The confrontation between general relativity and experiment

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Abstract. We review the experimental evidence for Einstein's general relativity. Tests of the Einstein equivalence principle support the postulates of curved space-time and bound variations of fundamental constants in space and time, while solar system experiments strongly confirm weak-field general relativity. The binary pulsar provides tests of gravitational wave damping and of strong-field general relativity. Future experiments, such as the gravity probe B gyroscope experiment, a satellite test of the equivalence principle, and tests of gravity at short distance to look for extra spatial dimensions could further constrain alternatives to general relativity. Laser Interferometric Gravitational Wave Observatories on Earth and in space may provide new tests of scalar–tensor gravity and graviton-mass theories via the properties of gravitational waves.

Keyword. Tests of general relativity.

PACS No. 04.80.Cc

1. Introduction

Because of its elegance and simplicity, and because of its empirical success, general relativity has become the foundation for our understanding of the gravitational interaction. Yet modern developments in particle theory suggest that it is probably not the entire story and that modification of the basic theory may be required at some level. String theory generally predicts a proliferation of scalar fields that could result in alterations of general relativity reminiscent of the Brans–Dicke theory of the 1960s. In the presence of extra dimensions, the gravity that we feel on our four-dimensional 'brane' of a higher dimensional world could be somewhat different from a pure four-dimensional general relativity. Some of these ideas have motivated the possibility that fundamental constants may actually be dynamical variables and hence may vary in time or in space. However, any theoretical speculation along these lines must abide by the best current empirical bounds. Decades of high-precision tests of general relativity have produced some very tight constraints. In this article we review some of these experimental constraints. We begin with an

overview of experimental tests of general relativity. For further discussion of topics in this section, and for references to the literature, the reader is referred to 'Theory and Experiment in Gravitational Physics' [1] and to the 'living' review article [2]. We end with a discussion of possible future tests of gravitational theory that will exploit the direct detection of gravitational radiation using ground-based and space-based interferometers [3].

2. Empirical status of general relativity

2.1 *The Einstein equivalence principle*

The Einstein equivalence principle (EEP) is a powerful and far-reaching principle, which states that (i) test bodies fall with the same acceleration independently of their internal structure or composition (weak equivalence principle, or WEP), (ii) the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed (local Lorentz invariance, or LLI), and (iii) the outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed (local position invariance or LPI).

The Einstein equivalence principle is the heart of gravitational theory, for it is possible to argue convincingly that if EEP is valid, then gravitation must be described by 'metric theories of gravity', which state that (i) space-time is endowed with a symmetric metric, (ii) the trajectories of freely falling bodies are geodesics of that metric, and (iii) in local freely falling reference frames, the non-gravitational laws of physics are those written in the language of special relativity.

General relativity is a metric theory of gravity, but so are many others including the Brans–Dicke theory. In this sense, superstring theory is not metric, because of residual coupling of external, gravitation-like fields to matter. Theories in which varying non-gravitational constants are associated with dynamical fields that couple to matter directly are also not metric theories.

A direct test of WEP is the comparison of the acceleration of two laboratory-sized bodies of different composition in an external gravitational field. A measurement or limit on the fractional difference in acceleration between two bodies yields a quantity $\eta \equiv 2|a_1 - a_2|/|a_1 + a_2|$, called the 'Eötvös ratio'.

The best limit on η currently comes from the 'Eöt-Wash' experiments carried out at the University of Washington, which used a sophisticated torsion balance tray to compare the accelerations of bodies of different compositions toward the Earth, the Sun and the galaxy. Another strong bound comes from lunar laser ranging (LURE), which checks the equality of free fall of the Earth and the Moon toward the Sun. The results are

$$\eta_{\text{Eöt-Wash}} < 4 \times 10^{-13}, \quad \eta_{\text{LURE}} < 5 \times 10^{-13}. \tag{1}$$

In fact, by using laboratory materials whose composition mimics that of the Earth and the Moon, the Eöt-Wash experiments [4] permit one to infer an unambiguous bound from lunar laser ranging on the universality of acceleration of gravitational

binding energy at the level of 1.3×10^{-3} (test of the Nordtvedt effect – see §2.2 and table 1).

The best tests of local Lorentz invariance are the 'mass anisotropy' or 'Hughes–Drever' experiments. A simple way of interpreting these experiments is to suppose that a non-metric coupling to the electromagnetic interactions results in a change in the speed of electromagnetic radiation c relative to the limiting speed of the material test particles c_0. In other words, $c \neq c_0$. Such a Lorentz non-invariant electromagnetic interaction would cause shifts in the energy levels of atoms and nuclei that depend on the orientation of the quantization axis of the state relative to our velocity in the rest-frame of the universe and on the quantum numbers of the state with sizes proportional to $\delta \equiv |(c_0/c)^2 - 1|$. Searches for such energy anisotropies using laser-cooled trapped atom techniques have placed the strong bound $\delta < 10^{-21}$. For further discussion, see [5]; for recent alternative viewpoints on testing Lorentz invariance see [6,7].

Local position invariance requires, among other things, that the internal binding energies of atoms be independent of location in space and time, when measured against some standard atom. This means that a comparison of the rates of two different kinds of clocks should be independent of location or epoch and that the frequency shift between two identical clocks at different locations is simply a consequence of the apparent Doppler shift between a pair of inertial frames momentarily co-moving with the clocks at the moments of emission and reception respectively. The relevant parameter is $\alpha \equiv \partial \ln E_{\mathrm{B}}/\partial(U/c^2)$, where E_{B} is the atomic or nuclear binding energy and U characterizes the external gravitational potential. The best bounds come from a 1976 rocket red-shift experiment using hydrogen masers, and a 1993 clock intercomparison experiment (a 'null' red-shift experiment). The results are

$$\alpha_{\mathrm{maser}} < 2 \times 10^{-4}, \quad \alpha_{\mathrm{null}} < 10^{-3}. \tag{2}$$

Clock intercomparisons have also bounded a time variation of the fine structure constant over cosmological time-scales; from rubidium fountain clocks compared with cesium clocks, one finds the bound $\dot{\alpha}/\alpha < 8 \times 10^{-15}$ yr^{-1} [8]. A better bound comes from analysis of fission yields of the Oklo natural reactor, which occurred in Africa two billion years ago, namely $(\dot{\alpha}/\alpha)_{\mathrm{Oklo}} < 6 \times 10^{-17}$ yr^{-1} [9]. These and other bounds on variations of constants, including reports of positive evidence for variations from quasar spectra, are discussed by Martins and others in ref. [10]. For a review of these and other tests of EEP, see [2].

2.2 *Bounds on the post-Newtonian parameters of metric theories*

When we confine our attention to metric theories of gravity and further focus on the slow-motion, weak-field limit appropriate to the solar system and similar systems, it turns out that, in a broad class of metric theories, only the numerical values of a set of parameters vary from theory to theory. This framework, called the parametrized post-Newtonian (PPN) formalism, is a convenient tool for classifying alternative metric theories of gravity, for interpreting the results of experiments, and for suggesting new tests of metric gravity. The framework contains ten PPN

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Table 1. Current limits on the PPN parameters.

Parameter	Effect	Limit	Remarks
$\gamma - 1$	(i) Time delay	2.3×10^{-5}	Cassini Doppler
	(ii) Light deflection	3×10^{-4}	VLBI
$\beta - 1$	(i) Perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from
			helioseismology
	(ii) Nordtvedt effect	5×10^{-4}	$\eta = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	Gravimeter data
α_1	Orbital polarization	10^{-4}	Lunar laser ranging
			PSR J2317+1439
α_2	Solar spin	4×10^{-7}	Alignment of sun
	precession		and ecliptic
α_3	Pulsar acceleration	2×10^{-20}	Pulsar $\dot{P}$ statistics
η^1	Nordtvedt effect	10^{-3}	Lunar laser ranging
ζ_1	–	2×10^{-2}	Combined PPN bounds
ζ_2	Binary motion	4×10^{-5}	$\ddot{P}_p$ for PSR 1913+16
ζ_3	Newton's third law	10^{-8}	Lunar acceleration
ζ_4	–	–	Not independent

Here $\eta = 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 - 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$.

parameters: γ, related to the amount of spatial curvature generated by mass; β, related to the degree of non-linearity in the gravitational field; ξ, α_1, α_2, and α_3, which determine whether the theory violates local position invariance or local Lorentz invariance in gravitational experiments (violations of the strong equivalence principle); and ζ_1, ζ_2, ζ_3 and ζ_4, which describe whether the theory has appropriate momentum conservation laws. The parameter γ, or more precisely, the combination $(1 + \gamma)/2$, governs the deflection of light and the Shapiro time delay in ranging. The combination $(2 + 2\gamma - \beta)/3$ governs the perihelion advance. The combination $4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 - 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$ governs the Nordtvedt effect, a possible violation of WEP induced by gravitational self-energy of massive bodies. In general relativity, $\gamma - 1$, $\beta - 1$, and the remaining parameters all vanish.

Three decades of experiments, ranging from the standard light-deflection and perihelion-shift tests to lunar laser ranging, planetary and satellite tracking, and geophysical and astronomical observations, have placed bounds on the PPN parameters consistent with general relativity. The results are summarized in table 1. Scalar–tensor theories of gravity are characterized by a coupling function $\omega(\phi)$ whose size is inversely related to the 'strength' of the scalar field relative to the metric. In the solar system, the parameter $|\gamma - 1|$, for example is equal to $1/(2 + \omega(\phi_0))$, where ϕ_0 is the value of the scalar field today outside the solar system. Solar-system experiments (primarily recent Doppler tracking of the Cassini spacecraft [11]) constrain $\omega(\phi_0) > 40000$. See [2] for an up-to-date review.

2.3 *The binary pulsar*

The binary pulsar PSR 1913+16, discovered in 1974, provided important new tests of general relativity, specifically of gravitational radiation and of strong-field gravity.

Table 2. Parameters of the binary pulsar PSR 1913+16.

Parameter	Symbol	Value
Keplerian parameters		
Eccentricity	e	0.6171308(4)
Orbital period	P_b (d)	0.322997462736(7)
Post-Keplerian parameters		
Mean rate of periastron advance	$\langle \dot\omega \rangle$ ($^\circ$yr^{-1})	4.226621(11)
Gravitational red-shift/time dilation	γ' (m s)	4.295(2)
Orbital period derivative	$\dot P_b$ (10^{-12})	$-2.422(6)$

Numbers in parentheses denote errors in last digit.

Through precise timing of the pulsar 'clock', the important orbital parameters of the system could be measured with exquisite precision. These included non-relativistic 'Keplerian' parameters, such as the eccentricity e, and the orbital period (at a chosen epoch) P_b, as well as relativistic 'post-Keplerian' parameters, such as the mean advance of periastron, the effect of time-dilation and gravitational red-shift on pulses, and the rate of decrease of the orbital period (table 2). The last effect is a result of gravitational radiation damping (apart from a small correction due to galactic differential rotation). According to GR, all three post-Keplerian effects depend on e and P_b, which are known, and on the two stellar masses which are unknown. By combining the observations with the GR predictions, one obtains both a measurement of the two masses, and a test of GR, since the system is overdetermined. The results are

$$m_1 = 1.4411 \pm 0.0007 M_\odot, \quad m_2 = 1.3873 \pm 0.0007 M_\odot,$$
$$\dot P_b^{\mathrm{GR}}/\dot P_b^{\mathrm{OBS}} = 1.002 \pm 0.003. \tag{3}$$

The results also test the strong-field aspects of GR in the following way: the neutron stars that comprise the system to have very strong internal gravity, contributing as much as several tenths of the rest mass of the bodies (compared to the orbital energy, which is only 10^{-6} of the mass of the system). Yet in general relativity, the internal structure is 'effaced' as a consequence of the strong equivalence principle, and the orbital motion and gravitational radiation emission depend only on the masses m_1 and m_2. By contrast, in alternative metric theories, SEP is not valid in general, and internal structure effects can lead to significantly different behavior, such as the emission of dipole gravitational radiation. Unfortunately, in the case of scalar–tensor theories of gravity, because the neutron stars are so similar in PSR 1913+16 (and in other double-neutron star binary pulsar systems), dipole radiation is suppressed by symmetry; the best bound on the coupling parameter $\omega(\phi_0)$ from PSR 1913+16 is in the hundreds. However, the recent discovery of the relativistic neutron star/white dwarf binary pulsar J1141-6545, with a 0.19 day orbital period, may ultimately lead to a very strong bound on dipole radiation, and thence on scalar–tensor gravity [12]. For further discussion of binary pulsar tests, see [2].

2.4 *Future experimental tests*

There are a number of possibilities for future tests of alternative gravitation theories. (i) High-precision WEP experiments can test super-string inspired models of scalar–tensor gravity or theories with varying fundamental constants in which weak violations of WEP can occur via non-metric couplings. The project MICRO-SCOPE, designed to test WEP to a part in 10^{15} has been approved by the French space agency CNES for a possible 2008 launch. A proposed NASA–ESA satellite test of the equivalence principle (STEP) may reach the level of $\eta < 10^{-18}$. In addition, laboratory tests of the inverse square law of Newtonian gravitation at millimeter scales and below are underway to search for such additional couplings or for the presence of large extra dimensions; the challenge of these experiments is to distinguish gravitation-like interactions from electromagnetic and quantum mechanical (Casimir) effects [13,14]. (ii) The NASA relativity mission called gravity probe-B, will measure the precession of an array of gyroscopes in Earth orbit. Although its primary science goal is a 1% measurement of the gravitomagnetic dragging of inertial frames caused by the rotation of the Earth, it will also measure the larger geodetic precession caused by space curvature, and could thus measure γ to better than a part in 10^4, giving a bound on ω of 10^4 or higher. Successful launch of the mission took place on April 20, 2004. (iii) Orbiting optical interferometers to measure the deflection of light at the micro-arcsecond level could bound ω at the level of 10^5 or greater.

3. Gravitational wave tests of gravitation theory

The detection of gravitational radiation by either laser interferometers or resonant cryogenic bars will, it is widely stated, usher in a new era of gravitational wave astronomy [15,16]. Furthermore, it will yield new and interesting tests of general relativity (GR) in its radiative regime [3].

3.1 *Polarization of gravitational waves*

A laser-interferometric or resonant bar gravitational wave detector measures the local components of a symmetric 3×3 tensor which is composed of the 'electric' components of the Riemann tensor, R_{0i0j}. These six independent components can be expressed in terms of polarizations (modes with specific transformation properties under null rotations). Three are transverse to the direction of propagation, with two representing quadrupolar deformations and one representing an axisymmetric 'breathing' deformation. Three modes are longitudinal, with one an axially symmetric stretching mode in the propagation direction, and one quadrupolar mode in each of the two orthogonal planes containing the propagation direction. General relativity predicts only the first two transverse quadrupolar modes, independently of the source, while scalar–tensor gravitational waves can in addition contain the transverse breathing mode. More general metric theories predict up to the full complement of six modes. A suitable array of gravitational antennas could delineate or

limit the number of modes present in a given wave. If distinct evidence were found of any mode other than the two transverse quadrupolar modes of GR, the result would be disastrous for GR. On the other hand, the absence of a breathing mode would not necessarily rule out scalar–tensor gravity, because the strength of that mode depends on the nature of the source.

3.2 *Speed of gravitational waves*

According to GR, in the limit in which the wavelength of gravitational waves is small compared to the radius of curvature of the background space-time, the waves propagate along null geodesics of the background space-time, i.e., they have the same speed, c, as light. In other theories, the speed could differ from c because of coupling of gravitation to 'background' gravitational fields. For example, in some theories with a flat background metric $\boldsymbol{\eta}$, gravitational waves follow null geodesics of $\boldsymbol{\eta}$, while light follows null geodesics of $\mathbf{g}$ [1].

Another way in which the speed of gravitational waves could differ from c is if gravitation were propagated by a massive field (a massive graviton), in which case v_{g} would be given by, in a local inertial frame,

$$\frac{v_{\mathrm{g}}^2}{c^2} = 1 - \frac{m_{\mathrm{g}}^2 c^4}{E^2} \approx 1 - \frac{1}{2}\frac{c^2}{f^2 \lambda_{\mathrm{g}}^2}, \tag{4}$$

where m_{g}, E and f are the graviton rest mass, energy and frequency, respectively, and $\lambda_{\mathrm{g}} = h/m_{\mathrm{g}}c$ is the graviton Compton wavelength ($\lambda_{\mathrm{g}} \gg c/f$ assumed). An example of a theory with this property is the two-tensor massive graviton theory of Visser [17].

The most obvious way to test for a massive graviton is to compare the arrival times of a gravitational wave and an electromagnetic wave from the same event, e.g. a supernova. For a source at a distance D, the resulting bound on the difference $|1 - v_{\mathrm{g}}/c|$ or on λ_{g} is

$$\left|1 - \frac{v_{\mathrm{g}}}{c}\right| < 5 \times 10^{-17} \left(\frac{200 \text{ Mpc}}{D}\right)\left(\frac{\Delta t}{1 \text{ s}}\right), \tag{5}$$

$$\lambda_{\mathrm{g}} > 3 \times 10^{12} \text{ km} \left(\frac{D}{200 \text{ Mpc}}\frac{100 \text{ Hz}}{f}\right)^{1/2}\left(\frac{1}{f\Delta t}\right)^{1/2}, \tag{6}$$

where $\Delta t \equiv \Delta t_{\mathrm{a}} - (1 + Z)\Delta t_{\mathrm{e}}$ is the 'time difference', where Δt_{a} and Δt_{e} are the differences in arrival time and emission time, respectively, of the two signals, and Z is the red-shift of the source. In many cases, Δt_{e} is unknown, so that the best one can do is to employ an upper bound on Δt_{e} based on observation or modelling.

However, there is a situation in which a bound on the graviton mass can be set using gravitational radiation alone [18]. That is the case of the inspiralling compact binary. Because the frequency of the gravitational radiation sweeps from low frequency at the initial moment of observation to higher frequency at the final moment, the speed of the gravitons emitted will vary, from lower speeds initially to higher speeds (closer to c) at the end. This will cause a distortion of the observed

Table 3. Potentially achievable bounds on λ_g from gravitational wave observations of inspiralling compact binaries.

$m_1(M_\odot)$	$m_2(M_\odot)$	Distance (Mpc)	Bound on λ_g (km)
Ground-based (LIGO/VIRGO)			
1.4	1.4	300	4.6×10^{12}
1.4	10	630	5.4×10^{12}
10	10	1500	6.0×10^{12}
Space-based (LISA)			
10^7	10^7	3000	6.9×10^{16}
10^6	10^6	3000	5.4×10^{16}
10^5	10^5	3000	2.3×10^{16}
10^4	10^4	3000	0.7×10^{16}

phasing of the waves and result in a shorter than expected overall time Δt_a of passage of a given number of cycles. Furthermore, through the technique of matched filtering, the parameters of the compact binary can be measured accurately [19], and thereby the emission time Δt_e can be determined accurately.

A full noise analysis using proposed noise curves for the advanced LIGO ground-based detectors, and for the proposed space-based LISA yields potentially achievable bounds that are summarized in table 3. These potential bounds can be compared with the solid bound $\lambda_g > 2.8 \times 10^{12}$ km, derived from solar system dynamics, which limit the presence of a Yukawa modification of Newtonian gravity of the form $V(r) = (GM/r) \exp(-r/\lambda_g)$ [20], and with the model-dependent bound $\lambda_g > 6 \times 10^{19}$ km from consideration of galactic and cluster dynamics [17].

3.3 *Tests of scalar–tensor gravity*

Scalar–tensor theories generically predict dipole gravitational radiation, in addition to the standard quadrupole radiation, which results in modifications in gravitational-radiation back-reaction, and hence in the evolution of the phasing of gravitational waves from inspiralling sources. The effects are strongest for systems involving a neutron star and a black hole. Double neutron star systems are less promising because the small range of masses near $1.4M_\odot$ with which they seem to occur results in the suppression of dipole radiation by symmetry. Double black-hole systems turn out to be observationally identical in the two theories, because black holes by themselves cannot support scalar 'hair' of the kind present in these theories. Dipole radiation will be present in black hole neutron-star systems, however.

For example, for a $1.4M_\odot$ neutron star and a $10M_\odot$ $(3M_\odot)$ black hole at 200 Mpc observed by an advanced LIGO detector, the bound on ω could be 600 (1800). The bound increases linearly with signal-to-noise ratio, although the event rate decreases as the inverse cube of S/N [21].

Significantly stronger bounds could be obtained using observations of low-frequency gravitational waves by a space-based LISA-type detector. For example,

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Table 4. Potentially achievable bounds on ω for NS–BH systems using LISA data analysis. Signal-to-noise ratio $= 10$, integration time is one year prior to the innermost stable circular orbit.

$m_{\mathrm{BH}}(M_\odot)$	Bound on ω
1000	191000
5000	53700
10000	28600
50000	4100
100000	1200

observations of a $1.4 M_\odot$ NS inspiralling to a $10^3 M_\odot$ BH with a signal-to-noise ratio of 10 could yield a bound of $\omega > 1.9 \times 10^5$, substantially greater than the current experimental bound of $\omega > 40000$ [22,23]. The results are summarized in table 4.

For a follow-on mission to LISA whose peak sensitivity to gravitational waves would be at frequencies between those of LISA and the ground-based detectors, the bounds could be even stronger: for a NS inspiralling into a $200 M_\odot$ BH, the bound could exceed 12 million. Recent discoveries of such intermediate-mass black holes in globular clusters make such tests potentially feasible [24].

Acknowledgements

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