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Brane-world cosmology and inflation

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Abstract. There has been substantial progress in brane-world cosmology in recent years. Much attention has been particularly paid to the second Randall–Sundrum (RS2) scenario in which a single positive-tension brane is embedded in a five-dimensional space-time, called the bulk, with a negative cosmological constant. This brane-world scenario is quite attractive because of the non-trivial geometry in the bulk and because it successfully gives four-dimensional general relativity in the low energy limit. After reviewing basic features of the RS2 scenario, we consider a brane-world inflation model driven by the dynamics of a scalar field living in the five-dimensional bulk, the so-called bulk inflaton model. An intriguing feature of this model is that the projection of the bulk inflaton on the brane behaves just like an ordinary inflaton in four dimensions in the low energy regime, $H^2\ell^2 \ll 1$, where H is the Hubble expansion rate of the brane and ℓ is the curvature radius of the bulk. We then discuss the cosmological perturbation on superhorizon scales in this model. We find that, even under the presence of spatial inhomogeneities, the model is indistinguishable from the standard four-dimensional inflation to $O(H^2\ell^2)$. That is, the difference may appear only at $O(H^4\ell^4)$.

Keywords. Brane world; cosmology; inflation.

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1. Introduction

Recent progress in particle physics has given the idea that our universe is a (mem)brane in a higher dimensional space-time and all the interactions but gravity are confined on the brane, the so-called brane-world scenario [1,2]. In particular, the scenario proposed by Randall and Sundrum [3], in which a single positive tension brane is embedded in the five-dimensional anti-de Sitter space (AdS_5) with Z_2-symmetry (the RS2 scenario), has attracted much attention as an alternative to the standard four-dimensional cosmology [4] because of its attractive feature that the standard four-dimensional gravity is recovered on the brane in the low energy limit [3,5]. The inflationary cosmology is also widely accepted because of its success in explaining cosmological observations [6]. It is therefore natural to consider the inflationary cosmology from the point of view of the brane-world scenario [7]. Brane inflation in the quantum cosmological context is also of great interest [8–10].

Here, we first give a brief review on recent progress in brane-world cosmology, focusing on the RS2 scenario. We then incorporate a bulk scalar field and consider a model of brane-world inflation driven by the dynamics of a scalar field ϕ in the five-dimensional bulk [11–15] and present some recent results on superhorizon scale cosmological perturbations in this model [16].

2. Brane-world cosmology in RS2 scenario

In 1999, Randall and Sundrum wrote two very interesting papers [2,3] on a possible low-energy realization of brane world. In [2], they found an interesting Z_2-symmetric solution of the five-dimensional Einstein equations with a negative cosmological constant. In this solution, two boundary branes with positive and negative tensions are embedded in the five-dimensional anti-de Sitter (AdS) space and the tensions of the branes are chosen so that the effective cosmological constant on the branes vanishes and the four-dimensional Minkowski space is realized on the branes. Note that the brane tension corresponds to the vacuum energy on the brane, but it is not equal to the cosmological constant on the brane. This suggests a possible new solution to the cosmological constant problem, though we do not discuss it here.

Randall and Sundrum argued that the mass-hierarchy problem in particle physics may be solved if we live on the negative tension brane. This work has received much attention from the particle physics community, and subsequently a large number of papers have been published on it. However, it was soon realized that there exists the so-called radion mode that describes fluctuations of the distance between the two branes, and this mode causes an unacceptable modification of the effective gravitational theory on the negative tension brane, unless the radion is stabilized rather artificially [5,17].

In the second paper [3], Randall and Sundrum showed that the negative tension brane may actually be absent if we live on the positive tension brane. They found that because of the curvature of AdS, even if the extra dimension is infinite, gravity on the brane is nicely confined around the brane and the Einstein gravity is effectively recovered on the brane [3,5,18]. As a result, although the hierarchy problem remains unsolved, this model has attracted much attention from the relativity/cosmology community, and the brane-world cosmology has boomed [4].

A simple cosmological extension of the RS2 brane world is realized by embedding a brane in the five-dimensional AdS–Schwarzschild space-time [19], where the bulk metric is given by

$$\mathrm{d}s^2 = -A(R)\mathrm{d}T^2 + \frac{\mathrm{d}R^2}{A(R)} + R^2\mathrm{d}\Omega_K^2, \tag{1}$$

$$A(R) = K + \frac{R^2}{\ell_0^2} - \frac{\alpha^2}{R^2}\,, \tag{2}$$

where $\mathrm{d}\Omega_K^2$ is the metric on the constant curvature 3-space with curvature $K = \pm 1$ or 0, ℓ_0 is the AdS curvature radius given by $\ell_0 = \sqrt{6/|\Lambda_5|}$ (where Λ_5 is the five-dimensional cosmological constant), and α^2 is the mass parameter of the black hole ($\alpha^2 = 2G_5M$). On this bulk space-time, we embed a brane along a trajectory

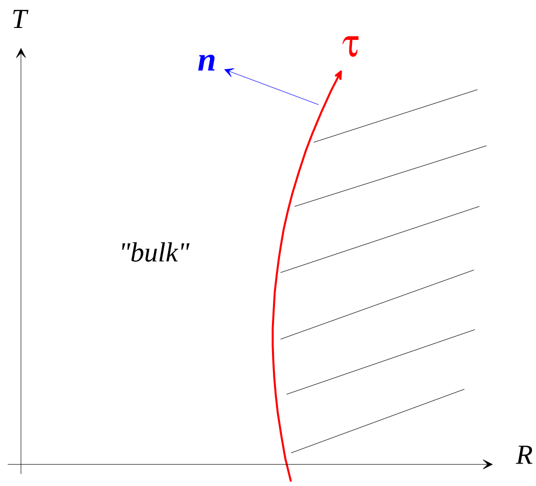

Figure 1. A cosmological brane world as an embedded time-like singular hypersurface in a bulk space-time. The vector n^a is the unit normal to the brane. The Z_2-symmetry is realized by cutting out the shaded region and glueing the two identical space-times at the location of the brane.

$T = T(\tau)$, $R = R(\tau)$, as shown in figure 1. One of the simplest but interesting examples is the case of a de Sitter brane. For $\alpha^2 = 0$ and $K = 1$, the embedding,

$$\begin{aligned} R(\tau) &= \ell_0 \sinh(r_0/\ell_0) \cosh(H\tau)\,, \\ T(\tau) &= \ell_0 \arctan\big[\tanh(r_0/\ell_0)(H\tau)\big]\,, \end{aligned} \tag{3}$$

gives a de Sitter brane with the Hubble parameter H, where $H^2\ell_0^2 = 1/\sinh^2(r_0/\ell_0)$. Assuming Z_2-symmetry, this is realized by the energy–momentum tensor on the brane in the vacuum form,

$$S_{\mu\nu} = -\sigma\, q_{\mu\nu}\,; \quad \sigma = \frac{3}{4\pi G_5 \ell_0} \coth(r_0/\ell_0)\,, \tag{4}$$

where $q_{\mu\nu}$ is the induced metric on the brane. Here and in what follows, the Greek indices $\{\mu, \nu, \ldots\}$ run over four dimensions, while the Latin indices $\{a, b, \ldots\}$ run over five dimensions. Note that the RS2 flat brane is recovered in the limit $r_0 \to \infty$, that is, $H\ell \to 0$.

We note that there exists a Euclidean version of the above de Sitter brane solution, and we may consider it as a gravitational instanton describing the creation of an inflating brane world [8,9]. It also means that an inflationary universe is a natural initial state of the universe in the brane-world scenario.

In a general case when $S_{\mu\nu}$ is not in the vacuum form, the dynamics of a brane is determined as follows. For a brane trajectory $(T, R) = (T(\tau), R(\tau))$, the induced metric on the brane is

$$\mathrm{d}s^2|_{\mathrm{brane}} = \left(-A(R)\dot{T}^2 + \frac{\dot{R}^2}{A(R)}\right)\mathrm{d}\tau^2 + R^2(\tau)\mathrm{d}\Omega_K^2. \tag{5}$$

For convenience, let us choose τ to be the proper time on the brane, $-A(R)\dot{T}^2 + \dot{R}^2/A(R) = -1$, which gives

$$A^2(R)\dot{T}^2 = \dot{R}^2 + A(R)\,. \tag{6}$$

Let n^a be the vector unit normal to the brane. Then Israel's junction condition [20] under Z_2-symmetry gives

$$[K_{\mu\nu}]_\pm = 2K_{\mu\nu}(+0) = -8\pi G_5 \left(S_{\mu\nu} - \frac{1}{3} S g_{\mu\nu} \right) , \tag{7}$$

where $[Q]_\pm = Q(+0) - Q(-0)$ is the discontinuity of Q across the brane, and $K_{\mu\nu}$ is the extrinsic curvature of the brane,

$$K_{\mu\nu} = \frac{1}{2} \pounds_n q_{\mu\nu}\,; \quad n_a = (\dot{R}, -\dot{T}, 0, 0, 0)\,. \tag{8}$$

Without loss of generality, we put $S_{\mu\nu}$ in the form,

$$S^\mu{}_\nu = \mathrm{diag}(-\rho, p, p, p) - \sigma_0\, \delta^\mu{}_\nu\,; \quad \sigma_0 = \frac{3}{4\pi G_5 \ell_0}\,, \tag{9}$$

where the vacuum energy offset σ_0 is chosen to be the value for the RS flat brane. Then the junction condition (7) gives

$$\frac{A(R)}{R}\,\dot{T} = \frac{4\pi G_5}{3}(\rho + \sigma_0) = \frac{4\pi G_5}{3}\rho + \frac{1}{\ell_0}\,. \tag{10}$$

Combining eqs (6) and (10), we arrive at

$$\left(\frac{\dot{R}}{R} \right)^2 + \frac{K}{R^2} = \frac{8\pi G_4}{3}\rho + \ell_0^2 \left(\frac{4\pi G_4}{3}\rho \right)^2 + \frac{\alpha^2}{R^4}\,, \tag{11}$$

where $G_4 = G_5/\ell_0$ is the effective four-dimensional gravitational constant. This is the Friedmann equation on the brane, first derived by Binetruy *et al* [21].

The characteristic feature is the presence of the term $\propto \rho^2$. The ρ^2-term dominates in the early universe, $H \propto \rho$, while it reduces to the conventional Friedmann equation for $\ell_0^2 G_4 \rho \ll 1$ ($\Leftrightarrow \rho \ll \sigma_0$). Another feature is the presence of the term α^2/R^4 that behaves like radiation. It arises from the five-dimensional Weyl tensor and is called the dark radiation or Weyl fluid [22]. Since $\alpha^2 = 2G_5 M$ where M is the mass of the black hole in the bulk, it is argued that this is a particular realization of the AdS/CFT correspondence [8,23], and may describe the Hawking radiation from the bulk black hole [24].

3. Bulk inflaton model

So far, we have assumed that the bulk is vacuum except for the presence of the cosmological constant. It is however natural to expect that some kind of bulk fields are present from a higher dimensional theory point of view. For example, in string theory, there are gravitational scalar fields such as dilaton or moduli fields. These scalar fields will introduce non-trivial dynamics in the bulk and affect cosmology on the brane. In particular, since potential of a scalar field acts like the cosmological constant term if it is sufficiently slowly varying both spatially and temporally, there will appear an effective non-zero cosmological constant on the brane and the brane may undergo inflation solely due to the dynamics in the bulk [11].

It is therefore interesting to investigate the possibility of inflation on the brane without introducing an inflaton on the brane. For this purpose, let us for simplicity consider a minimally coupled scalar field in the bulk,

$$L = -\frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi - U(\phi), \tag{12}$$

and derive the effective gravitational equation on the brane.

We choose the Gaussian normal coordinates,

$$\mathrm{d}s^2 = (n_a n_b + q_{ab}) \mathrm{d}x^a \mathrm{d}x^b = \mathrm{d}r^2 + q_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu\,, \tag{13}$$

in the vicinity of the brane, and assume that the brane is located at $r = r_0$. The Einstein equations in the bulk take the form

$${}^{(5)}G_{ab} + \Lambda_5 g_{ab} = 8\pi G_5 \left(T_{ab}\left[\phi\right] + S_{ab} \delta(r - r_0) \right), \tag{14}$$

where $T_{ab}\left[\phi\right]$ is the energy–momentum tensor of the bulk scalar field, and S_{ab} is the energy–momentum tensor on the brane for which we assume the vacuum form,

$$S_{ab} = -\sigma_0\, q_{ab}\,, \tag{15}$$

where σ_0 is the RS brane tension given in eq. (9). We assume that there is no coupling of the bulk scalar field to the brane tension. Inclusion of it is straightforward [25,26], but we choose not to do so in order to avoid unnecessary complications. The scalar field then satisfied the Neumann boundary condition at the brane,

$$\partial_r \phi|_{r=r_0} = 0\,. \tag{16}$$

From the junction condition (7), we obtain the four-dimensional effective gravitational equations on the brane [11,18,25]

$${}^{(4)}G_{\mu\nu} = 8\pi G_5 T^{(\mathrm{b})}_{\mu\nu} - E_{\mu\nu}, \tag{17}$$

where

$$T^{(\mathrm{b})}_{\mu\nu} = \frac{2}{3}\left[T_{cd} q^c_a q^d_b + \left(T_{cd} n^c n^d - \frac{1}{4} T \right) q_{ab} \right], \tag{18}$$

$$E_{\mu\nu} \equiv {}^{(5)}C_{abcd}\, q^a_\mu\, q^c_\nu n^b n^d, \tag{19}$$

with ${}^{(5)}C_{abcd}$ being the five-dimensional Weyl tensor.

At first glance, it appears that the dynamics on the brane will be quite different from the conventional four-dimensional theory because of the peculiar form of the

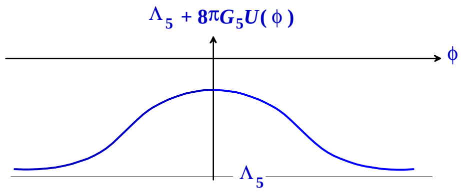

Figure 2. A tachyonic potential for the bulk inflaton model. We assume $\Lambda_5 + 8\pi G_5 U_0 < 0$ so that the effective cosmological constant at $\phi = 0$ is negative.

projection of the bulk energy–momentum tensor on the brane as well as the presence of the projected Weyl tensor term $E_{\mu\nu}$. However, as we shall see below, it turns out that these two terms, when combined together, give an effective four-dimensional Einstein scalar system in the conventional form in the low energy regime $H^2\ell_0^2 \ll 1$.

In general, the projected Weyl tensor term $E_{\mu\nu}$ cannot be determined without solving the bulk dynamics. However, for a spatially homogeneous and isotropic brane, $E_{\mu\nu}$ on the brane may be evaluated without solving the bulk. By using the four-dimensional contracted Bianchi identities, one finds [11]

$$E_{tt} = \frac{4\pi G_5}{a^4(t)} \int^t \mathrm{d}t' a^4(t') \Big(\partial_r^2 \phi + \frac{\dot{a}}{a} \dot{\phi} \Big), \tag{20}$$

where $a(t)$ is the cosmic scale factor on the brane and t the cosmic proper time. The other components of $E_{\mu\nu}$ are determined by the isotropy of the brane and the traceless nature of $E_{\mu\nu}$.

To proceed further, we assume that the potential $U(\phi)$ takes the quadratic form with a tachyonic mass,

$$U(\phi) = U_0 + \frac{1}{2} m^2 \phi^2 \,; \quad U_0 > 0\,, \quad m^2 < 0\,, \tag{21}$$

in the vicinity of $\phi = 0$ (see figure 2). We tacitly assume that there exists a minimum of the potential somewhere at $\phi = \phi_{\rm min} \neq 0$ at which $U(\phi_{\rm min}) = 0$, where the RS2 flat brane is recovered.

For the potential of the form (21), we then focus on the region $|m^2|\phi^2 \ll U_0$ and consider the perturbation with respect to the amplitude of ϕ. In the zeroth order, the background metric is just AdS_5. We have

$$\begin{aligned} &\mathrm{d}s^2 = \mathrm{d}r^2 + b^2(r)\big(-\mathrm{d}t^2 + a(t)^2 \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j\big), \\ &b(r) = H\ell \sinh(r/\ell)\,, \quad a(t) = \frac{\mathrm{e}^{Ht}}{H}\,, \end{aligned} \tag{22}$$

where $H\ell = 1/\sinh(r_0/\ell)$ and ℓ is the curvature radius of this AdS metric given by

$$\ell = \sqrt{\frac{6}{|\Lambda_5 + 8\pi G_5 U_0|}}\,. \tag{23}$$

Note that $\ell > \ell_0$. This implies that the effective four-dimensional gravitational constant is smaller (the effective Planck energy is larger) at this stage of inflation. The brane world inflates with the Hubble expansion rate H given by

$$H^2 = \frac{8\pi G_5}{3}\frac{U_0}{2}\,. \tag{24}$$

In the first order in ϕ, the evolution of the scalar field is determined by the field equation on the background (22). In this AdS_5 bulk with a de Sitter brane, the field equation is separable and it is possible to express ϕ in the sum over all possible modes,

$$\phi = u_0(r)\,\phi_0(t) + \int_0^\infty \mathrm{d}p\, u_p(r)\,\psi_p(t)\,, \tag{25}$$

where $\phi_0(t)$ has the effective four-dimensional mass-squared given by [11,14]

$$M_{\rm eff}^2 = \frac{m^2}{2} \quad \text{for} \quad H^2\ell^2 \ll 1 \tag{26}$$

and $\phi_p(t)$ are called the Kaluza–Klein modes with mass-squared $M_p^2 = (p^2{+}9/4)H^2$. It has been found that the late time behavior of ϕ on the brane is dominated by the bound-state mode $\phi_0(t)$ [14]. In this case, one has

$$\partial_r^2\phi|_b = \frac{m^2}{2}\phi = -\ddot\phi - 3H\dot\phi\,, \tag{27}$$

on the brane. For the slow-roll inflation to last sufficiently long on the brane, we hereafter assume that $|m^2| \ll H^2$.

We now turn to the effective four-dimensional equation (17) at second order. Using eq. (27), E_{tt} is evaluated as

$$E_{tt} = -\frac{4\pi G_5}{a^4}\int^t \mathrm{d}t\, a^4\dot\phi(\ddot\phi + 2H\dot\phi) = -2\pi G_5\dot\phi^2 + \frac{C}{a^4}\,, \tag{28}$$

where C is an integration constant which depends on the initial condition. The term C/a^4 is just the dark radiation term. In the present case, since we are interested in the late time behavior at the inflationary stage, the dark radiation term can be safely neglected. Then, the effective Friedmann equation becomes

$$3\left(\frac{\dot a}{a}\right)^2 = 8\pi G_4\,\rho^{(\rm b)} - E_{tt} \equiv 8\pi G_4\,\rho_{\rm eff}\,, \tag{29}$$

where

$$\rho^{(\rm b)} = \ell_0\left(\frac{1}{4}\dot\phi^2 + \frac{1}{2}V(\phi)\right)\,, \tag{30}$$

and

$$\rho_{\rm eff} = \ell_0\left(\frac{1}{2}\dot\phi^2 + \frac{1}{2}V(\phi)\right)\,. \tag{31}$$

Interestingly, this is perfectly consistent with the late time behavior of the first-order solution for ϕ that behaves effectively as a four-dimensional field with mass-squared $M_{\rm eff}^2 = m^2/2$.

Introducing the effective four-dimensional field φ by

$$\varphi = \sqrt{\ell_0}\phi\,, \tag{32}$$

the system is equivalent to the four-dimensional Einstein scalar system with the potential,

$$V_{\rm eff}(\varphi) = \frac{\ell_0}{2} U(\varphi/\sqrt{\ell_0})\,. \tag{33}$$

Thus the effective field φ behaves as a conventional inflaton on the brane, and the conventional slow-roll inflation is realized on the brane in the low energy regime up through $O(H^2\ell^2)$ [11,14].

4. Cosmological perturbations on superhorizon scales

We have seen that the isotropic and homogeneous cosmology on the brane is indistinguishable from the four-dimensional theory in the low energy regime of the bulk inflaton model. It is then important to see whether there appears any sign of the bulk dynamics in the cosmological perturbations. Some interesting features have been found in the tensor-type perturbations in an inflating brane world [8,27]. However, here we focus on the scalar-type cosmological perturbations.

To investigate cosmological perturbations on the brane, it is necessary to solve the five-dimensional dynamics. This is essentially a problem involving partial differential equations [28], and it is generally formidable to solve it. However, if we focus on the perturbations on superhorizon scales, the problem simplifies considerably [29]. Based on a low-energy effective action approach [30], a formalism particularly suited to this situation was recently developed by Kanno and Soda [31]. According to them, the effective four-dimensional equation at low energy is given by

$$\begin{aligned}
&{}^{(4)}G_{\mu\nu} = 8\pi G_4\, T^{\rm eff}_{\mu\nu}(\varphi) + X_{\mu\nu}\,;\\
&T^{\rm eff}_{\mu\nu} \equiv \nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{2}q_{\mu\nu}\Big(\nabla^\alpha\varphi\nabla_\alpha\varphi + 2V_{\rm eff}(\varphi)\Big),\\
&X_{\mu\nu} \equiv -E_{\mu\nu} - \frac{8\pi G_4}{3}\left(\nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{4}q_{\mu\nu}\nabla^\alpha\varphi\nabla_\alpha\varphi\right),
\end{aligned} \tag{34}$$

where φ is the effective four-dimensional field defined by eq. (32), and $T^{\rm eff}_{\mu\nu}$ and $X_{\mu\nu}$ are conserved separately to $O(H^2\ell^2)$,

$$\nabla^\mu T^{\rm eff}_{\mu\nu} = O\big(H^4\ell^4\big)\,, \quad \nabla^\mu X_{\mu\nu} = O\big(H^4\ell^4\big)\,. \tag{35}$$

Note that $X^\mu{}_\mu = 0$. Thus $X_{\mu\nu}$ describes effectively the dark radiation, although a part of it is due to the scalar field, and it contains the information of the bulk dynamics. Because $X_{\mu\nu}$ is traceless, the conservation equations $\nabla^\mu X_{\mu\nu} = 0$ determines the behavior of the components $X^0{}_\mu$ and $X^i{}_i$. However, the anisotropic stress part of $X_{\mu\nu}$,

$$X_{ij}^{\rm aniso} \equiv X_{ij} - \frac{1}{3} g_{ij} X^k{}_k \,, \tag{36}$$

cannot be determined from the four-dimensional equations. Since

$$T_{\mu\nu}^{\rm eff,\, aniso}(\varphi) = 0 \tag{37}$$

at linear order, we have

$$X_{\mu\nu}^{\rm aniso} = -E_{\mu\nu}^{\rm aniso} \,. \tag{38}$$

Therefore, the information of the bulk dynamics is totally contained in the anisotropic part of $E_{\mu\nu}$, and our task is to solve the five-dimensional dynamics of it.

To evaluate $E_{\mu\nu}$, we take the following approach [16]. We assume the metric and the scalar field in the form,

$$\mathrm{d}s^2 = \mathrm{d}r^2 + b^2(r,t)(-\mathrm{d}t^2 + a^2(r,t)\mathrm{d}\Omega_{(3)}^2) \,, \quad \phi = \phi(r,t), \tag{39}$$

where

$$b(r,t) = b(r) + O(\phi^2) \,, \quad a(r,t) = a(t) + O(\phi^2) \,, \tag{40}$$

and consider the perturbation to $O(\phi^2)$ in the background scalar field amplitude as well as the linear perturbation to $O(\delta\phi)$ for inhomogeneities.

The lowest order background is approximated by AdS_5, eq. (22). Then we have

$$E_{\mu\nu} = O(\phi^2) \,, \quad \delta E_{\mu\nu} = O(\phi\delta\phi) \quad (\leftrightarrow \delta g_{\mu\nu} = O(\phi\delta\phi)) \,. \tag{41}$$

To $O(\phi^2)$, $\delta\phi$ then satisfies the linearized field equation in AdS_5:

$$\left(-\Box_{\mathrm{AdS}_5} + m^2\right) \delta\phi = 0 \,, \tag{42}$$

with the Neumann boundary condition (16) at the brane. The perturbation of the projected Weyl tensor $\delta E_{\mu\nu}$ also satisfies a similar equation in AdS_5 [16]:

$$\mathcal{L}[\delta E_{ab}] = S_{\mu\nu}, \tag{43}$$

where $\mathcal{L}$ is a five-dimensional d'Alembertian-like operator and $S_{\mu\nu}$ is the source term given by the scalar field in a quadratic form, of $O(\phi\delta\phi)$. The boundary condition at the brane is in the form

$$\partial_r(b^2 \delta E_{\mu\nu}) = \delta\sigma_{\mu\nu} \,, \tag{44}$$

where $\sigma_{\mu\nu}$ is again of $O(\phi\delta\phi)$.

To solve eq. (43), we proceed as follows:

1. Solve eq. (42) for $\delta\phi$ in the AdS bulk.

2. Solve $\mathcal{L}\,[\delta E_{\mu\nu}] = S_{\mu\nu}[\delta\phi]$ by the Green function method:

$$\delta E(x) \sim \int_{\rm bulk} {\rm d}^5x'\, G(x,x')\delta S(x') + \int_{\rm brane} {\rm d}^4x'\, \partial_r(b^2\delta E(x'))G(x,x')$$

3. Analyse the late time behavior of $\delta E_{\mu\nu}$ on the brane on superhorizon scales.

This procedure is straightforward, though it is technically rather involved. After a long and tedious calculation, we find [16]

$$\begin{aligned}\delta X_{\mu\nu} &\equiv -\delta E_{\mu\nu} - \frac{8\pi G_4}{3}\\ &\qquad\times\left(\nabla_\mu\varphi\nabla_\nu\delta\varphi + \nabla_\mu\delta\varphi\nabla_\nu\varphi - \frac{1}{2}g_{\mu\nu}\nabla^\alpha\varphi\nabla_\alpha\delta\varphi\right)\\ &= O(H^4\ell^4).\end{aligned} \tag{45}$$

That is, the anisotropic stress part, $\delta X^i{}_j - \frac{1}{3}\delta^i_j\,\delta X^k{}_k$, decays out and no effect of the bulk dynamics persists at late times.

Thus, up through $O(H^2\ell^2)$, the bulk inflaton model is found to be completely equivalent to the conventional four-dimensional slow-roll inflation. The difference appears only at $O(H^4\ell^4)$ or when $H^2\ell^2 \gtrsim 1$.

5. Summary

After a brief review on the RS2-type brane-world cosmology, we discussed the possible brane-world inflationary scenario driven by the dynamics of a bulk scalar field, the so-called bulk inflaton model. In the leading non-trivial order of the bulk effect, that is, of $O(H^2\ell^2)$, where H is the Hubble parameter on the brane and ℓ is the AdS curvature radius in the bulk, we found that the bulk inflaton model is completely equivalent to the conventional four-dimensional model of inflation. This means that the bulk inflaton model is perfectly as viable as the standard inflation model. However, this also means that it is almost impossible to find cosmological evidence for the brane-world scenario. Nevertheless, there is still a hope that we can find some non-trivial evidence for the brane world. Here, we considered the case in which H is almost constant in time. In reality, H varies in time, and this introduces mixing between the bound state mode (with $M^2_{\rm eff} = m^2/2$ and the Kaluza–Klein modes (with $M^2 \geq 9H^2/4$). Further, if the time variation of H is sufficiently large, as at the stage of reheating, this mode decomposition into the bound state and Kaluza–Klein modes becomes meaningless. In such a case, there may occur phenomena specific to the brane world, and some imprints of them may remain until today, for example, in the cosmic microwave background anisotropy. Recently, a family of exactly soluble bulk inflaton models has been found [32], which may help us better understand the relation between the bulk dynamics and brane inflation. Some progress has been made in understanding the brane dynamics from the bulk point of view [33]. Investigations on the effect of a time-varying H have also appeared [34]. But we are still far from having good understanding of the effect of the bulk dynamics on the brane. More studies are apparently needed to clarify if there can indeed be cosmological evidence for the brane world.

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