body interaction with its parameters given in table 1. We have used conventional choice for single nucleon energies, i.e., bound energy below the Fermi sea and free energy above the Fermi sea. The σ-nucleon coupling is modified using eq. (1).

The results are shown in figure 1. We see that the EOS becomes more and more stiff with increasing values of α. The effect of reduction of $g_{\sigma\mathrm{NN}}$ is highly repulsive. The saturation density decreases from 0.25 fm^{-3} to 0.08 fm^{-3} as α is increased from 0.0 to 0.08. Though at some intermediate value of α, the saturation density matches with the experimental value, the energy per particle is always higher than the experimental value.

Thus, we see that by changing the σ-nucleon coupling alone as given in eq. (1), it is not possible to reproduce the saturation density and energy per nucleon simultaneously.

Now, we turn towards the other parameter, i.e., mass of σ-meson. We expect that the attractive effect generated in nuclear matter EOS due to reduction of σ-meson mass may bring down the energy per nucleon to its experimental value. So, in addition to the modification of coupling constant given in eq. (1) we modify σ-meson mass, m_σ, as given below.

$$m^*_\sigma = m_\sigma\,(1 - \beta\,\tilde{\rho})\,, \tag{2}$$

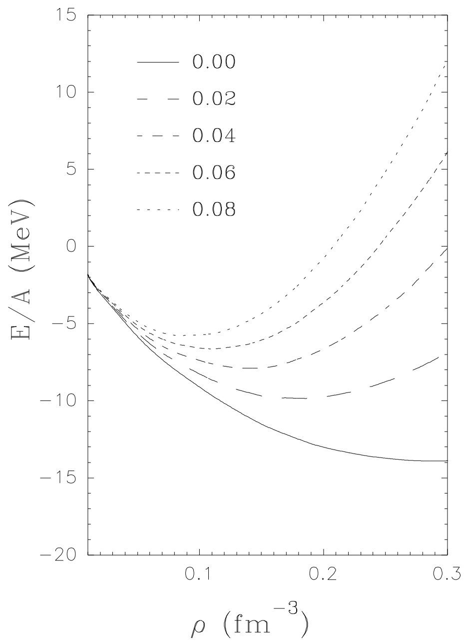

Figure 1. EOS of symmetric nuclear matter when σ-nucleon coupling constant of Bonn potential is modified according to eq. (1). Various curves correspond to different values of α of eq. (1).

where m_σ^* is the σ-meson mass in nuclear medium at density ρ. We have taken for this parameter also a linear dependence introducing a parameter β which determines the modification of m_σ at saturation density.

Now, we calculate the EOS again by taking the Bonn potential as two-body interaction modifying its σ-nucleon coupling and σ-meson mass as given in eqs (1) and (2) respectively. For a given value of α in the range from 0.0 to 0.20 we fix β in such a way that the EOS gives saturation density at 0.17 fm^{-3}. The reduction of mass of σ-meson causes much more attraction than the repulsion generated by the reduction of σ-nucleon coupling. We find that repulsion generated by 20% reduction of σ-nucleon coupling is balanced by attraction generated by only 4% reduction of σ-meson mass. We determine the energy per nucleon and incompressibility from the EOS so obtained. We find from the EOS curves obtained for various values of α and β that the energy per nucleon and the incompressibility cannot be matched with their corresponding experimental values for any set of values of α and β.

Thus, we see again that it is not possible to reproduce the energy per nucleon and the incompressibility by changing the σ-nucleon coupling given in eq. (1) and σ-meson mass given in eq. (2), though saturation density can be matched with its experimental value.

To improve the situation, we discard the linear dependence of in-medium σ-meson given in eq. (2) and take a density-independent form for in-medium σ-meson mass as given below.

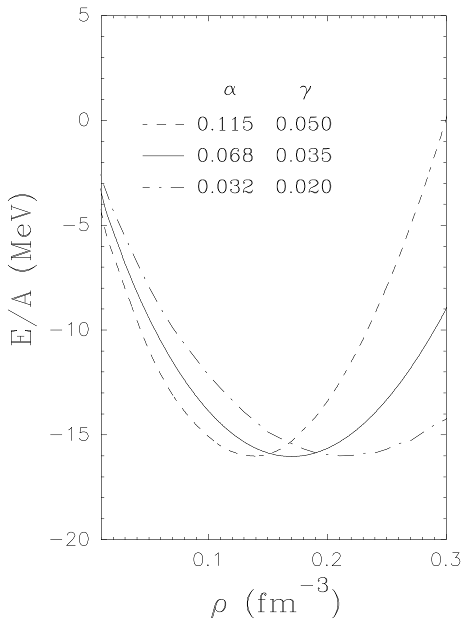

Figure 2. EOS of symmetric nuclear matter when σ-nucleon mass and coupling constant of Bonn potential is modified according to eqs (1) and (3) respectively. Various curves correspond to different values of α and γ.

$$m_\sigma^* = m_\sigma\,(1-\gamma)\,, \tag{3}$$

where m_σ^* is, as before, the σ-meson mass in nuclear medium. Choice of such a density-independent form has been guided by the following considerations. Spectral function of σ-meson may have two components [16]. Only one of those, which can be represented by a two-pion correlator and which is susceptible to in-medium modification due to pion polarization to nucleon–hole and delta–hole states, gets modified in the medium, while the other part is density independent. Our ansatz is that medium-dependent part of the spectral function saturates very fast.

Now, we calculate the EOS once again by taking Bonn potential as two-body interaction modifying its σ-nucleon coupling and σ-meson mass as given in eqs. (1) and (3) respectively. The results are shown in figure 2. We see that for $\alpha = 0.032$, calculated EOS gives energy per nucleon at -16 MeV for $\gamma = 0.020$. However, the saturation density comes out at 0.21 fm^{-3} which is more than the experimental value. If we take $\alpha = 0.068$ we get, for $\gamma = 0.035$, both the energy per nucleon and saturation density at their corresponding experimental values of -16 MeV and 0.17 fm^{-3} respectively. The incompressibility obtained from this EOS is 215 MeV. Thus, we see that it is possible to understand all three saturation observables of symmetric nuclear matter by incorporating modification of mass and coupling of σ-meson by a very little extent. We also show the EOS obtained for $\alpha = 0.115$. In this case, calculated EOS gives energy per nucleon at -16 MeV for $\gamma = 0.050$. However, the saturation density comes out at 0.13 fm^{-3} which is less than the experimental value.

Thus, we conclude from the above analysis that the saturation density, energy per nucleon and incompressibility of symmetric nuclear matter can be understood in the Brueckner–Bethe–Goldstone framework with Bonn potential by modifying its σ-meson parameters alone to a very small extent. We find that the σ-nucleon coupling constant and σ-meson mass are to be reduced only by 7% and 3.5% respectively of the corresponding values required to understand the nucleon–nucleon phase shifts and deuteron properties. Modification of mass required is much smaller than that of QCD models [19,20]. We need only two parameters to understand three saturation observables. Though we have analysed by taking modifications on mass and coupling of σ-meson alone, effects due to in-medium modification of masses and couplings of other mesons, ω in particular should be included. Also, we need to know in-medium modification of the spectral function from microscopic calculation. We are looking into these aspects for complete analysis.

Before we end, a few comments are in order.

1. The ansatz (3), which may appear to be *ad hoc*, is guided by the following considerations: Due to pion polarization effect, modification of the width of the in-medium σ-meson spectral function is much more than that of the peak position. In some model calculations [21], modification of the peak position of the spectral function is very small. Thus, in-medium σ-meson mass, which is related to the peak position of the spectral function, does not depend strongly on density, and it attains a density-independent value very fast. However, the effect of in-medium density-dependent width is reflected through density-dependent σ-nucleon coupling.
2. To determine the parameter ρ_0 self-consistently, we first calculate the EOS with $\alpha = 0$ (1) and $\gamma = 0$ (3), then identify ρ_0 from the EOS as the density where E/A is minimum. For the next calculation with non-zero values of γ and α, we take ρ_0 which was found previously. Once again, we calculate ρ_0 from the calculated EOS and put this new ρ_0 in the next calculation. This procedure is continued till all the satuartion observables match with their corresponding experimental values. Thus, though ρ_0 appears to be a parameter, its value (0.17 fm^{-3}) has been determined self-consistently.
3. Though the primary aim of this paper is to investigate the effect of modification of σ-meson parameters alone, we find, by including modification of parameters of all other mesons with a variety of modification schemes, it is not possible in the framework of non-relativistic Brueckner calculation to match simultaneously all the three saturation observables, namely, the saturation density, binding energy and incompressibility to their corresponding experimental values, by taking any other choice of meson parameter modification scheme than given here. However, if we restrict only to the saturation density and binding energy, it is possible to match these two observables with their corresponding experimental values by taking other meson parameter modification scheme. In this sense, the meson parameter modification scheme presented here is robust.

Acknowledgements

The authors are grateful to Ruprecht Machleidt for giving them his codes of Bonn potential and nuclear matter calculations. ABS acknowledges hospitality of INFN-LNS where the work was started.

References

[1] R Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989)
[2] T Hamada and I D Johnston, *Nucl. Phys.* **34**, 382 (1962)
[3] H A Bethe and M B Johnson, *Nucl. Phys.* **A230**, 1 (1974)
[4] R V Reid, *Ann. Phys. (N.Y.)* **50**, 411 (1968)
[5] R B Wiringa, R A Smith and T L Ainsworth, *Phys. Rev.* **C29**, 1207 (1984)
[6] M Lacombe, B Loiseau, J M Richard, R Vinh Mau, P Pires and R de Toureil, *Phys. Rev.* **C21**, 861 (1980)
[7] K Hollinde and R Machleidt, *Nucl. Phys.* **A247**, 495 (1975)
[8] G Schierholz, *Nucl. Phys.* **B40**, 332 (1972)
[9] T Ueda, M Nack and A E S Green, *Phys. Rev.* **C8**, 2061 (1973)
[10] B ter Haar and R Malfliet, *Phys. Rep.* **149**, 208 (1987)
[11] C H Lee, T T S Kuo, G Q Li and G E Brown, *Phys. Rev.* **C57**, 3448 (1998)
[12] G E Brown, W Weise, G Baym and J Speth, *Comm. Nucl. Part. Phys.* **17**, 39 (1987)
[13] B Friedman and V R Pandharipande, *Nucl. Phys.* **A361**, 502 (1981)
[14] R B Wiringa, V Fiks and A Fabocini, *Phys. Rev.* **C38**, 1010 (1998)
[15] M Baldo, I Bombaci and G F Burgio, *Ap. J.* **328**, 274 (1997)
[16] R Rapp, R Machleidt, J W Durso and G E Brown, *Phys. Rev. Lett.* **82**, 1827 (1999)
[17] A B Santra and U Lombardo, *Phys. Rev.* **C62**, 018202 (2000)
[18] G E Brown and M Rho, *Phys. Rev. Lett.* **66**, 2720 (1991)
[19] K Saito, K Tsushima and A W Thomas, *Phys. Rev.* **C55**, 2637 (1997)
[20] T Hatsuda and S H Lee, *Phys. Rev.* **C46**, R34 (1992)
[21] P Schuck, Z Aouissat, G Chanfray and J Wambach, nucl-th/0002031
[22] S Typel and H H Wolter, *Nucl. Phys.* **A656**, 331 (1999)