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Propagation of dust-acoustic waves in weakly ionized plasmas with dust-charge fluctuation*

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Abstract. For an unmagnetized partially ionized dusty plasma containing electrons, singly charged positive ions, micron-sized massive negatively charged dust grains and a fraction of neutral atoms, dispersion relations for both the dust-ion-acoustic and the dust-acoustic waves have been derived, incorporating dust charge fluctuation. The dispersion relations, under various conditions, have been exhaustively analysed. The explicit expressions for the growth rates have also been derived.

Keywords. Dust-acoustic wave; dust-ion-acoustic wave; dust-charge fluctuation; dispersion relation.

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1. Introduction

In recent years, research on various low-frequency electrostatic waves in dusty plasmas is gaining momentum. Rao *et al* [1] have demonstrated theoretically a low-frequency acoustic-like mode in unmagnetized dusty plasma, called dust-acoustic (DA) mode. For the excitation of this kind of wave, the dynamics of the dust grains has essentially to be considered because inertia is provided by the mass of the dust particles. Moreover, the phase velocity of these waves is extremely small compared to the electron- and ion-thermal velocity. Shukla and Silin [2] also have reported another low-frequency acoustic-like mode called dust-ion-acoustic (DIA) mode supported by a dusty plasma with negatively charged stationary dust grains. These waves have been found to have phase velocity much smaller than electron-thermal velocity and much greater than ion-thermal velocity.

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It is worth mentioning that dusty plasma cannot support the usual ion-acoustic wave. For the DIA waves, the ion dynamics is important because the ion mass provides the inertia whereas the restoring force comes from the pressure of the inertialess electrons and the stationary dust grains form the equilibrium charge neutralizing background. Both the dust-acoustic (DA) and the dust-ion-acoustic (DIA) waves have been observed in laboratory experiments [3–5]. However, in weakly ionized laboratory and astrophysical plasmas, the ionization of neutral gas plays a very significant role. Subsequently, the ionization processes have been incorporated in the physics of wave phenomena in partially ionized dusty plasmas. Shukla and Morfill [6] have shown that by employing multi-fluid dusty plasma model, both the DA and DIA waves can be spontaneously excited by an ionization instability in multicomponent weakly ionized dusty plasmas. In their investigation they have found that the dust-acoustic and the dust-ion-acoustic waves are subjected to an ionization instability provided that the ionization cross-section is sufficiently rapid to overcome the damping effects caused by the collisions between the charged particles and the neutral atoms, the ion viscosity and the charge fluctuation of the dust grains [7–9]. But they have not incorporated the last one in their study. In our investigation, we have reformulated the problem, taking dust-charge variation into account and derived linear dispersion relations for both the DA and DIA waves and explicit expressions for the growth rates under different situations. The paper is organized as follows: In §2, we have presented the plasma model and the basic governing equations and then in §3, we have derived the linear dispersion relations for both the DA and DIA waves. We have also deduced explicit expressions for the damping or growth rates under various conditions. In §4, we have made discussion on the modifications of the findings of Shukla and Morfill [6] due to charge fluctuation of the dust particles.

2. Model and basic equations

We consider a uniform, isotropic, unmagnetized, partially ionized dusty plasma consisting of electrons, singly charged ions, micron-sized extremely massive negatively charged dust grains, and a fraction of neutral atoms. The dust particles are assumed to be spherical and are of the same radius a, the grain size and the intergrain separation are supposed to be much smaller than the effective dust Debye length. The charge neutrality condition, in the absence of charge fluctuation is given by $n_{\mathrm{i0}} = n_{\mathrm{e0}} + z_{\mathrm{d0}} n_{\mathrm{d0}}$ where n_{j0} (j = e, i, d for electrons, ions and dust particles, respectively) denotes the unperturbed number densities of the plasma species and z_{d0} represents the equilibrium number of charges residing on the negatively charged dust grains. The charge neutrality condition can also be expressed as $n_{\mathrm{i0}} = n_{\mathrm{e0}}(1 + P)$ where $P = z_{\mathrm{d0}} n_{\mathrm{d0}}/n_{\mathrm{e0}}$. In connection with charging of the dust grains due to attachment of the electrons and ions on the dust grains, the case $P \gg 1$ corresponds to the situation where the background electrons get depleted completely.

For one-dimensional wave propagation along the z-axis, the basic fluid equations which govern the dynamics of the ions and the cold dust are obtained from the following set of equations [10]:

For the ions:

$$\frac{\partial n_\mathrm{i}}{\partial t} + \frac{\partial}{\partial z}(n_\mathrm{i} v_\mathrm{i}) = \nu_\mathrm{I}(n_\mathrm{i} - n_{\mathrm{i}0}), \tag{1}$$

$$\frac{\partial v_\mathrm{i}}{\partial t} + v_\mathrm{i}\frac{\partial v_\mathrm{i}}{\partial z} = -\frac{e}{m_\mathrm{i}}\frac{\partial \phi}{\partial z} - \frac{K_\mathrm{B}T_\mathrm{i}}{m_\mathrm{i} n_\mathrm{i}}\frac{\partial n_\mathrm{i}}{\partial z} - \nu_\mathrm{i} v_\mathrm{i} + \mu_\mathrm{i}\frac{\partial^2 v_\mathrm{i}}{\partial z^2}. \tag{2}$$

For the dust particles:

$$\frac{\partial n_\mathrm{d}}{\partial t} + \frac{\partial}{\partial z}(n_\mathrm{d} v_\mathrm{d}) = 0, \tag{3}$$

$$\frac{\partial v_\mathrm{d}}{\partial t} + v_\mathrm{d}\frac{\partial v_\mathrm{d}}{\partial z} = -\frac{q_\mathrm{d}}{m_\mathrm{d}}\frac{\partial \phi}{\partial z}, \tag{4}$$

where n_i and n_d stand for the sum of the equilibrium and the perturbed number densities of the ions and dust particles respectively, v_i and v_d denote velocities of the ion fluid and the dust fluid respectively, m_i and m_d are the masses of the respective species, e is the magnitude of the electronic charge, ϕ is the electrostatic potential, T_i is the ion temperature, ν_i is the ion-neutral collision frequency, K_B is the Boltzmann constant, ν_I is the ionization-collision frequency and μ_i is the coefficient of kinematic viscosity.

The Boltzmannian distribution of electrons assumed to form the neutralizing background is

$$n_\mathrm{e} = n_{\mathrm{e}0}\exp\left(\frac{e\phi}{K_\mathrm{B}T_\mathrm{e}}\right), \tag{5}$$

where $n_{\mathrm{e}0}$ and T_e denote the unperturbed electron number density and electron temperature respectively. The description is closed by the electrostatic Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e\left[n_\mathrm{e} - \frac{q_\mathrm{d} n_\mathrm{d}}{e} - n_\mathrm{i}\right]. \tag{6}$$

The charge fluctuations obey the current balance equation

$$\frac{\partial q_\mathrm{d}}{\partial t} + v_\mathrm{d}\frac{\partial q_\mathrm{d}}{\partial z} = I_\mathrm{e} + I_\mathrm{i}, \tag{7}$$

where I_e and I_i are the electron current and the ion current given by

$$\begin{aligned} I_\mathrm{e} &= -\pi a^2 e n_\mathrm{e}(\phi)\sqrt{\frac{8K_\mathrm{B}T_\mathrm{e}}{m_\mathrm{e}}}\exp\left(\frac{e\psi}{K_\mathrm{B}T_\mathrm{e}}\right), \\ I_\mathrm{i} &= \pi a^2 e n_\mathrm{i}(\phi)\sqrt{\frac{8K_\mathrm{B}T_\mathrm{i}}{m_\mathrm{i}}}\left(1 - \frac{e\psi}{K_\mathrm{B}T_\mathrm{i}}\right), \end{aligned} \tag{8}$$

where a is the radius of the dust grains which are assumed to be spherical, ψ denotes the grain surface potential relative to the ambient plasma potential defined by $\psi = q_d/a$; q_d is the dust charge which can be looked upon as a dynamical plasma variable.

The current expressions (8) may be rewritten as

$$I_e = -a^2 e\sqrt{8\pi} n_e v_{te} \exp\left(\frac{eq_d}{aK_B T_e}\right),$$
$$I_i = a^2 e\sqrt{8\pi} n_i v_{ti}\left(1 - \frac{eq_d}{aK_B T_i}\right), \tag{9}$$

where v_{te}, v_{ti} are the electron (ion) thermal velocity given by $v_{i,\alpha} = \sqrt{K_B T_\alpha / m_\alpha}$ ($\alpha = \mathrm{e}, \mathrm{i}$ for electrons and ions respectively). The equilibrium values of the currents are given by

$$I_{e0} = -a^2 e\sqrt{8\pi} n_{e0} v_{te} \exp\left(\frac{eq_{d0}}{aK_B T_e}\right),$$
$$I_{i0} = a^2 e\sqrt{8\pi} n_{i0} v_{ti}\left(1 - \frac{eq_{d0}}{aK_B T_i}\right). \tag{10}$$

Here, q_{d0} represents the equilibrium value of the dust charge which is determined from

$$n_{i0} v_{ti}\left(1 - \frac{eq_{d0}}{aK_B T_i}\right) = n_{e0} v_{te} \exp\left(\frac{eq_{d0}}{aK_B T_e}\right). \tag{11}$$

3. Dispersion relations

In order to derive the linear dispersion relations for DA and DIA waves, we have to linearize eqs (1)–(7). Expanding the dynamical plasma variables like n_i, n_e, v_i, v_d etc. about their unperturbed state as $n_\alpha = n_{\alpha 0} + n_{\alpha 1}$, $v_\alpha = v_{\alpha 1}$, $q_d = q_{d0} + q_{d1}$ and $\phi = \phi_1$ ($\alpha = \mathrm{i}, \mathrm{e}, \mathrm{d}$ for ion, electron and dust grain respectively) and assuming the first-order perturbations to vary as $\exp[i(kz - \omega t)]$ we have the following equations:

$$-i\omega n_{i1} + ikn_{i0} v_{i1} = \nu_I n_{i1}, \tag{12}$$

$$(-i\omega + \Gamma_i) n_{i0} v_{i1} + ikv_{ti}^2 n_{i1} = -\left(\frac{iken_{i0}}{m_i}\right)\phi_1; \quad \Gamma_i = \nu_i + k^2\mu_i, \tag{13}$$

$$-\omega n_{d1} + kn_{d0} v_{d1} = 0, \tag{14}$$

$$\omega v_{d1} = \left(\frac{kq_{d0}}{n_{d0} m_d}\right)\phi_1, \tag{15}$$

$$n_{e1} = \left(\frac{en_{e0}}{K_B T_e}\right)\phi_1, \tag{16}$$

$$k^2\phi_1 = 4\pi e\left[\frac{1}{e}(q_{d0} n_{d1} + n_{d0} q_{d1}) + n_{i1} - n_{e1}\right], \tag{17}$$

$$-i\omega q_{\rm d1} = I_{\rm e1} + I_{\rm i1}, \tag{18}$$

where

$$I_{\rm e1} = -\frac{I_0}{n_{\rm e0}} n_{\rm e1} - \left(\frac{I_0 e}{aK_{\rm B}T_{\rm e}}\right) q_{\rm d1}$$

and

$$I_{\rm i1} = \frac{I_0}{n_{\rm i0}} n_{\rm i1} - \left[\left(\frac{I_0 e}{aK_{\rm B}T_{\rm i}}\right) \Big/ \left(1 - \frac{eq_{\rm d0}}{aK_{\rm B}T_{\rm i}}\right)\right] q_{\rm d1}.$$

Here $I_0 = -I_{\rm e0} = I_{\rm i0}$ is the equilibrium value of the electron or the ion current. If the expressions for the first-order perturbations of the electron-current and the ion-current are substituted, then eq. (18) turns out to be

$$q_{\rm d1} = i\left(\frac{1}{\omega + i\Omega_{\rm c}}\right) I_0 \left(\frac{n_{\rm i1}}{n_{\rm i0}} - \frac{n_{\rm e1}}{n_{\rm e0}}\right), \tag{19}$$

where $\Omega_{\rm c}$ is the charging frequency defined as

$$\Omega_{\rm c} = \frac{eI_0}{aK_{\rm B}T_{\rm e}}\left(1 + \frac{\sigma}{1 - (e\psi_0/K_{\rm B}T_{\rm i})}\right); \quad \sigma = \frac{T_{\rm e}}{T_{\rm i}} \text{ and } \psi_0 = \frac{q_{\rm d0}}{a}$$

(the equilibrium value of the grain surface potential relative to the plasma potential) which is identical with eq. (3.7) of Takaishi and Nishikawa [11]. If eqs (12)–(15) are combined, then the first-order perturbations in the number densities of the ions and the dust particles come out to be

$$n_{\rm i1} = \frac{(n_{\rm i0}k^2 e/m_{\rm i})\phi_1}{\omega^2 + i\omega(\Gamma_{\rm i} - \nu_{\rm I}) + \Gamma_{\rm i}\nu_{\rm I} - k^2 v_{\rm ti}^2} \tag{20}$$

and

$$n_{\rm d1} = \left(\frac{k^2 n_{\rm d0} q_{\rm d0}}{m_{\rm d}\omega^2}\right)\phi_1. \tag{21}$$

The perturbed dust charge, i.e. $q_{\rm d1}$ can be easily determined from eqs (16) and (20). If now $n_{\rm i1}$, $n_{\rm e1}$, $n_{\rm d1}$ and $q_{\rm d1}$ are substituted in the linearized Poisson's equation (17), then we obtain the dispersion relation as

$$\begin{aligned} &1 + \frac{1}{k^2\lambda_{\rm De}^2} - \frac{\omega_{\rm pi}^2}{\omega^2 + i\omega(\Gamma_{\rm i} - \nu_{\rm I}) + \Gamma_{\rm i}\nu_{\rm I} - k^2 v_{\rm ti}^2} - \frac{\omega_{\rm pd}^2}{\omega^2} \\ &= \frac{\varepsilon I_0}{e}\frac{i}{\omega + i\Omega_{\rm c}}\left[\frac{\omega_{\rm pi}^2}{\omega^2 + i\omega(\Gamma_{\rm i} - \nu_{\rm I}) + \nu_{\rm I}\Gamma_{\rm i} - k^2 v_{\rm ti}^2} - \frac{\omega_{\rm pi}^2}{\sigma k^2 v_{\rm ti}^2}\right]. \end{aligned} \tag{22}$$

Here $\lambda_{\rm De} = \sqrt{T_{\rm e}/4\pi n_{\rm e0}e^2}$ denotes the electron Debye length, $\omega_{\rm pi} = \sqrt{4\pi n_{\rm i0}e^2/m_{\rm i}}$ is the ion-plasma frequency and $\omega_{\rm pd} = \sqrt{4\pi n_{\rm d0} z_{\rm d0}^2 e^2/m_{\rm d}}$ represents the dust-plasma frequency and $\varepsilon = n_{\rm d0}/n_{\rm i0}$. The term occurring in the right-hand side of eq. (22) which is the linear dispersion relation, arises solely due to charge fluctuation of the dust particles in the absence of which it would immediately reduce into eq. (10) of Shukla and Morfill [6]. Now we shall consider two cases in succession. Firstly, if the dust particles in the plasma are stationary ($\omega_{\rm pd} = 0$), then we obtain from eq. (22)

$$\frac{1}{\omega_{\rm DIA}^2} - \frac{1}{\omega^2 + i\omega(\Gamma_{\rm i} - \nu_{\rm I}) + \Gamma_{\rm i}\nu_{\rm I} - k^2 v_{\rm ti}^2} = \frac{\varepsilon I_0}{e}\frac{i}{\omega + i\Omega_{\rm c}}\left[\frac{1}{\omega^2 + i\omega(\Gamma_{\rm i} - \nu_{\rm I}) + \nu_{\rm I}\Gamma_{\rm i} - k^2 v_{\rm ti}^2} - \frac{1}{\sigma k^2 v_{\rm ti}^2}\right], \tag{23}$$

where

$$\omega_{\rm DIA}^2 = \frac{k^2 c_{\rm s}^2}{1 + k^2\lambda_{\rm De}^2}\frac{n_{\rm i0}}{n_{\rm e0}}$$

is the dust-ion-acoustic frequency [2]. It is worth mentioning that this wave frequency reduces to the ion-acoustic wave frequency defined by $\omega_{\rm IA} = kc_{\rm s}$ for $n_{\rm i0} = n_{\rm e0}$ and in the long-wavelength approximation where $c_{\rm s}$ is the ion-sound velocity. Equation (23) represents an oscillatory instability. Solutions of both eq. (22) and eq. (23) are complex. So the wave frequency (ω) may be expressed as $\omega = \omega_{\rm r} + i\gamma$ where γ is the growth rate and $-\gamma$ corresponds to the damping rate. If this expression for complex ω is substituted in eq. (23), the real and the imaginary parts are equated, then we have $\omega_{\rm r}$ and γ given by

$$\omega_{\rm r}^2 = \lfloor \gamma^2 + 2\gamma(\gamma - \nu_{\rm I} + \Gamma_{\rm i}) + k^2 v_{\rm ti}^2 - \Gamma_{\rm i}\nu_{\rm I} + \omega_{\rm DIA}^2 \rfloor + \left[\Omega_{\rm c}(2\gamma + \Gamma_{\rm i} - \nu_{\rm I}) + \frac{\varepsilon I_0}{\sigma e}\frac{1}{v_{\rm ti}^2}\frac{\omega_{\rm DIA}^2}{k^2}(2\gamma + \Gamma_{\rm i} - \nu_{\rm I})\right] \tag{24}$$

and

$$\left[(3\gamma + \Gamma_{\rm i} - \nu_{\rm I} + \Omega_{\rm c})k^2 + \frac{\varepsilon I_0}{\sigma e}\frac{1}{v_{\rm ti}^2}\omega_{\rm DIA}^2\right]\omega_{\rm r}^2 = k^2\gamma^2(\gamma + \Omega_{\rm c}) + k^2\gamma(\gamma + \Omega_{\rm c})(\Gamma_{\rm i} - \nu_{\rm I}) - k^2(\gamma + \Omega_{\rm c})\Gamma_{\rm i}\nu_{\rm I} + k^4(\gamma + \Omega_{\rm c})v_{\rm ti}^2 + k^2(\gamma + \Omega_{\rm c})\omega_{\rm DIA}^2 + \frac{\varepsilon I_0}{\sigma e}\frac{1}{v_{\rm ti}^2}\omega_{\rm DIA}^2[(\sigma + 1)k^2 v_{\rm ti}^2 + \gamma^2 + \gamma(\Gamma_{\rm i} - \nu_{\rm I}) - \Gamma_{\rm i}\nu_{\rm I}]. \tag{25}$$

These two coupled equations will come into effect in the linear analysis of propagation characteristics and stability of the DIA waves. Moreover, the equations can be used to study whether there will be growing or decaying instability and how the phase velocity depends on various plasma parameters when charge variation of the dust grains is taken care of. It is interesting to note here that the relations (24) and (25) get reduced into eqs (12) and (13) of Shukla and Morfill [6] if dust charge is assumed to be constant. So, these relations may be looked upon as the generalized and more realistic forms of the equations which Shukla and Morfill had derived.

Secondly, assuming $k^2 v_{\rm ti}^2 \gg \omega(\Gamma_{\rm i} - \nu_{\rm I}) \gg \omega^2 \gg \Gamma_{\rm i}\nu_{\rm I}$ we get from eq. (22),

$$1 + \frac{1}{k^2\lambda_{\rm D}^2} + i\frac{1}{k^2\lambda_{\rm Di}^2}\left[\frac{\omega(\Gamma_{\rm i} - \nu_{\rm I})}{k^2 v_{\rm ti}^2} + \frac{\varepsilon I_0}{e}\frac{1}{\omega + i\Omega_{\rm c}}\left(\frac{1 + \sigma}{\sigma}\right)\right] - \frac{\omega_{\rm pd}^2}{\omega^2} = 0. \tag{26}$$

This is the dispersion relation associated with the dust acoustic wave which yields eq. (14) of Shukla and Morfill [6] in the absence of charge fluctuation. Here $\lambda_{\rm D}^{-2} = \lambda_{\rm De}^{-2} + \lambda_{\rm Di}^{-2}$ and $\lambda_{\rm Di} = \sqrt{T_{\rm i}/4\pi n_{\rm i0}e^2}$ is the ion-Debye length. It is obvious that solutions of (26) are complex also. So, one can make the substitution $\omega = \omega_{\rm R} + i\gamma_{\rm D}$ in (26) and assume $\gamma_{\rm D} \ll \omega_{\rm R}$ to have (equating the real and the imaginary parts respectively)

$$\left(1+\frac{1}{k^2\lambda_{\rm D}^2}\right)\omega_{\rm R}(\omega_{\rm R}^2 - 2\Omega_{\rm c}\gamma_{\rm D}) - \frac{1}{k^2\lambda_{\rm Di}^2}\left[\frac{(\Gamma_{\rm i}-\nu_{\rm I})(4\gamma_{\rm D}+\Omega_{\rm c})\omega_{\rm R}^3}{k^2 v_{\rm ti}^2} + 2\omega_{\rm R}\gamma_{\rm D}\frac{\varepsilon 1_0}{e}\left(\frac{1+\sigma}{\sigma}\right)\right] = \omega_{\rm R}\omega_{\rm pd}^2 \tag{27}$$

and

$$\left(1+\frac{1}{k^2\lambda_{\rm D}^2}\right)\omega_{\rm R}^2(3\gamma_{\rm D}+\Omega_{\rm c}) + \frac{1}{k^2\lambda_{\rm Di}^2}\left[(\omega_{\rm R}^4 - 3\omega_{\rm R}^2\Omega_{\rm c}\gamma_{\rm D})\left(\frac{\Gamma_{\rm i}-\nu_{\rm I}}{k^2 v_{\rm ti}^2}\right) + \omega_{\rm R}^2\frac{\varepsilon I_0}{e}\left(\frac{1+\sigma}{\sigma}\right)\right] = \omega_{\rm pd}^2(\gamma_{\rm D}+\Omega_{\rm c}). \tag{28}$$

These equations in $\omega_{\rm R}$ and $\gamma_{\rm D}$ may be employed for linear analysis also. Elimination of $\omega_{\rm r}$ from eqs (24) and (25) leads us to have a cubic equation in γ of the form

$$A\gamma^3 + B\gamma^2 + C\gamma + D = 0, \tag{29}$$

where

$$\begin{aligned}
A &= 8,\\
B &= 8[(\Gamma_{\rm i}-\nu_{\rm I}) + \Omega_{\rm cl}]; \quad \Omega_{\rm cl} = \Omega_{\rm c} + \frac{\varepsilon I_0}{\sigma e}\frac{1}{v_{\rm ti}^2}\frac{\omega_{\rm DIA}^2}{k^2}\\
C &= 2\lfloor(\Gamma_{\rm i}-\nu_{\rm I})^2 + (k^2 v_{\rm ti}^2 - \Gamma_{\rm i}\nu_{\rm I} + \omega_{\rm DIA}^2) + 3(\Gamma_{\rm i}-\nu_{\rm I})\Omega_{\rm cl} + \Omega_{\rm cl}^2\rfloor\\
D &= (k^2 v_{\rm ti}^2 - \Gamma_{\rm i}\nu_{\rm I} + \omega_{\rm DIA}^2)(\Gamma_{\rm i} - \nu_{\rm I} + \Omega_{\rm cl} - \Omega_{\rm c})\\
&\quad + \Omega_{\rm cl}(\Gamma_{\rm i}-\nu_{\rm I})(\Gamma_{\rm i}-\nu_{\rm I}+\Omega_{\rm cl}) + (\Omega_{\rm cl}-\Omega_{\rm c})\Gamma_{\rm i}\nu_{\rm I} - \frac{\varepsilon I_0}{\sigma e}(\sigma+1)\omega_{\rm DIA}^2.
\end{aligned}$$

4. Results and discussion

Equation (29) which has been obtained on elimination of $\omega_{\rm r}$ from eqs (24) and (25), has been solved numerically for argon dusty plasma. The parameter values used in the numerical computation are: $n_{\rm d0} = 5\times10^3$ cm^{-3}, $n_{\rm i0} = 10^9$ cm^{-3}, a (radius) $= 10^{-3}$ cm, $T_{\rm e} = 2\times10^4$ K, $T_{\rm i} = 2\times10^3$ K, $v_{\rm ti} = 6.4\times10^4$ cm/s, $z_{\rm d0} = 1.39\times10^4$ [12] and $\nu_{\rm i} = 10^7$/s, $\mu_{\rm i} = 1.2\times10^3$ cm^2/s, $\nu_{\rm I} = 6\times10^5$/s. All the roots of eq. (29) have come out to be negative which indicate decaying modes of the wave. Let the roots be denoted by γ_1, γ_2 and γ_3. Figure 1 in which $-\gamma_1$, $-\gamma_2$ and $-\gamma_3$ have been plotted against k, the wave vector (magnitude) in cm^{-1}, shows modes of variation of the decay rates with k. It is very interesting to note that up to a value of k around 18, the decaying instability of the wave corresponding to the root γ_1 gradually diminishes whereas the decaying instability corresponding to the root γ_2 continues to increase and opposite thing happens to the instability after $k = 18$. Hence, so far as the decaying instabilities represented by the roots γ_1, γ_2 are concerned, there is transition in the mode of variation of the decay rate around $k = 18$. But the decay rate as predicted by the third root, i.e., γ_3 is found to increase almost linearly with k. In this context, a comparison can be made between the results obtained in our case and obtained by Shukla and Morfill [6]. Equation (13) in Shukla and Morfill [6] indicates only one kind of decaying instability, in particular, ionization instability whereas in our case, eq. (29) gives rise to three different modes of ionization instabilities. This novelty is obviously due to charge variation of the dust grains.

Furthermore, the decay rate as predicted by eq. (13) in Shukla and Morfill [6] and for the same parameter values, has also been plotted against k. This decay rate is observed to increase almost in a linear fashion. It is important to note here that the first part of the plot gets superimposed on the first part of the $-\gamma_2$ vs. k plot and the last part of the plot coincides with the last part of the $-\gamma_1$ vs. k plot. In figure 2, variations of ω_r (the real part of the angular frequency) corresponding to the roots of eq. (29), with k have been displayed.

5. Concluding remarks

Shukla and Morfill [6] have studied the ionizing instability of the electrostatic dust-acoustic waves in weakly ionized dusty plasmas by using the multi-fluid model. In our paper, the work of Shukla and Morfill [6] has been extended with the consideration of a salient feature of dusty plasma, that is, charge fluctuation which causes damping together with that due to ion viscosity, collision between the charged particles and neutral atoms.

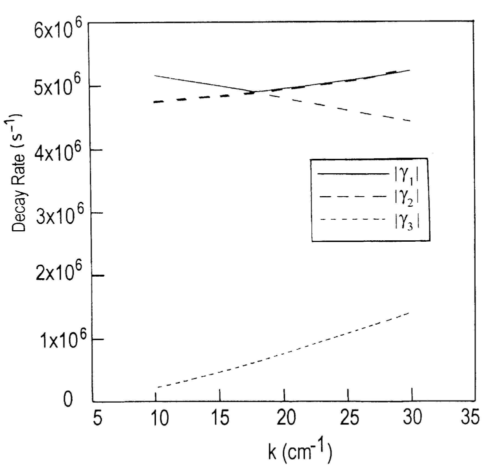

Figure 1. Variation of decay rate with wave number for argon dusty plasma.

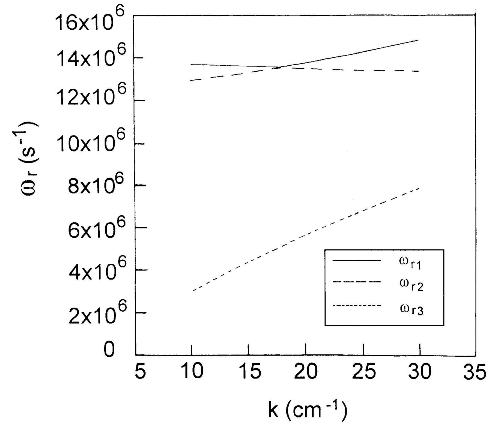

Figure 2. Variation of real part of the angular frequency with wave number for argon dusty plasma.

Equations (24) and (25) involving $\omega_{\rm r}$, the dust-ion-acoustic wave frequency and γ, the growth rate are too much complicated. So, the only way out to study growing or damping of the waves and their dependence on various plasma parameters is to do numerical computation. Similarly, the relations (27) and (28) in which $\omega_{\rm R}$ and $\gamma_{\rm D}$ are coupled, are also heavily complicated. Numerical computation can reveal some insight concerning instability of the dust-acoustic waves in the plasma. Moreover, effects of charge fluctuation on the instability of both the dust-acoustic and the dust-ion-acoustic waves can be investigated numerically. Equation (29) has been solved numerically and the roots have been found to correspond to decaying instabilities of the DIA waves. Another numerical computation for $\gamma_{\rm D}$ may be done from eqs (27) and (28). Variation of $\gamma_{\rm D}/\gamma$ with different plasma parameters can also be studied.

Streaming of the dust grains has been considered neither in Shukla and Morfill [6] nor in this paper. But it is found that dust streaming has considerable effect on the formation of dust-acoustic solitary wave structures in both dust-ion and dust-ion-electron plasmas. There are many situations in space and astrophysical plasmas where the dust-streaming effects are taken into account. For example, two-stream instabilities were investigated in dusty plasmas in planetary magnetospheres and cometary tails. Recently, a linear instability in the drift frequency range has been pointed out due to a dusty plasma flow and in the non-linear vortex solutions has been obtained. So, our work may be extended incorporating streaming of the dust particles.

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