PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 63, No. 6 December 2004 pp. 1367–1379

Working group report: Quantum chromodynamics

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Abstract. This is the report of the QCD working sub-group at WHEPP-8 which was part of the QCD and QGP working group. Discussion and work on some aspects of resummation and parton distribution are reported.

Keywords. Quantum chromodynamics; resummation; polarised scattering; parton distributions.

PACS No. 12.38.-t

1. Introduction

The participants of the QCD working sub-group are: Rahul Basu, D Indumathi, E Laenen, Swapan Majhi, Prakash Mathews, Anuradha Misra, Asmita Mukherjee, R Ratabole, V Ravindran and W Vogelsang.

The main focus of this working group had been to concentrate on some issues in resummation which are essential to extend the predictive power of QCD by summing large logarithmic corrections to all orders in perturbation theory. Resummation in connection with observable at TeV colliders and RHIC spin physics were considered. Some aspects of parametrisation of parton distributions where also discussed.

The working group activity was structured around talks followed by discussions. The emphasis in this working group was on these discussions, which were usually moderated by the speaker. The talks as part of this working group activity are listed as follows:

Plenary talks

- Resummation for observable at TeV colliders — E Laenen
- QCD spin physics: Status and potential at RHIC — W Vogelsang

The above talks by Laenen and Vogelsang are part of these proceedings.

Working group talks

- Threshold and joint resummation techniques — E Laenen
- Photon content of the polarised and unpolarised nucleon — A Mukherjee
- Recent QCD results for Higgs searches at LHC — V Ravindran
- Physics prospects at eRHIC — W Vogelsang
- Resummations for various observables — W Vogelsang

This report summarises the work by participants in this working group.

An essential ingredient of perturbative QCD is that the cross-section can be factorised into a hard part and a soft part. The hard part quantifies the interaction of the partons and is IR safe and calculable in perturbative theory provided the coefficients of the perturbative expansion are small. But these coefficients could be enhanced at kinematic edges of phase space. The cancellation of IR divergences occurs between the real and virtual corrections. In the case of virtual corrections the integration spans all energies but in the real corrections the kinematics determines the phase space region and the cancellation is exact only for completely inclusive case. Kinematical constraints could lead to incomplete cancellations resulting in large logarithms and the series would have to be resummed to all orders to improve the predictive power. Consequently, the resummation formalism depends on the observable under consideration.

This working group was particularly interested in the following: (i) Extend the next-to-leading logarithm resummation of threshold enhancements for the prompt photon transverse momentum distribution to include leading flavor-diagonal collinear effects. Their impact is assessed by comparing with the purely threshold-resummed, as well as joint-resummed distribution, for both fixed-target and collider kinematics. (ii) With the spin physics program at RHIC it is important to extend the resummation framework to polarised process to look at polarised observables. (iii) To study error propagation in parametrising parton distributions using orthogonal polynomials.

2. Soft gluon resummation in SVC approximation for prompt photon hadroproduction

Rahul Basu, E Laenen, S Majhi, A Misra and W Vogelsang

2.1 *Introduction*

Effects of soft gluon resummation in prompt photon hadroproduction have been computed in recent years at next-to-leading logarithmic (NLL) resummed level

[1–5] and expansions thereof [6,7]. The general technique for resummation involves going over to Mellin space or N space, where N is the parameter conjugate to the kinematic variable that measures the distance from threshold. In Melllin space the threshold region corresponds to the limit $N \to \infty$ and NLL accuracy contributions to $O(\alpha_{\rm s}^n \ln^{n+1} N)$ and $O(\alpha_{\rm s}^n \ln^n N)$ in the exponent are resummed.

We have extended the resummation formalism for prompt photon hadroproduction to include subdominant contribution in large-z limit. These terms are proportional to $\ln^k(1-z)$ and in Mellin space they appear as $\frac{1}{N}\ln^k N$. These are usually neglected in the limit $N \to \infty$, but have been shown to be phenomenologically important for Higgs production cross-sections [8–10].

These terms have a purely collinear origin and the flavor-diagonal leading terms can be incorporated in a threshold-resummation formalism by including the regular part of Altarelli-Parisi splitting functions in the integrand of the resummed exponent [8]. Even the non-diagonal terms can be derived by replacing the flavor-diagonal anomalous dimension by the full one [11,12]. However, in this write-up we restrict ourselves to the flavor-diagonal effects, reserving a more complete study for the future.

2.2 *Collinear enhanced exponents*

In Mellin space, the resummed radiative factor factorises into a product of independent radiative factors for two initial and one final parton appearing in the Born process and a factor for soft gluon interference between initial and final states,

$$\sigma^{(\rm res)}_{ab\to d\gamma} = \alpha\alpha_{\rm s}\sigma^{(0)}_{ab\to d\gamma,N} C_{ab\to d\gamma}\Delta^{ab\to d\gamma}_{N+1}, \tag{1}$$

where $\Delta^{ab\to d\gamma}_{N+1}$ is the soft gluon radiative factor given by

$$\begin{aligned}\Delta^{ab\to d\gamma}_{N}\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2};\frac{Q^2}{\mu_F^2}\right) &= \Delta^{a}_{N}\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2};\frac{Q^2}{\mu_F^2}\right)\\ &\times\Delta^{b}_{N}\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2};\frac{Q^2}{\mu_F^2}\right)\Delta^{({\rm int})ab\to d\gamma}_{N}\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2}\right)\\ &\times J^{d}_{N}\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2}\right),\end{aligned} \tag{2}$$

where Δ^a_N and Δ^b_N are the resummed factors for incoming partons, J^d_N is the radiative factor for the final state parton and $\Delta^{({\rm int})ab\to d\gamma}_N$ is the interference term. These radiative factors are given by the following expressions:

$$\begin{aligned}\Delta^{a}_{N}\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2};\frac{Q^2}{\mu_F^2}\right) &= \exp\Bigg[\int_0^1 {\rm d}z\frac{z^{N-1}-1}{1-z}\\ &\times\int_{\mu_F^2}^{(1-z)^2Q^2}\frac{{\rm d}q^2}{q^2}A_a(\alpha_{\rm s}(q^2)) + O(\alpha_{\rm s}(\alpha_{\rm s}\ln N)^k)\Bigg],\end{aligned} \tag{3}$$

$$J_N^d\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2};\frac{Q^2}{\mu_F^2}\right)=\exp\Bigg[\int_0^1 {\rm d}z\frac{z^{N-1}-1}{1-z}\times\int_{(1-z)Q^2}^{(1-z)Q^2}\frac{{\rm d}q^2}{q^2}A_d(\alpha_{\rm s}(q^2))+\frac{1}{2}B_d(\alpha_{\rm s}((1-z)Q^2))\Bigg], \quad (4)$$

$$\Delta_N^a\left(\alpha_{\rm s}(\mu^2),\frac{Q^2}{\mu^2}\right)=\exp\Bigg[\int_0^1 {\rm d}z\frac{z^{N-1}-1}{1-z}D_{ab\to d\gamma}(\alpha_{\rm s}((1-z)Q^2))+O(\alpha_{\rm s}(\alpha_s\ln N)^k)\Bigg]. \quad (5)$$

The radiative factors have been evaluated in ref. [2] up to NLLO and are given by

$$\ln\Delta_N^a=\ln N\,h_a^{(1)}(\lambda)+h_a^{(2)}\left(\lambda,\frac{Q^2}{\mu^2},\frac{Q^2}{\mu_F^2}\right)+O(\alpha_{\rm s}(\alpha_{\rm s}\ln N)^k), \quad (6)$$

$$\ln J_N^d=\ln N\,f_a^{(1)}(\lambda)+f_a^{(2)}\left(\lambda,\frac{Q^2}{\mu^2}\right)+O(\alpha_{\rm s}(\alpha_{\rm s}\ln N)^k), \quad (7)$$

$$\ln\Delta_N^{({\rm int})ab\to d\gamma}=\frac{D^{(1)}_{ab\to d\gamma}}{2\pi b_0}\ln(1-2\lambda)+O(\alpha_{\rm s}(\alpha_{\rm s}\ln N)^k). \quad (8)$$

Here $h_a^{(1)}$, $f_a^{(1)}$ and $h_a^{(2)}$, $f_a^{(2)}$ are the LL and NLL functions given in terms of perturbative coefficients $A_a^{(1)}$, $A_a^{(2)}$ and $B_a^{(1)}$ as

$$h_a^{(1)}=+\frac{A_a^{(1)}}{2\pi b_0\lambda}[2\lambda+(1-2\lambda)\ln(1-2\lambda)], \quad (9)$$

$$\begin{aligned}h_a^{(2)}=&+\frac{A_a^{(1)}}{2\pi b_0^3}\left[2\lambda+(1-2\lambda)\ln(1-2\lambda)+\frac{1}{2}\ln^2(1-2\lambda)\right]\\&-\frac{A_a^{(1)}\gamma_E}{\pi b_0}\ln(1-2\lambda)-\frac{A_a^{(2)}}{2\pi^2b_0^2}[2\lambda+\ln(1-2\lambda)]\\&+\frac{A_a^{(1)}}{2\pi b_0}[2\lambda+\ln(1-2\lambda)]\ln\frac{Q^2}{\mu^2}-\frac{A_a^{(1)}}{\pi b_0}\lambda\ln\frac{Q^2}{\mu_F^2},\end{aligned} \quad (10)$$

$$f_a^{(1)}=-\frac{A_a^{(1)}}{2\pi b_0\lambda}[(1-2\lambda)\ln(1-2\lambda)-2(1-\lambda)\ln(1-\lambda)], \quad (11)$$

$$f_a^{(2)}=-\frac{A_a^{(1)}b_1}{2\pi b_0^3}\Bigg[\ln(1-2\lambda)-2\ln(1-\lambda)$$

$$
\begin{aligned}
&+\frac{1}{2}\ln^2(1-2\lambda)-\ln^2(1-\lambda)\Big]\\
&+\frac{B_a^{(1)}}{2\pi b_0}\ln(1-\lambda)-\frac{A_a^{(1)}\gamma_E}{\pi b_0}[\ln(1-\lambda)-\ln(1-2\lambda)]\\
&-\frac{A_a^{(2)}}{2\pi^2 b_0^2}[2\ln(1-\lambda)-\ln(1-2\lambda)]\\
&+\frac{A_a^{(1)}}{2\pi b_0}[2\ln(1-\lambda)-\ln(1-2\lambda)]\ln\frac{Q^2}{\mu^2}, \qquad (12)
\end{aligned}
$$

where

$$\lambda = b_0\alpha_{\rm s}(\mu^2)\ln N \qquad (13)$$

and the z integration is performed by setting

$$z^{N-1}-1=-\Theta\left(1-z-\frac{\exp(-\gamma_E)}{N}\right). \qquad (14)$$

These expressions are obtained by expanding the perturbative functions $A_a(\alpha_{\rm s})$, $B_d(\alpha_{\rm s})$ and $D_{ab\to d\gamma}$ in powers of $\alpha_{\rm s}$,

$$A_a(\alpha_{\rm s})=\frac{\alpha_{\rm s}}{\pi}A_a^{(1)}+\left(\frac{\alpha_{\rm s}}{\pi}\right)^2 A_a^{(2)}+O(\alpha_{\rm s}^3) \qquad (15)$$

and so on. To include the leading collinear enhancement in Δ_a, one can perform the following replacement in the exponent:

$$\frac{z^{N-1}-1}{1-z}A_q^{(1)}\to\left[\frac{z^{N-1}-1}{1-z}-z^{N-1}\right]A_q^{(1)}+\mathcal{O}\left(\frac{1}{N^2}\right) \qquad (16)$$

if the initial parton is a quark and

$$\frac{z^{N-1}-1}{1-z}A_g^{(1)}\to\left[\frac{z^{N-1}-1}{1-z}-2z^{N-1}\right]A_g^{(1)}+\mathcal{O}\left(\frac{1}{N^2}\right) \qquad (17)$$

if the initial parton is a gluon. The extra contributions to the quark and gluon radiative factors are then given by $\Delta'_{q,N}$ and $\Delta'_{g,N}$ where

$$
\begin{aligned}
\ln\Delta'_{q,N}=&-\int_0^1 {\rm d}z z^{N-1}\int_{\mu_F^2}^{(1-z)^2Q^2}\frac{{\rm d}q^2}{q^2}A_q^{(1)}\frac{\alpha_{\rm s}}{\pi}\\
&+A_q^{(2)}\left(\frac{\alpha_{\rm s}}{\pi}\right)^2+O(\alpha_{\rm s}(\alpha_{\rm s}\ln N)^k) \qquad (18)
\end{aligned}
$$

and

$$
\begin{aligned}
\ln\Delta'_{g,N}=&-2\int_0^1 {\rm d}z z^{N-1}\int_{\mu_F^2}^{(1-z)^2Q^2}\frac{{\rm d}q^2}{q^2}A_g^{(1)}\frac{\alpha_{\rm s}}{\pi}\\
&+A_g^{(2)}\left(\frac{\alpha_{\rm s}}{\pi}\right)^2 O(\alpha_{\rm s}(\alpha_{\rm s}\ln N)^k). \qquad (19)
\end{aligned}
$$

These integrals can be evaluated by noting that

$$z^{N-1} = \frac{z^{N-1} - 1 - (z^N - 1)}{1 - z} \tag{20}$$

and therefore $\Delta'_{q,N}$ can be expressed as

$$\begin{aligned}\ln \Delta'_{q,N} = &-\int_0^1 \frac{\mathrm{d}z}{1-z}\left[\Theta\left(1 - z - \frac{\exp(-\gamma_E)}{N}\right)\right.\\ &\left.-\Theta\left(1 - z - \frac{\exp(-\gamma_E)}{N+1}\right)\right]\\ &\times \int_{\mu_F^2}^{(1-z)^2 Q^2} \frac{\mathrm{d}q^2}{q^2}\left(A_q^{(1)}\frac{\alpha_\mathrm{s}}{\pi} + A_q^{(2)}\left(\frac{\alpha_\mathrm{s}}{\pi}\right)^2\right.\\ &\left.+O(\alpha_\mathrm{s}(\alpha_\mathrm{s}\ln N)^k)\right).\end{aligned} \tag{21}$$

This integral can be evaluated in the standard way and leads to

$$\ln \Delta'_{q,N} = -\frac{A_q^{(1)}}{\pi b_0}\exp\left(-\frac{\lambda}{\alpha_\mathrm{s} b_0}\right)\ln(1-2\lambda). \tag{22}$$

Similarly,

$$\ln \Delta'_{g,N} = -\frac{2A_g^{(1)}}{\pi b_0}\exp\left(-\frac{\lambda}{\alpha_\mathrm{s} b_0}\right)\ln(1-2\lambda). \tag{23}$$

To get the full leading $\ln N/N$ dependence, one must also add to this the contribution which comes from taking the Mellin transform of plus distributions and which is usually ignored in soft-virtual approximation. Taking these contributions into account, the radiative factors are modified to

$$\begin{aligned}\ln \Delta_N^a = &\ln N\, h_a^{(1)}(\lambda) + h_a^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}, \frac{Q^2}{\mu_F^2}\right)\\ &+h'_a(\lambda,\alpha_\mathrm{s}) + O(\alpha_\mathrm{s}(\alpha_\mathrm{s}\ln N)^k),\end{aligned} \tag{24}$$

$$\begin{aligned}\ln J_N^d = &\ln N\, f_a^{(1)}(\lambda) + f_a^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right)\\ &+f'_a(\lambda,\alpha_\mathrm{s}) + O(\alpha_\mathrm{s}(\alpha_\mathrm{s}\ln N)^k),\end{aligned} \tag{25}$$

where

$$h'_q = -\frac{A_q^{(1)}}{2\pi b_0}\exp\left(-\frac{\lambda}{\alpha_\mathrm{s} b_0}\right)\ln(1-2\lambda) \tag{26}$$

$$h'_g = -\frac{3A_g^{(1)}}{2\pi b_0}\exp\left(-\frac{\lambda}{\alpha_\mathrm{s} b_0}\right)\ln(1-2\lambda) \tag{27}$$

and

$$f'_q = \frac{A_q^{(1)}}{2\pi b_0} \exp\left(-\frac{\lambda}{\alpha_s b_0}\right) [\ln(1-2\lambda) - \ln(1-\lambda)] \tag{28}$$

$$f'_g = \frac{3A_g^{(1)}}{2\pi b_0} \exp\left(-\frac{\lambda}{\alpha_s b_0}\right) [\ln(1-2\lambda) - \ln(1-\lambda)]\,. \tag{29}$$

2.3 *Numerical studies*

We now study the impact of the new terms numerically. To get an impression of the typical size of the collinear corrections, we show in figure 1 the result of our preliminary analysis for prompt photon cross-section for fixed-target pN collisions at $\sqrt{s} = 31.5$ GeV. The plot shows the threshold resummed cross-section with and without $\ln N/N$ contribution as well as an unsubtracted and unmatched (with 5 GeV recoil cut-off) joint resummation [13] calculation. Inclusion of $\ln N/N$ terms leads to an enhancement comparable to the enhancement in joint resummation. At higher $p_{\rm T}$ values the collinear-enhanced threshold resummation result is larger than the joint resummed one.

For Tevatron collider kinematics ($\sqrt{S} = 1960$ GeV), we see that the collinear effects are again substantial, whereas those of joint resummation (with 15 GeV recoil cut-off) are marginal.

We conclude that a rather straightforward collinear enhancement to threshold-resummation yields remarkably large corrections, warranting a more extensive look in the near future in which effects of flavor-non-diagonal terms together with appropiate matching are included.

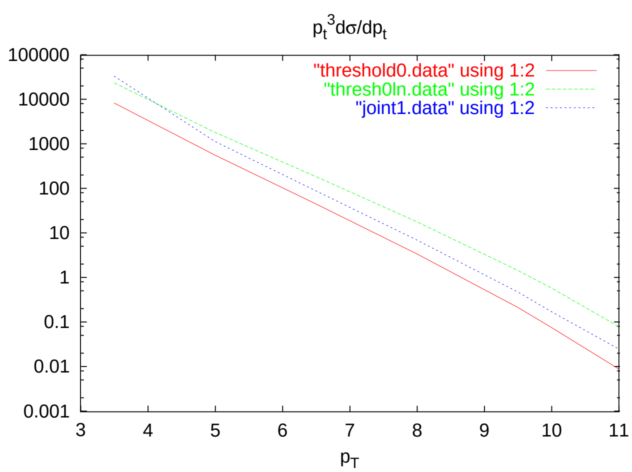

Figure 1. Effect of $\ln N/N$ compared to pure threshold and to joint resummation for E706 kinematics.

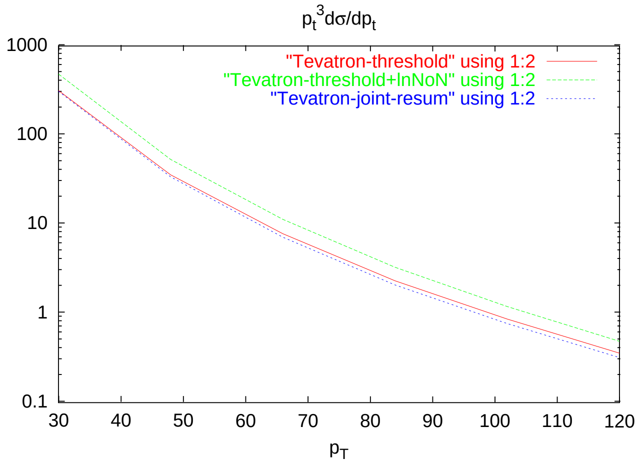

Figure 2. Effect of $\ln N/N$ compared to pure threshold and to joint resummation for Tevatron kinematics.

3. Spin asymmetries at RHIC and resummation

Rahul Basu, D Indumathi, Prakash Mathews, A Mukherjee, V Ravindran and W Vogelsang

The radiation of soft gluons in hard scattering processes with near-elastic kinematics generates large logarithmic higher order corrections to the partonic cross-sections. A classic process exemplifying this is Drell–Yan di-muon production, for which threshold corrections of the form $\alpha_{\rm s}^n \ln^{2n-1}(1-z)/(1-z)$ grow large in the limit $z = Q^2/\hat{s} \to 1$, i.e. when the partonic center-of-mass energy $\hat{s}$ approaches the di-muon mass Q. Similarly, recoil corrections appearing in the di-muon transverse momentum distributions, $\alpha_{\rm s}^n \ln^{2n-1}(Q^2/Q_{\rm T}^2)/Q_{\rm T}^2$, become large when the pair is produced at small transverse momentum $Q_{\rm T} \ll Q$. These large corrections may be resummed, i.e., calculated to all orders in the strong coupling. Such resummations have been carried out not only for Drell–Yan type processes, but also for many cross-sections of interest in hadronic collisions.

With the advent of the world's first polarized *pp* collider, RHIC at Brookhaven, it becomes possible to study hard scattering processes with polarized proton beams [14]. The main emphasis here is to study the spin structure of the nucleon. To do this at a quantitative level, a solid and well-developed theoretical framework is needed, which includes perturbative higher-order QCD corrections. Therefore, it will be important to extend the resummation framework to interactions of polarized particles, and to investigate resummation effects on spin asymmetries.

One example of particular relevance is the single-spin asymmetry in W boson production,

$$A_{\rm L} = \frac{\sigma_+^W - \sigma_-^W}{\sigma_+^W + \sigma_-^W}, \tag{30}$$

where the signs denote the helicities of the incoming polarized protons. This process has been shown to give first-hand information on the polarization of sea quarks in the proton. However, even at RHIC's highest energy of $\sqrt{s} = 500$ GeV threshold effects in W production are still expected to be quite important, in view of the large W mass of $M_W \approx 80$ GeV. Therefore, threshold resummation is expected to be very relevant, which motivates this project.

4. Structure functions

4.1 *Parametrising parton distributions with orthogonal polynomials*

D Indumathi

The x dependence of parton density distributions is parametrised as

$$q(x) \sim P(x)x^{\beta}\,(1-x)^{\alpha}\,. \tag{31}$$

Both exponents β and α can be estimated from theory (Regge behaviour at small x and counting rules at large x respectively). The polynomial $P(x)$ is sensitive to the intermediate-x values and has been variously parametrised. In some cases, Pade-improved exponential terms have also been included, as also terms with fractional powers of x. While most available parametrisations [15] show good agreement with existing data, they have very different behaviour when extrapolated to a region of (x, Q^2) which is outside the domain of available data. Moreover, valence and sea quark distribution and gluons are parametrised individually; hence there are substantial differences between the individual parametrisations while the combinations that are compared with data may still agree.

We use orthogonal polynomials to parametrise $P(x)$. This has been used in different contexts in the literature earlier [16]. We have

$$P(x) = \sum_{k=0}^{N} a_k(Q^2)\theta_k^{\alpha\beta}(x), \tag{32}$$

where $\theta_k^{\alpha\beta}$ are Jacobi polynomials of order k:

$$\theta_k^{\alpha\beta}(x) = \sum_{j=0}^{k} c_{kj}(\alpha,\beta)x^j\,. \tag{33}$$

The input distributions are specified at $Q^2 = Q_0^2$ by specifying a_k. The evolution of individual terms in the distribution function are independent of each other owing to the orthogonality of the polynomials.

We focus on the singlet sector. We use the parametrisation of eq. (31) for both sea distributions and gluons. We use $\alpha = 4, 7$ and $\beta = -1.08$ for gluon and sea quarks respectively. Evolution of each of the input densities can be independently performed, at every order in k, to individually compute the input sea and gluon contributions to the sea $(S/S, g/S)$ and gluon $(S/g, g/g)$ densities, on evolution.

Furthermore, terms with different θ_k will evolve separately as well. This can be done without fixing $a_k(Q_0^2)$, up to which (unknown) constant(s), the relative contributions of each of these terms can be evaluated. Programs exist at both leading order (LO) and next-to-leading order (NLO); some results for LO for $k = 0, 1$ are shown below. All results are quoted setting $a_k(Q_0^2) = 1$; comparison with data should eventually lead to a fit to these parameters.

Choice of Q_0^2: The ZEUS Collaboration [17], in its analysis of its data, used $Q_0^2 = 7$ GeV^2, which is unusually large. One of their results was that the best-fit gluon distribution function turned negative on backwards evolution to low Q^2. Such a gluon distribution is unphysical since it represents a density distribution. We studied the effect of choice of Q_0^2 on this issue. Results are shown in the figures.

The contributions to the gluon on backwards evolution starting from two different $Q_0^2 = 4, 10$ is shown in figure 3. The thicker lines are for $k = 0$ and the thinner lines for $k = 1$. Solid (dashed) lines indicate the g/g (S/g) contribution. It is seen that the g/g term becomes negative at small x for $Q_0^2 = 10$. Typically the sea is about three times smaller than the gluon at small x and such Q^2; hence the total contribution at a given k is dominated by the gluon input. Furthermore, the dominant ($k = 0$) contribution from the sea quarks is negative at small x; hence this cannot compensate for the negative gluon term to give a net positive gluon distribution at this Q^2. At larger Q^2, of course, the gluon density increases at low x and remains positive definite.

In contrast, the behaviour of the sea densities is shown in figure 4. Now the solid lines correspond to the S/S term and the dashed lines to the g/S term. It is seen that the evolved densities are not very sensitive to the starting Q_0^2. Even if the $k = 0$ contribution from the gluon is multiplied by a factor of three, the net contribution from the dominant $k = 0$ sea and gluon inputs together is still positive down to $x = 10^{-4}$ and likely to remain so down to even smaller x values.

In short, evolution is dominated by $k = 0$ terms, unless $a_k, k \neq 0$, are unusually large. On backwards evolution, the dominant input sea contribution to the evolved sea densities increases away from Q_0^2 while that of the gluons is negative. The

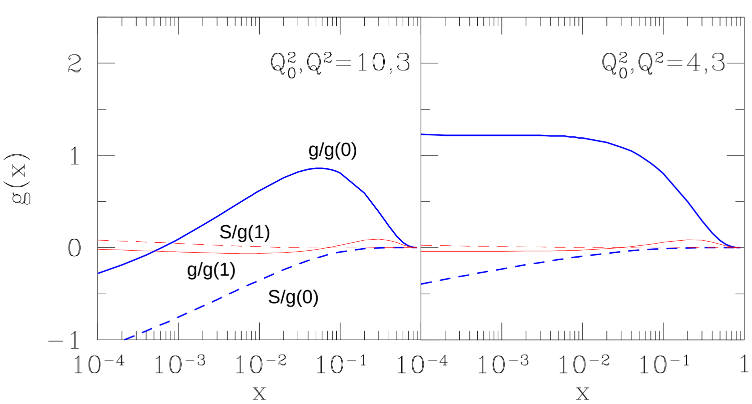

Figure 3. Comparison of evolution to $Q^2 = 3$ GeV^2 of an input gluon distribution at $Q_0^2 = 10, 4$ GeV^2. For details, see the text.

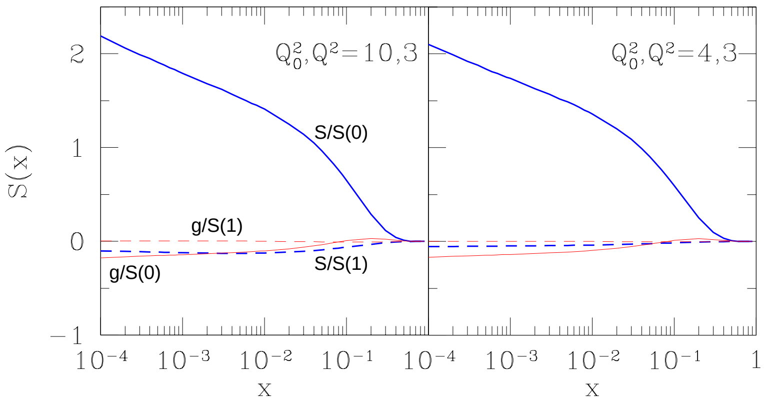

Figure 4. Comparison of evolution to $Q^2 = 3$ GeV^2 of an input sea distribution at $Q_0^2 = 10, 4$ GeV^2. For details, see the text.

combination, weighted with the proper coefficients a_0^g and a_0^S, can remain positive definite for a large range of these coefficients. On the other hand, the dominant (gluon) input contribution to the gluon distribution (g/g) itself decreases with decreasing Q^2 and quickly turns negative at small x. Since S/g is also negative here, there is no compensation possible and the net gluon distribution is thus no longer positive definite for such small Q^2.

Note, however, that the operating scale is not just Q_0^2; the evolution is in fact determined by the scale

$$s = \log \frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)} \ . \tag{34}$$

Lowering Q_0^2, therefore, ensures that the gluon distribution remains positive definite down to lower values of Q^2 and may thus be preferable. It appears that the gluon distribution remains positive at small x for $s > -0.1$ or so.

The main aim of using the orthogonal polynomials was to study error propagation. The results discussed above were incidental to such a study. This work is still in progress, in collaboration with J Pasupathy and C R Das.

4.2 *Impact parameter dependent parton distributions for a relativistic composite system*

Asmita Mukherjee

Generalized parton distributions (GPDs) have attracted a lot of theoretical and experimental attention recently [18]. They are universal non-perturbative objects which give non-trivial information about the quark–gluon correlation in the nucleon which is not accessible by the conventional forward distributions. Experimental determination of GPDs will give valuable information on how the nucleon spin is carried by its constituents. The impact parameter dependent parton distributions have been introduced recently as the Fourier transform of the GPDs [19]. When the momentum transfer is purely transverse, they describe the distribution of partons in the transverse plane. For a transversely polarized nucleon, the distribution of partons gets distorted in the transverse plane and the Fourier transform of the helicity-flip GPD E_q directly gives this distortion, which can be used to develop an intuitive explanation for various transverse single spin asymmetries [20]. Using the overlap representation of the GPDs in terms of the light-cone wave functions [21], we are investigating the impact parameter dependent PDFs for a relativistic composite system like a dressed electron in light-front QED. In fact such a simple model can give useful information about the general properties of the GPDs [21] and form factors [22] and can act as a template for more complex systems like the proton wave function.

This work is in progress in collaboration with Dipankar Chakrabarty and the details will be reported elsewhere.

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