PRAMANA
— journal of
physics

Vol. 63, No. 2
August 2004
pp. 375–379

On measuring the neutron coherent scattering length with ultrahigh precision

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Abstract. We propose an order of magnitude improvement in the present five parts in 10^5 precision of a nondispersive interferometric measurement of the neutron coherent scattering length b_c. For this purpose we make a judicious selection of the Bragg angle for the interferometer and the sample thickness. The precision is further improved by an optimal choice of the Bragg reflection (and a consequent neutron wavelength). By performing the experiment in vacuum, errors arising from possible variations in the pressure, composition or humidity of the ambient air can be eliminated. On attaining such precision, we ought to account for the neutron beam refraction at the sample-ambient interfaces, to infer the correct b_c from the observed phase. The formula for the phase used hitherto is approximate and would significantly overestimate b_c. The refractive index for neutrons can thus be determined to a phenomenal precision of a few parts in 10^{12}.

Keywords. Neutron scattering length; neutron interferrometry; nondispersive phase.

PACS Nos 03.75.Dg; 07.60.Ly; 61.12.-q

Perfect crystal interferometry affords precise determination of the coherent scattering length [1,2] of samples. With a parallel-faced sample slab, of thickness D and atomic density N, placed normal to one subbeam in the interferometer, neutrons acquire the phase

$$\Phi = -\left(Nb_c - N_a b_a\right) D\lambda. \tag{1}$$

Here λ denotes the incident neutron wavelength and subscript 'a' stands for the corresponding properties of ambient. This variation of Φ with λ over the spread $\Delta\lambda$ in the incident wavelengths reduces the interference contrast. The consequent loss in phase precision limits the attainable b_c precision to about 1 part in 10^3.

Rauch *et al* [3] raised the precision to about 4.7 parts in 10^4 by following Scherm's suggestion to insert the sample with its surface parallel to the Bragg planes of the interferometer. Neutrons of each wavelength from the beamsplitter are then incident at the corresponding Bragg angle θ_B to the sample and the phase

$$\Phi \approx -\left(Nb_c - N_a b_a\right) D\lambda/\sin\theta_B = -2\left(Nb_c - N_a b_a\right) Dd\,, \tag{2}$$

is *nondispersive*. Here d symbolizes the Bragg planar spacing. However, here the phase varies sharply with the inclination θ of the sample (cf. Φ_{0-I} and Φ_{II-0}

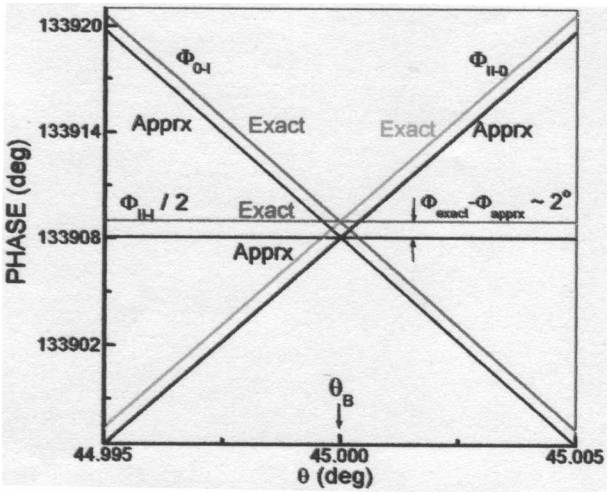

Figure 1. Exact and approximate nondispersive phases in air.

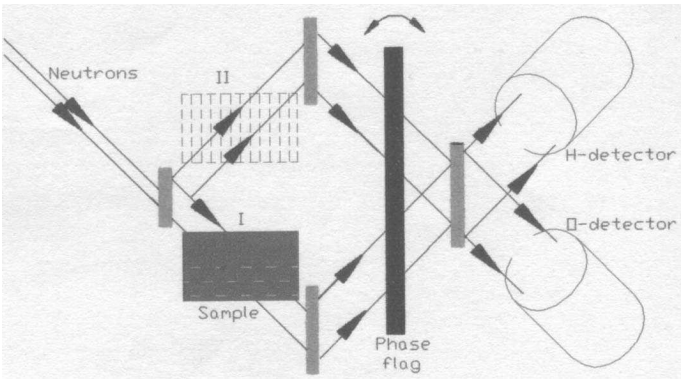

Figure 2. Our proposal.

curves in figure 1). The nondispersivity condition therefore requires the sample to be aligned with arcsecond precision.

Ioffe *et al* [4] overcame this limitation by measuring the phase shift between interferograms recorded with the sample placed alternately in subbeams I and II (figure 2). This eliminates the first order variation of the phase (cf. $\Phi_{\mathrm{II-I}}$ curve in figure 1) with the horizontal misalignment $\Delta\theta$ from θ_{B}. The sample alignment thus requires only arcminute precision to locate the minimum in $\Phi_{\mathrm{II-I}}$, occurring at the intersection of $\Phi_{0-\mathrm{I}}$ and $\Phi_{\mathrm{II}-0}$ curves.

The nondispersive phase shift

$$\Phi_{\mathrm{I-II}} \approx -\frac{(Nb_{\mathrm{c}} - N_{\mathrm{a}}b_{\mathrm{a}})\,Dd}{\cos\Delta\gamma} 2\left(2 + (\Delta\theta)^2\left\{1 + 2\cot^2\theta_{\mathrm{B}}\right\}\right), \tag{3}$$

then determines the coherent scattering length

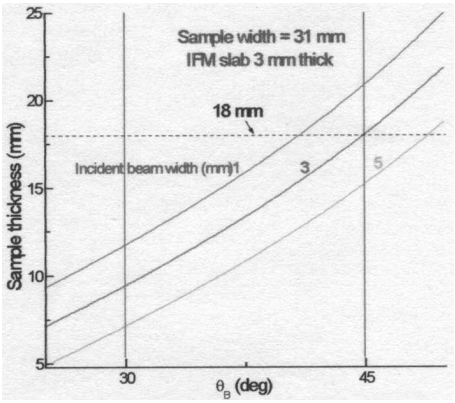

Figure 3. Variation of the allowed sample thickness with Bragg angle.

$$b_{\mathrm{c}} \approx -\frac{\Phi_{\mathrm{I-II}} \cos \Delta\gamma}{4NDd\,\left(1+\frac{\Delta\theta^2}{2}\left(1+2\cot^2\theta_{\mathrm{B}}\right)\right)} + \frac{N_{\mathrm{a}} b_{\mathrm{a}}}{N}, \tag{4}$$

where $\Delta\gamma$ denotes the vertical misalignment of the sample. The experiment [4] achieved a precision $\Delta b_{\mathrm{c}}/b_{\mathrm{c}}$ of 5.1×10^{-5}, whose source-wise constituents are listed on the LHS of table 1. By far the most predominant contribution arises from the relative variation $\Delta D/D$ in the sample thickness.

The precision can thus be improved by increasing D and reducing its variation ΔD. An increase in D dictates a large Bragg angle (figure 3). For practical reasons, we limit θ_{B} to 45° (figure 2) allowing $D = 18$ mm for 3 mm wide incident neutron beam. The width of the interferometer becomes rather large, about 9 cm, at this θ_{B}. Attaining $\Delta D = 0.1$ μm with a precision grinding and polishing machine would yield about an order of magnitude reduction in the $\Delta D/D$ contribution to $\Delta b_{\mathrm{c}}/b_{\mathrm{c}}$. In addition, the phase also increases by the same factor as D, reducing the $\Delta\Phi/\Phi$ contribution. Further, we can maximise d to 3.14 Å by choosing the $\{1\,1\,1\}$ Bragg reflection for the interferometer (hence $\lambda = 4.43$ Å) to further enhance Φ and reduce $\Delta\Phi/\Phi$. A thermal enclosure around and vibration isolation of the interferometer reduces the phase drift to a fraction of a degree over a day [2,4]. The effect of this phase drift over a typical measurement duration of a few hours, is minimised by recording the O and H detector intensities (figure 2) for the three positions (I, II and Out) of the sample in succession at each angular setting of the phase flag. A phase error of about 0.3°, thus routinely achieved in interferometric experiments, is included in table 1. The contribution from the uncertainty in the refractive index of air, dependent on variations in the temperature, pressure and relative humidity, can be larger than that assumed in [4], viz. $N_{\mathrm{a}} b_{\mathrm{a}}/N = 9.137(9) \times 10^{-3}$ fm. This can be eliminated by performing the experiment in vacuum. With a crystalline silicon sample ($Nd = 1.57 \times 10^{15}$ cm^{-2}), our proposed phase $\Phi_{\mathrm{I-II}} = -267816.2°$ will yield b_{c} with ultrahigh precision as shown on the RHS of table 1.

When such ultrahigh precision is achieved, it becomes necessary to account for neutron refraction at the ambient–sample interface. Conservation of the tangential component of the neutron wave vector across the interface yields the exact phase

Table 1. Comparison between various $\Delta b_c/b_c$ contributions at the present [4] and the proposed experiment.

Present	Source	Proposed
5×10^{-5}	Thickness: $\Delta D \to 0.1$ μm (precision grinding)	5.6×10^{-6}
9×10^{-6}	Phase: $\Delta\Phi = 0.3^\circ$, typical	1.1×10^{-6}
2.2×10^{-6}	Air: $\Delta\left(\frac{N_a b_a}{N}\right) = 9 \times 10^{-6}$ $\downarrow$ eliminate: vacuum expt	2.2×10^{-6}
1.1×10^{-7}	$\Delta\theta \approx 0.01^\circ$, typical	5×10^{-8}
1.4×10^{-7}	$\Delta\gamma \approx 0.01^\circ$, typical	2×10^{-8}
4×10^{-9}	$\Delta\{Nd\}_{Si} = 6 \times 10^6$ cm^{-2}	4×10^{-9}
5.1×10^{-5}	Total	6×10^{-6}
	Vacuum expt:	5.7×10^{-6}

$$\Phi_{I-II} = \frac{4\pi D}{\lambda}\left(\sqrt{n^2 - n_a^2\cos^2\theta_B} - n_a\sin\theta_B\right)$$
$$= 4\pi D\left(\sqrt{\frac{-(Nb_c - N_a b_a)}{\pi} + \frac{n_a^2}{4d^2}} - \frac{n_a}{2d}\right), \quad (5)$$

n denoting the refractive index. The exact and approximate (eq. (3)) phases for $\Delta\gamma = 0$ in our proposal are plotted in figure 1. The exact phase is greater by about 2° at $\theta = \theta_B$. The exact phase (eq. (5)) is rigorously nondispersive only in vacuum, i.e. when $n_a = 1$. However, since the refractive index of air differs from unity only by about 1.4×10^{-8}, the phase is nondispersive to an excellent approximation even in air, to better than three parts in 10^{10} for an incident wavelength spread $\Delta\lambda/\lambda$ of 1%. Equation (5) yields the coherent scattering length

$$b_c = -\frac{n_a\Phi_{I-II}}{4NDd} - \frac{\Phi_{I-II}^2}{16\pi ND^2} + \frac{N_a b_a}{N}. \quad (6)$$

Therefore the correction to the inferred b_c due to the refraction effects

$$\frac{\Delta b_c}{b_c} \approx -\frac{Nb_c d^2}{\pi} = -6.5 \times 10^{-6},$$

slightly exceeds the proposed precision in magnitude, underscoring the importance of refraction effects.

The refractive index, $n = (1 - Nb_c\lambda^2/\pi)^{1/2} \approx 1 - Nb_c\lambda^2/(2\pi)$, of silicon for thermal neutrons equals unity to within about one part in 10^6. Our proposal can thus determine the refractive power, $n - 1 \approx 10^{-6}$, with a relative precision of a few parts in 10^6, and hence the refractive index to a phenomenal precision of a few parts in 10^{12}.

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