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Status of numerical relativity

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Abstract. I describe the current status of numerical relativity from my personal point of view. Here, I focus mainly on explaining the numerical implementations necessary for simulating general relativistic phenomena such as the merger of compact binaries and stellar collapse, emphasizing the well-developed current status of such implementations that enable simulations for several astrophysical phenomena. Some of our latest results for simulation of binary neutron star mergers are briefly presented.

Keywords. Numerical relativity; gravitational waves; black hole; neutron star.

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1. Introduction

In the universe, there are many dynamical phenomena in which general relativity plays a crucial role. Stellar core collapse of massive stars to a neutron star or a black hole, formation of a supermassive black hole, non-axisymmetric deformation of a rapidly rotating neutron star, and the merger of binary compact objects such as binary neutron stars and black holes are typical examples. Since they are highly relativistic systems, any approximation of general relativity breaks down and Einstein's equations have to be solved without any approximation for the theoretical study. Also, since they are dynamical phenomena, analytical study is not feasible. Therefore, to clarify their detailed processes theoretically, we need to carry out numerical simulations.

General relativistic and dynamical phenomena in non-spherical systems are also strong emitters of gravitational waves. The frequency of gravitational waves from rotating stellar core collapses and from the late stage of coalescing, relativistic binaries are likely to be in the range between ∼10 Hz and ∼1 kHz. This implies that these phenomena are promising sources of kilometer-size laser interferometric detectors such as LIGO, TAMA, GEO, and VIRGO [1,2]. For the detection of gravitational waves by the laser interferometers, it is necessary to prepare theoretical templates of gravitational wave-forms, which are required to be computed numerically.

Since the phenomena that I mentioned above are non-axisymmetric or susceptible to deformation to non-axisymmetric objects even if they are initially axisymmetric, simulations have to be done assuming no symmetry in space and time. In the past decades, there have been much effort to complete numerical implementations for numerical relativity in three dimensions in space, and we have had great progress particularly in the last decade. The main purpose of this paper is to review this progress.

This article is organized as follows: In §2, I summarize the necessary implementations in numerical relativity and describe the latest progress in each implementation, emphasizing that simulations for several astrophysical phenomena such as merger of binary neutron stars and stellar collapses are now feasible. In §3, I present some of our latest numerical results for merger of binary neutron stars. Section 4 is devoted to a summary. Throughout this review, the geometrical units in which $c = G = 1$ are adopted.

2. Current status of numerical relativity

To obtain scientific results from numerical simulations for dynamical and general relativistic phenomena, several implementations are required to prepare. In the following, I describe several key implementations and explain the current status of their constructions individually.

2.1 *Initial condition*

One needs to prepare a realistic initial condition to produce scientific results. Here, I pay attention to two representative cases.

One is the case of rotating collapse of massive stellar cores, supermassive stars, and supramassive neutron stars. In this case, the initial condition should be a general relativistic rotating star in an approximate equilibrium. Numerical techniques for the computation of such an equilibrium have been developed in the past two decades and almost completed. It is now possible to compute equilibrium rotating stars for a wide variety of equations of state and rotation laws [3,4].

The other is the case of the merger of binary neutron stars or black holes. For these systems, the time-scale of gravitational wave emission is longer than the orbital period even in an orbit close to the innermost stable circular orbit (ISCO), and hence, the radiation reaction may be safely neglected. This implies that the assumption of the quasi-equilibrium is appropriate, and, therefore, general relativistic quasi-equilibrium states in close circular orbits are the appropriate initial conditions.

In computing quasi-equilibrium states, one may assume the existence of a helical Killing vector as $\xi^\mu \equiv (\partial/\partial t)^\mu + \Omega(\partial/\partial\varphi)^\mu$ where Ω is the orbital angular velocity. For the case of binary neutron stars, the most popular approach currently is to adopt the so-called conformally flat approximation (or Isenberg–Wilson–Mathews (IWM) formalism) [5]. In this formalism, the three-metric is assumed to be conformally flat. Then, one solves selected Einstein's equations; the constraint equations and a trace part of the Einstein's evolution equation with the maximal slicing condition. It should be noted that with this formalism, only five components of Einstein's equations are satisfied. The other five are violated, but one usually believes that the violation is small as long as the emissivitty of gravitational radiation is small and the self-gravity is not extremely strong. For example, this approximation can provide a very accurate approximate solution for a single compact rotating neutron star [6].

Since neutron stars are not as compact as black holes, the IWM formalism could be a good approximation. From this motivation, quasi-equilibrium states of binary neutron stars have been computed in the IWM formalism. For binaries of neutron stars, in addition to the field equations, the hydrostatic equations have to be solved. Realistic velocity field of binary neutron stars is almost irrotational because of negligible viscous effects inside the neutron stars [7]. For the irrotational velocity field, a spatial component of the fluid four-velocity is described using a scalar velocity potential, and as a result, the continuity equation reduces to an elliptic-type equation for the velocity potential. Furthermore, in the space-time with the helical Killing vector, the Euler equation can be integrated to give the first integral [8]. Thus, these two equations are the basic hydrostatic equations for binary neutron stars to be solved.

Adopting these coupled equations, many studies have been carried out in the last five years with different equations of state, compactness of neutron stars, and binary mass ratios mainly by Japanese [9,10] and French groups [11]. Both groups have shown that the numerical solutions satisfy a first thermodynamical law of binary system and a virial relation [12,13] with a high accuracy. Thus, one can conclude that a good initial condition for the merger of binary neutron stars is ready. The point to be improved in this issue is to develop a formulation beyond the conformal flatness formalism since the conformal flatness does not hold strictly [13].

In the past decade, there have also been several programmes for computing quasi-equilibria of binary black holes [14–16] adopting a conformally flat spatial metric. Among them, [16] presents the first work in terms of the IWM formalism that seems to give more plausible quasi-equilibria. However, this approximation would break down when applied to the space-time of binary black holes in close orbits in which the self-gravity is extremely strong. Indeed, the binding energy derived in this approximation does not agree with that obtained by post-Newtonian approximations near the innermost stable circular orbits [17,18]. Moreover, the conformal flatness approximation cannot be applied for a system of rotating black holes, since a Kerr solution does not have conformally flat spatial metric [19]. This implies that for a solution of binary black holes that are in general spinning, this approximation might be inadequate. It seems obvious that more robust formulation for computation of quasi-equilibrium states of binary black holes is necessary.

2.2 *Solving evolution equations*

One needs to find an appropriate formulation which enables stable and accurate integration of Einstein's field equations. In numerical relativity, Einstein's equations are usually solved using 3+1 formalism (or Arnowitt–Deser–Misner (ADM) formalism) [20,21]. In this formalism, the equations are divided into the evolution equations and the constraint equations. In the standard methods, the evolution equations are integrated and the constraint equations are used to check the accuracy of the numerical solution. However, it has been found that with inappropriate formalisms, a slight numerical error is significantly amplified, and falls into significant constraint violation to produce an inaccurate numerical solution and/or to crash the computation. To avoid such instability, a robust formulation is necessary.

Until early 1990s, many people seemed to believe that the ADM formalism is adequate [20,21]. Unfortunately, this formalism does not permit stable and long-term numerical simulations as first pointed out by Nakamura *et al* [22]. Many alternative methods have been proposed in the last several years. One of the promising formulations is the so-called Baumgarte–Shapiro–Shibata–Nakamura (BSSN) formalism that was originally proposed by Nakamura *et al* in 1987 [22], and subsequently modified by Shibata *et al* [23,24] and by Baumgarte, Shapiro and Yo [25,26]. In this formalism, the ADM equations are slightly modified by introducing auxiliary variables such as $F_i \equiv \delta^{jk}\partial_k\tilde{\gamma}_{ij}$, $\mathrm{tr}(\gamma_{ij})$, and K_k^k, in addition to the three-metric and the extrinsic curvature. The formalism has been tested for a long-term evolution of linear gravitational waves [23,25] and subsequently used for a wide variety of purposes such as to simulate the non-linear evolution of gravitational waves [23,27], the evolution of a rotating black hole [26,28], the grazing collision of two black holes [29], a short-term simulation of orbiting binary black holes [30], a long-term simulation of rotating neutron stars [31–35], the collapse of a rotating star to a black hole [31,32,34,36–38] and the merger of binary neutron stars [24,39,40], demonstrating its robustness.

As an alternative, hyperbolic formulations have also been developed [41–44]. In particular, Kidder, Scheel, and Teukolsky (KST) have recently developed the so-called KST formulation. In this formulation, Einstein's evolution equations are rewritten to 30–40 evolution equations with 12 parameters that can be freely chosen. KST have shown that with an appropriate choice of parameter value, it is feasible to carry out numerical simulation for a very long time-scale (several $1000M$ where M denotes the ADM mass of a system) even in the presence of a black hole. This seems to demonstrate that the KST formulation is one of the promising formulations in numerical relativity, although unfortunately they have not succeeded in a long-term simulation of binary black holes. In addition to the formulation itself, Lindblom and Scheel [44] have recently developed a method to analyse the stability property of hyperbolic formulations. Such a method is very helpful to find a robust formulation in numerical relativity.

2.3 *Gauge conditions*

Since general relativity is a covariant theory, there exist four degrees of freedom for the choice of coordinates. This freedom is called the gauge freedom. Although the physics does not, the magnitude of metric components depends significantly on the choice of gauge. This implies that if one chooses an inappropriate gauge condition, some of the components of the metric will grow monotonically to diverge (i.e., to hit coordinate singularities). Therefore, to perform a long-term numerical simulation avoiding coordinate singularities, it is necessary to suppress the growth of gauge modes. This is in particular important in simulating a system with angular momentum. When the system has angular momentum, the coordinate system twists due to the so-called Lense-Thirring effect. If one fails to choose an appropriate gauge condition to suppress this twisting, the computation crashes in a dynamical time-scale.

With regard to the slicing condition, the maximal slicing condition, in which the trace of the extrinsic curvature is set to be zero, has been recognized to be an appropriate choice for the singularity avoidance as well as for stable numerical simulation. However, to impose this condition, one needs to solve an elliptic-type equation, which is time-consuming. Moreover, it requires to impose boundary conditions. This introduces a difficulty in adopting the black hole excision technique (see below).

As an alternative, recently, dynamical lapse conditions, which are similar to the harmonic slicing [41] but slightly different, have been getting popular because of its simplicity for implementation [26,28,45]. Although the reason is not very clear at present, some types of the dynamical lapse conditions work well for the evolution of a single black hole and a rotating neutron star.

An appropriate spatial gauge condition in three-dimensional (3D) numerical relativity had not been investigated until quite recently. In 1978, Smarr and York [46] proposed the minimal distortion gauge condition by which growth rate of the coordinate distortion in the three-metric is likely to be minimized. The community of numerical relativity has recognized that this would be an appropriate gauge condition, but no attempt was made to improve it because one needs to solve a vector-elliptic-type equation, and hence, the computational cost is expensive. Recently, several attempts for imposing this condition have been made [47]. In particular, refs [31,39,40,47] demonstrate that a kind of minimal distortion gauge is very useful to perform a long-term simulation of binary systems.

A very serious drawback of the minimal distortion gauge is that the computational costs are quite expensive because solving the vector-elliptic-type equation is very time-consuming. To overcome this difficulty, recently several types of dynamical shift conditions have been proposed [28,37,45]. In these conditions, hyperbolic-type equations instead of elliptic-type one for the shift vector are solved. As a result, computational time is significantly saved. Furthermore, it is possible to reduce (transform) the equation to the form that provides a solution of the minimal-distortion-type gauge condition for stationary space-times. This implies that for quasi-stationary space-times, this gauge condition works well. Fortunately, even for highly dynamical space-times such as those for the stellar collapse to a black hole [37] and for the merger of binary neutron stars to a black hole [24], a dynamical shift condition works very well. Therefore, this gauge appears to be promising for many problems in numerical relativity.

2.4 *Solving general relativistic hydrodynamic equations*

To study the merger of neutron stars and the stellar collapse, one needs to prepare a general relativistic hydrodynamic code. For problems in which shock heating plays an important role such as stellar core collapse, a shock-capturing scheme should be implemented to perform accurate simulations. Also, it is necessary to take into account certain microphysical processes such as neutrino cooling to perform a physically realistic simulation.

In the past decade, there have been much progress in general relativistic hydrodynamic codes. The most popular and reliable approach now is to adopt high-resolution shock-capturing schemes by which shock waves can be captured very accurately and furthermore long term and accurate evolution of rapidly rotating and compact neutron stars is feasible (e.g., [34,35]). The details of this type of schemes are described in [48].

2.5 *Black hole finder*

In many simulations of numerical relativity, black holes are formed or exist from the beginning of simulation (e.g., in merger of binary black holes). To confirm the existence of black holes and to find their location, apparent horizon finder and/or event horizon finder have to be implemented.

Finding apparent horizons in 3D spaces with no symmetry used to be a difficult problem 10 years ago. However, in the past decade, many efficient methods have been developed. Thus, now it is easy to determine apparent horizons for any 3D spaces. Methods for finding apparent horizons are found in [49–52].

To determine event horizons, one needs to know the global structure of the space-time. This implies that it is necessary to deposit space-time data sets in computer disks. The data sets to be deposited for simulations with no spatial symmetry are, in particular, huge, and hence, finding event horizons has been one of the computational challenges. Thus, although methods for finding event horizons have been developed [53], they have been able to be applied only to axisymmetric space-times until quite recently. However, computational resources have been developed rapidly and now permit to deposit a huge amount of data sets. Recently, two works for finding event horizons for space-times of multiblack holes have been presented [54]. Although they determine the event horizons only for very simplified space-time of multiblack holes, their works certainly demonstrate that finding event horizons for arbitrary space-times is getting feasible.

2.6 *Black hole excision*

To evolve a system with black holes, one has to use a slicing condition of a singularity-avoidance property. Otherwise, the slice would hit the singularity in a dynamical time-scale and the computation would crash. However, in using such slicing conditions, the geometry around the black hole horizons is stretched. As a result, some components of metric become very steep and eventually form a cusp around the horizon. In other words, a coordinate singularity appears. Thus, in the presence of black holes, a slice will either hit a physical singularity or have a coordinate singularity. This is the reason why handling black holes is difficult in numerical relativity.

An important property of black holes is that no information propagates outward from the hole. This implies that in solving Einstein's equation for the outside of the black hole, one does not have to retain any information for the inside of the hole. From this idea, Unruh, and Seidel and Suen [55] have suggested that Einstein's equations should be solved only for the outside of the black holes excising their inside and imposing a boundary condition around the horizons.

There are many groups in the world that work in the evolution of black holes using this *excision* technique. So far, a number of groups has succeeded in evolving one black hole space-time using this idea (e.g., [26,28,43]). However, no group has succeeded in a long-term simulation for binary black holes (but see [30] for the latest progress). One of the reasons is the technical difficulty for handling the moving black holes in numerical computation. Also, it seems that an appropriate gauge condition for binary black holes has not been well-developed. Developing a numerical implementation for binary black holes with the excision technique is now considered to be one of the most important issues in numerical relativity.

2.7 *Extraction of gravitational wave-forms and computational resource*

One of the main purposes in numerical relativity is to compute gravitational wave-forms. To extract accurate wave-forms, which should be extracted in the wave zone, certain formulation and technique to distinguish the true dynamical degrees of freedom from the gauge modes are necessary. Gauge invariant perturbative methods have been developed in the last 15 years, and many works have shown that these are robust techniques [56]. Thus, one may conclude that the theoretical issue has been resolved. What remains is the technical issue: To compute gravitational waves in the wave zone, it is necessary to prepare a wide computational domain which is extended at least to the local wave zone. The location in the case of binary mergers is estimated as follows: Mergers of binaries set in when the orbital separation r between two stars becomes too small to keep quasi-circular orbits. Let me take the transition radius as $r \sim 10M$ where M is the total mass of the system. Assuming that the binary is in a circular orbit with a Kepler angular velocity Ω, one can approximately compute the wavelength of gravitational waves as

$$\lambda \approx \frac{\pi}{\Omega} \approx \pi\sqrt{\frac{r^3}{M}} \approx 100M\left(\frac{r}{10M}\right)^{3/2}. \tag{1}$$

Thus, the location of outer boundaries of the computation domain along each coordinate axis L has to be larger than $\sim$100M. On the other hand, to resolve individual stars or black holes accurately, the grid spacing should be smaller than one tenth of the Schwarzschild radius of the system $\sim$0.2M. This implies that if one uses a uniform grid, at least $N \sim 500 \times 2 = 1000$ grid points are required to prepare for one dimension (the factor 2 implies that one needs to cover both plus and minus directions). For $N \sim 10^3$, the memory required is $\sim$1 TBytes. Therefore, one needs a large-scale supercomputer.

Fortunately, computational resources have also been significantly improved in the last few years. Currently, it is possible to use a supercomputer of several 100 GBytes for memory and several 100 GFlops for computational speed. For example, Japanese astrophysicists currently use FACOM VPP5000 in which the maximum available memory is about 700 GBytes. With this resource, it is possible to perform a large-scale numerical simulation with a grid size as $(800, 800, 400)$ for (x, y, z) in $\sim$200 CPU hours. Even using a uniform grid covering the neutron star diameter by about 40 grid points, the outer boundaries are located at $\lesssim 100M$ for simulation of binary neutron star merger. This implies that one can locate outer boundaries in the local wave zone to compute scientific gravitational wave-forms within or of the order of error of $O(1\%)$.

However, from gravitational-astronomical point of view, the error should be much smaller than 1%. To compute such gravitational waves, we need much more powerful computers of memory larger than several Tbytes and of computational speed faster than several TFlops which enable to resolve compact objects with a better accuracy and to extend the outer boundaries in a wave zone.

3. Merger of binary neutron stars: Our work

Over the last five years, our group has been most interested in the simulations for the merger of binary neutron stars. Now, it is feasible to perform long-term simulations stably and accurately. We can determine the final outcome of the merger accurately. Furthermore, gravitational wave-forms can be computed with a good accuracy and radiation reaction to the evolution of the system is well taken into account. The latest results are presented in [24].

Here, I show some numerical results. In figures 1 and 2, the snapshot of the density contour curves and velocity vectors on the equatorial plane for the merger of unequal-mass neutron stars of mass-ratio $\sim$0.9 are shown for illustration. The initial conditions are in quasi-equilibrium states with $\Gamma = 2$ polytropic equation of state. The polytropic constant chosen for the spherical maximum mass of neutron stars is $\approx 1.68M_{\odot}$. The simulation is performed with a Γ-law equation of state of the form $P = (\Gamma - 1)\rho\varepsilon$ with $\Gamma = 2$ where P, ρ, and ε are pressure, baryon density, and specific internal energy.

In figure 1, the merger of two neutron stars of mass $\approx 1.35M_{\odot}$ and $\approx 1.45M_{\odot}$ is shown. In this case, the outcome is a very massive (hypermassive [57]) neutron star whose self-gravity is supported by a rapid and differential rotation. In figure 2, the merger of two neutron stars of mass $\approx 1.4M_{\odot}$ and $\approx 1.6M_{\odot}$ is shown. In this case, the outcome is a black hole. Because of a significant mass difference of the initial condition, a disk of mass $\sim$5% of the total mass is formed around the central black hole.

In figure 3, gravitational waves associated with the case shown in figure 1 are displayed. Here, I show gauge-invariant quantities of (l, m)=(2,2), (3,3), and (2,0). The (2,2) mode is the non-axisymmetric quadrupole mode which is most dominant and emitted by non-axisymmetric motion. In the late inspiral stage for $t \gtrsim P_{t=0}$ where $P_{t=0}$ is the orbital period of the initial condition, this mode is emitted by the orbital motion, and in the merger stage, it is excited due to the non-axisymmetric oscillations of the formed neutron star. The (3,3) mode is excited due to the mass asymmetry of two neutron stars. The (2,0) mode is the axisymmetric quadrupole

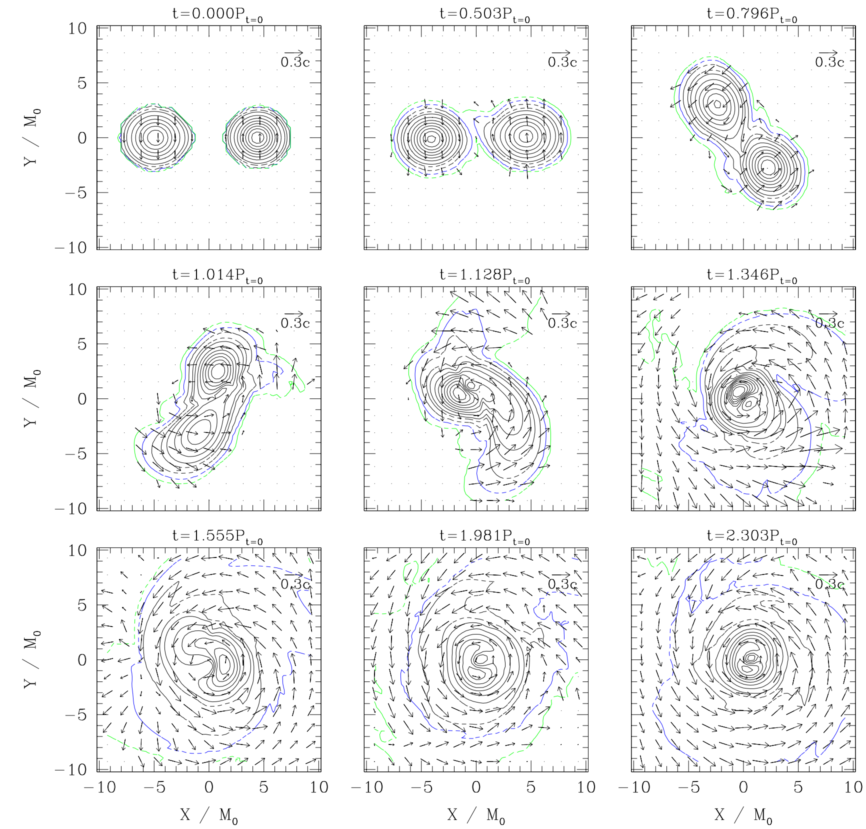

Figure 1. Snapshots of the density contour curves for ρ in the equatorial plane for the merger of two neutron stars of mass $1.35M_\odot$ and $1.45M_\odot$. The massive one is located for $x > 0$ at $t = 0$. The solid contour curves are drawn for $\rho/0.15 = 1 - 0.1j$ for $j = 0, 1, 2, \ldots, 9$, and the dashed–solid curves are for $\rho/0.15 = 0.05$, 0.01, 10^{-3} and 10^{-4}. Vectors indicate the local velocity field (v^x, v^y), and the scale is shown in the upper right-hand corner. $P_{t=0}$ denotes the orbital period of the quasi-equilibrium configuration given at $t = 0$ and $\approx$2.7 ms in this case. The length scale is shown in units of GM_0/c^2, where $M_0 \approx 2.8M_\odot$ is the ADM mass computed at $t = 0$.

mode which is excited mainly by the quasi-radial oscillation of the formed hypermassive neutron star.

The frequency of the (2,2) mode during the merger stage is typically $\sim$2.2 kHz in this case. This value is a little too high to be detected by ground-based laser interferometers in operation now. On the other hand, that of (2,0) mode which is induced by a quasi-periodic radial oscillation of the formed neutron star is $\sim$0.7 kHz. This value is in the range for detection by ground-based laser interferometers. Thus, such an oscillation mode may be an interesting source of gravitational waves.

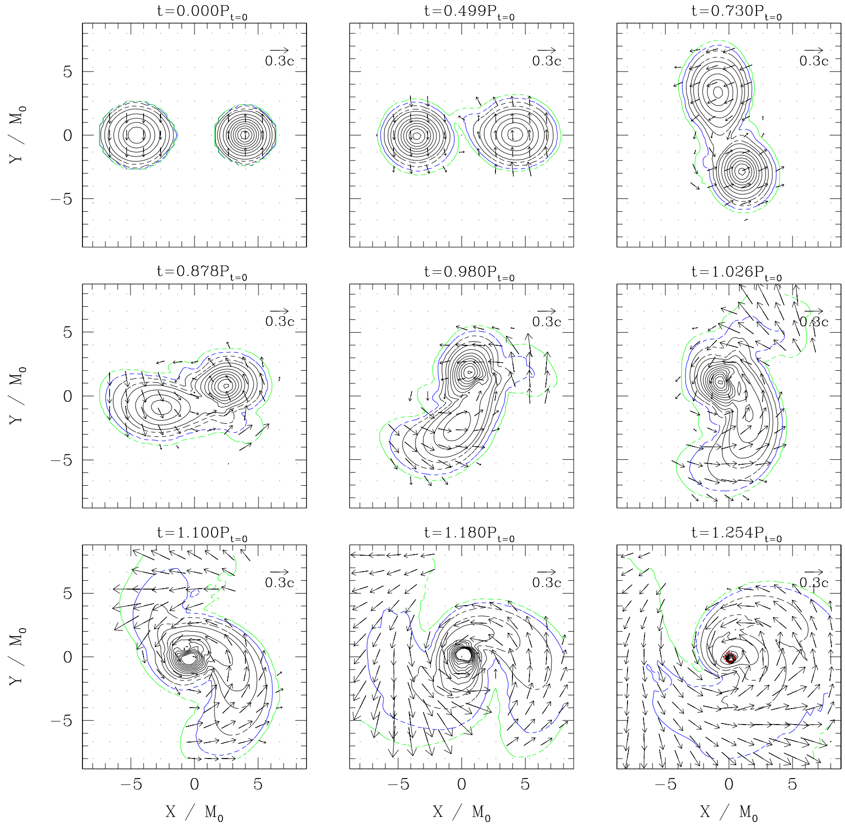

Figure 2. The same as figure 1 but for the merger of two neutron stars of mass $1.4M_\odot$ and $1.6M_\odot$. Here, the solid contour curves are drawn for $\rho/0.20 = 1 - 0.1j$ for $j = 0, 1, 2, \ldots, 9$, and the dashed–solid curves are for $\rho/0.20 = 0.05$, 0.01, 10^{-3} and 10^{-4}. $P_{t=0} \sim 2.6$ ms in this case. The thick dotted circle in the last panel of radius $r \sim 0.5M_0$ denotes the location of the apparent horizon. At $t = 0$, the primary neutron star is located at $x > 0$.

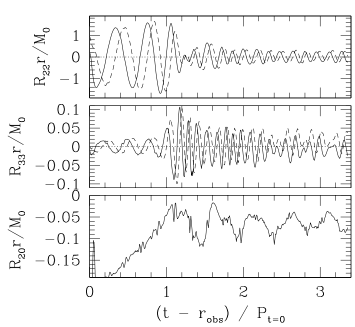

Figure 3. $R_{22\pm}r$, $R_{33\pm}r$, and $R_{20}r$ as a function of the retarded time for model M1315. The solid and dashed curves for R_{22} and R_{33} denote R_{lm+} and R_{lm-}.

4. Summary

As described in this review, long-term 3D simulations in full general relativity are now workable, as long as black holes are absent in the system. In the last five years, many hydrodynamic numerical simulations have been performed, including the merger of binary neutron stars [24,39,40], collapse of rotating supramassive neutron stars to black holes [32,37,38], non-axisymmetric deformation of neutron stars [32], and collapse of a supermassive star to a supermassive black hole [58].

There has been no scientific result for binary black hole merger so far. Many groups in the world have been developing numerical implementation and certain progress has been made in the last several years [28,30,43]. The most important issue to be resolved now seems to be to find a gauge condition and numerical techniques suited for handling orbiting black holes. When numerical relativity community resolves these issues, one will be able to perform simulations not only for binary black holes but also for a wide variety of interesting problems such as the merger between a black hole and the neutron star and runaway instability of self-gravitating accretion disks rotating around a black hole.

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