PRAMANA
— journal of
physics

Vol. 63, No. 5
November 2004
pp. 1047–1052

Spectroscopic factors for two-proton radioactive nuclei

CHINMAY BASU
Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700 064, India
E-mail: chinmay@hp1.saha.ernet.in

MS received 28 July 2003; revised 23 June 2004; accepted 31 July 2004

Abstract. Spectroscopic factors for two-proton emitting nuclei are discussed in the framework of the BCS (Bardeen–Cooper–Schriefer) model. Calculations carried out for the two-proton unstable ^{45}Fe,^{48}Ni and ^{54}Zn nuclei are presented.

Keywords. Two-proton emission; spectroscopic factor.

PACS Nos 21.10.Jx, 23.50.+z

Emission of one and two protons from the ground state of proton-rich nuclei have become a subject of recent interest. One-proton emitters have been studied extensively from both the experimental and theoretical point of view. For these nuclei, calculations of the decay width and spectroscopic factors can be found in the literature [1,2]. On the other hand, the phenomenon of two-proton emission has been less studied. An extremely neutron deficient nuclei may decay by simultaneous emission of two protons when single proton emission is energetically forbidden due to pairing interaction. A number of recent experimental observations of two-proton decay [3–5] and some theoretical works in this field have been carried out. The study of two-proton decay is important since the mechanism of decay is still not clearly understood. The calculations are based on either the two-body di-proton theory [6–9] or the three-body models [10,11]. To what extent the two emitted protons form a di-proton (^{2}He) while penetrating the Coulomb barrier is an intriguing question. Further study in this field is of utmost importance since it will offer new insights into clustering and three-body effects inside nuclei. All recent calculations in the field of two proton radioactivity concentrate in the calculation of decay width by solving the barrier penetration problem. However calculation of spectroscopic factors (which contain the structure effects in the decay process) for two-proton unstable systems are sparse. The only calculation in the recent past is by Brown [7] in the framework of the cluster overlap approximation [12–14]. This method is complex and requires a knowledge of the initial, final and di-proton wave functions.

The BCS (Bardeen–Cooper–Schriefer) approximation has been shown recently to be valid for drip-line nuclei [15–17] and can be conveniently used to calculate

spectroscopic factors for such nuclei. This method has been used in ref. [1] for the calculation of spectroscopic factors of one-proton emitting systems but so far has not been applied to the two-proton case. In this paper, we present calculations of spectroscopic factor for two-proton unstable nuclei in the framework of the independent quasiparticle BCS model. The di-proton model is used, as the calculation of spectroscopic factor in the three-body model is not known clearly [18]. The method is applied to ^{45}Fe, ^{48}Ni and ^{54}Zn nuclei. Whereas two-proton decay in ^{45}Fe has been observed recently, theoretical predictions show that ^{48}Ni and ^{54}Zn are the other best candidates for ground state two-proton emission [3].

In the evaluation of proton decay width, the knowledge of single-particle spectroscopic factor is required [1,7,10]. The basic aspects of spectroscopic factors may be found in several texts [19,20] and papers [21]. The single-particle spectroscopic factor is defined as

$$S_j = \langle \psi_{JM} | \phi_{JM}(j, J_0) \rangle^2 \tag{1}$$

where $j,\ m$ refers to the nucleon state (shell model state), $J,\ M$ the parent state and $J_0,\ M_0$ refers to the daughter state

$$\phi_{JM}(j, J_0) = \sum_{m, M_0} (jmJ_0M_0|JM) a^+_{jm} \psi_{J_0M_0} \tag{2}$$

and

$$S_j^{1/2} = \sum_{m, M_0} (jmJ_0M_0|JM) \langle \psi_{JM} | a^+_{jm} | \psi_{J_0, M_0} \rangle. \tag{3}$$

Equation (3) is used for the evaluation of spectroscopic factors of one-proton emission or one-nucleon transfer reaction. In the same way, the spectroscopic factor for two-nucleon emission or two-nucleon transfer is written as [22]

$$S_j^{1/2} = \sum_{\mu, M_0} (\chi\mu J_0M_0|JM) \langle \psi_{JM} | U^+(j_1, j_2, \chi) | \psi_{J_0, M_0} \rangle \tag{4}$$

with

$$U^+(j_1, j_2, \chi) = \frac{1}{\sqrt{1 + \delta j_1 j_2}} \sum_{m_1, m_2} (j_1 m_1 j_2 m_2 | \chi\mu) a^+_{j_1 m_1} a^+_{j_2 m_2}, \tag{5}$$

where χ is the combined angular momentum of the two nucleonic orbits and μ is its projection. In the framework of the di-proton theory [7,11] the two protons residing in the same shell (time reversed), interacting via pairing interaction reside in the $\chi = 0$ state with the maximum probability. Thus from eq. (5) with $j_1 = j_2 = j$, $m_1 = -m_2 = m$ and $\chi = 0$ we get

$$U^+(j, j, 0) = \frac{1}{\sqrt{2}} \sum_m (jmj - m|00) a^+_{jm} a^+_{j-m}. \tag{6}$$

In the BCS method, we consider the ground state of the parent and daughter as the zero quasi-particle BCS state and write the spectroscopic factor as

$$S_j^{1/2} = \sum_{\mu, M_0} (\chi \mu J_0 M_0 | JM) \langle \mathrm{BCS} | U^+(jj0) | \mathrm{BCS} \rangle. \tag{7}$$

Using the quasi-particle representation of the particle creation and destruction operator, the relation which describes the BCS state as the quasi-particle vacuum and the commutation relations for quasi-particle operators [19] in (7) we finally obtain

$$S_j = \left(j + \frac{1}{2} \right) U_j^2(A) V_j^2(A+2). \tag{8}$$

In eq. (8), $A+2$ refers to the parent nucleus and A refers to the daughter after the emission of two protons.

The evaluation of spectroscopic factor for two-proton emission thereby requires a calculation of the occupation probabilities U_j and V_j. In the BCS model the occupation probabilities are defined in terms of the single particle energy ϵ_j, the Fermi level λ and the pairing gap Δ [19,23]. The two parameters Δ and λ are obtained by numerically solving the BCS gap equations [23]

$$\frac{1}{2} |G_\mathrm{p}| \sum_j \frac{\Omega_j}{E_j} = 1, \tag{9}$$

$$\sum_j \Omega_j \left[1 - \frac{(\epsilon_j - \lambda)}{E_j} \right] = N, \tag{10}$$

where G_p is the pairing strength, $\Omega_j = (j + \frac{1}{2})$ is the shell degeneracy, N is the number of particles or holes in the unfilled shell and $E_j = \sqrt{(\epsilon_j - \lambda)^2 + \Delta^2}$.

To apply our method we first consider ^{48}Ni [24], which has been the most frequently discussed candidate for two-proton emission. If ^{48}Ni is considered as a doubly magic closed shell nucleus we take $V_j^2 = 1$ in eq. (8). In order to calculate U_j^2 for the residual ^{46}Fe, the method of Kisslinger and Sorenson [23] is used. This method was applied to Ni, Sn and Pb isotopes and also to $N = 28$, 50, 82 nuclei. The two holes in the closed shell of ^{48}Ni creates the ^{46}Fe nucleus. However, according to Brown [7] if a closed $f_{7/2}$ shell is considered for ^{48}Ni the spectroscopic factor is reduced by a factor of 4 in comparison to a full fp-shell calculation. To calculate the occupancies by considering the entire fp-shell we solve the BCS gap equations. The gap equations retain the same form as in eqs (9) and (10) in a more generalized BCS model applicable to drip-line nuclei [15]. In order to solve the gap equations, the single-particle excitation energies are required. For the extreme proton-rich ^{48}Ni and ^{46}Fe the experimental single-particle levels are not known. One way is to calculate the spectrum from Wood–Saxon-type potential and solve the BCS equations in a basis that includes both bound and resonant states [16]. We however adopt a simpler approach and estimate the single particle energies of the drip-line nuclei from their mirror partners [6]. In this work we use the experimental single particle excitation spectrum of ^{47}Ca [25]. The absolute energy values of the proton states of ^{47}Co are related to the neutron single-particle energies of its mirror through the empirical expression given by Goldansky [8] and Nazarewicz

Table 1. Table contains the single particle energies $\epsilon_{\rm p}$ and $\epsilon_{\rm n}$ and the excitation energies E^*, all expressed in MeV. Also shown are the shell spin parity and the calculated spectroscopic factors in the jj (S_j) and LS (S_{LS}) coupling schemes. S_{LS} depicted in the table are for the case $L = S = 0$.

j^π	$\epsilon_{\rm n}(^{47}{\rm Ca})$	$\epsilon_{\rm p}(^{48}{\rm Ni})$	$\epsilon_{\rm p}(^{46}{\rm Fe})$	E^*	S_j	S_{LS}
$\frac{7}{2}^-$	-9.94	-1.4	-1.83	0	0.4289	0.324
$\frac{3}{2}^-$	-7.93	0.61	0.18	2.01	0.1612	0.1316
$\frac{1}{2}^-$	-7.091	1.45	1.02	2.849	0.047	0.027
$\frac{5}{2}^-$	-6.673	1.87	1.44	3.267	0.1098	0.072

et al [6]. The derived single-particle energies and the corresponding excitation energies are given in table 1. Solution of the BCS gap equation is performed with the single-particle energies of fp-orbits in table 1 and proton pairing strength from the empirical expression $G_{\rm p} = \frac{1}{A}[17.9 + 0.176(N - Z)]$ of ref. [1]. The solution yields a value of $\lambda = 0.41$ and $\Delta_{\rm p} = 1.2$ for ^{46}Fe and $\lambda = 1.09$ and $\Delta_{\rm p} = 1.19$ for ^{48}Ni. A closed $f_{7/2}$ shell for ^{48}Ni on the other hand yields for ^{46}Fe $\lambda = -0.59$ and $\Delta_{\rm p} = 0.9$. The values of $\Delta_{\rm p}$ derived in this way is close to that from HFB+SKp calculations of [6]. The corresponding jj-coupling spectroscopic factors for the different shells are also depicted in table 1. The jj-coupling spectroscopic factors are converted to the LS-coupling scheme by the standard transformation described in Rose [26]. The LS-coupling spectroscopic factors for the spatially symmetric $L = S = 0$ is chosen (table 1) for a direct comparison with the calculations of ref. [7]. The calculations of [7] have been done in the cluster overlap approximation using the shell model code OXBASH [27]. A value of 0.1375 calculated by Brown [7] includes factors from the projection of the pair wave function onto the $0s$ state of relative motion and the centre of mass of the pair, relative to the residual nucleus wave functions. These factors are contained in the term $G^2(A/(A-k))^\lambda$ of [7] and are required when the actual wave functions are replaced by shell model wave functions [14,28]. Without these factors the spectroscopic factor calculated by Brown would be 0.341. We next apply our method to ^{45}Fe and ^{54}Zn. For ^{45}Fe and ^{43}Cr (daughter after two-proton decay) the single-particle basis is obtained from the experimental spectrum of ^{45}K and ^{43}K respectively. The values of gap parameters obtained are $\lambda = -1.18$, $\Delta_{\rm p} = 1.25$ for ^{45}Fe and $\lambda = -1.19$, $\Delta_{\rm p} = 1.90$ for ^{43}Cr. Similarly, for ^{54}Zn these parameters are $\lambda = -0.51$, $\Delta_{\rm p} = 0.87$ for ^{54}Zn and $\lambda = 0.54$, $\Delta_{\rm p} = 1.90$ for ^{52}Ni. All the experimental spectrum used are obtained from the nuclear structure data base [25]. The main inputs to the calculations are single-particle energies and so the accuracy of our results depend upon the value of single-particle levels. The LS coupling spectroscopic factors calculated in the present formalism for these nuclei are shown in table 2 along with ^{48}Ni. The result of [7] with and without the correction factor are also tabulated for comparison. It can be seen that, without the correction in [7] the results from the two methods are comparable. However, the correction factor depends on the position of the two protons in the shell model orbital which is not exactly known for the proton drip-line nuclei. Thus how much these corrections are exactly important for such nuclei is not known.

Table 2. Table shows calculation of spectroscopic factors for two-proton emitting nuclei from the present work (A), from ref. [7] without correction (B) and with correction (C).

	S_{LS} ($L = 0$, $S = 0$)		
2p emitter	A	B	C
^{48}Ni	0.324	0.341	0.1375
^{45}Fe	0.41	0.475	0.195
^{54}Zn	0.82	–	–

In this work we discuss the evaluation of spectroscopic factor for two-proton unstable nuclei in the BCS theory. The full fp-shell orbits are considered in the calculations of BCS gap parameters and shell occupancies. The single-particle states of the extremely proton-rich nuclei are obtained from the experimental levels of their mirror partners. A comparative discussion is done with the calculations of Brown in the cluster overlap approximation. Though the basic problem in the phenomenon of proton decay is solving the barrier penetration problem, the information for spectroscopic factors are also required. The experimentally measured lifetimes (decay width) should be compared with calculations including structure effects which are contained in these factors. As the existing calculations are complex our simple approach could be very useful to predict the spectroscopic amplitudes for two-proton unstable systems.

References

[1] Sven Aberg, Paul B Semmes and W Nazarewicz, *Phys. Rev.* **56**, 1762 (1997)
[2] C N Davids *et al*, *Phys. Rev.* **C55**, 2255 (1997)
[3] J Giovinazzo *et al*, *Phys. Rev. Lett.* **89**, 102501 (2002)
[4] M Pfutzner *et al*, *Euro. Phys. J.* **A14**, 279 (2002)
[5] G Del Campo, *Phys. Rev. Lett.* **86**, 43 (2001)
[6] W Nazarewicz *et al*, *Phys. Rev.* **C53**, 740 (1996)
[7] B Alex Brown, *Phys. Rev.* **C43**, R1513 (1991); **C44**, 924(E) (1991)
[8] V I Goldansky, *Nucl. Phys.* **19**, 482 (1960)
[9] W E Ormand, *Phys. Rev.* **C53**, 214 (1996)
[10] L V Grigorenko, *Phys. Rev. Lett.* **85**, 22 (2000)
[11] L V Grigorenko *et al*, *Phys. Rev.* **C64**, 054002 (2001)
[12] W Chung *et al*, *Phys. Lett.* **B79**, 381 (1978)
[13] J B McGrory, *Phys. Lett.* **B47**, 481 (1973)
[14] M Ichimura *et al*, *Nucl. Phys.* **A204**, 225 (1973)
[15] O Civitarese, R J Liotta and T Vertse, *Phys. Rev.* **C64**, 057305 (2001)
[16] A T Kruppa, P H Heenen and R J Liotta, *Phys. Rev.* **C63**, 044324 (2001)
[17] N Sandulescu, R J Liotta and R Wyss, *Phys. Lett.* **B394**, 6 (1997)
[18] B A Brown, F C Barker and D J Millener, *Phys. Rev.* **C65**, R051309 (2002)
[19] J M Eisenburg and W Griener, *Nuclear theory* (North Holland, 1986) vol. 3

[20] M A Preston, *Physics of the nucleus* (Addison Wesley Publishing Company Inc., 1965)
[21] S Yoshida, *Phys. Rev.* **123**, 2122 (1961)
[22] S Yoshida, *Nucl. Phys.* **33**, 685 (1962)
[23] L S Kisslinger and R A Sorenssen, *Mat. Fys. Medd. Dan. Vid. Selsk.* **32**, No. 9 (1960)
[24] B Blank *et al*, *Phys. Rev. Lett.* **84**, 1116 (2000)
[25] http://www.nndc.bnl.gov/nndcscrt/testwww/AR047CA.HTML
[26] M E Rose, *Elementary theory of angular momentum* (John Wiley & Sons, 1957)
[27] W D M Rae, A Etchegoyen and B A Brown, OXBASH, The Oxford-Buenos Aires-MSU shell-model code, Michigan State University Cyclotron Laboratory Report No. 524 (1988)
[28] N Anyas-Weiss *et al*, *Phys. Rep.* **C12**, 202 (1974)