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Current drive for rotamak plasmas*

ROBIN STORER and GUNHILD VON OERTZEN
School of Chemistry, Physics and Earth Sciences, Flinders University, G.P.O. Box 2100, Adelaide 5001, Australia
E-mail: robin.storer@flinders.edu.au

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Abstract. Experiments which have been undertaken over a number of years have shown that a rotating magnetic field can drive a significant non-linear Hall current in a plasma. Successful experiments of this concept have been made with a device called rotamak. In its original configuration this device was a field reversed configuration without a toroidal magnetic field but with a vertical field to establish the MHD equilibrium. However, modifications have shown that current can also be driven if a central current-carrying rod is used to provide an applied toroidal field. The new rotamak has then a spherical tokamak magnetic field structure.

Keywords. Plasmas; current drive; rotating field; rotamak.

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1. Introduction

The rotamak is a spherical device in which a plasma is created and driven by a rotating magnetic field (RMF) [1]. The current drive is essentially a non-linear Hall $\mathbf{J} \times \mathbf{B}$ current arising from the interaction between the rotating field, the plasma currents and the applied steady fields. The general principle has been applied to drive currents in a compact torus configuration in a spherical vessel [2], in a similar device with an applied toroidal field [3] and in an elongated field reversed configuration [4]. In all the cases the RMF needs to satisfy conditions in terms of two dimensionless parameters γ and λ [5,6]. Here γ is the ratio of the electron cyclotron frequency $\omega_{\rm ce}$ (associated with the rotating field: $\omega_{\rm ce} = eB_\omega/m_{\rm e}$) and the electron–ion collision frequency $\nu_{\rm ei} \approx ne^2\eta/m_{\rm e}$. (In this relation, η is the plasma resistivity and n is the electron number density.) Thus we take

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$$\gamma = \frac{\omega_{\rm ce}}{\nu_{\rm ei}} = \frac{B_\omega}{ne\eta}. \tag{1}$$

The dimensionless parameter λ is the ratio of the plasma size, a, to the classical skin depth, δ. Thus

$$\lambda = \frac{a}{\delta} = \sqrt{\frac{\omega\mu_0 a^2}{2\eta}}. \tag{2}$$

The condition $\gamma > \lambda$ ensures that the dominant electromagnetic effect is the non-linear Hall term, i.e., the second term in the Ohm's law:

$$\mathbf{E} = \eta\mathbf{J} + \frac{1}{ne}\mathbf{J}\times\mathbf{B}. \tag{3}$$

The rotamak at Flinders University was operated both in a reversed field configuration (rotamak-FRC) [1] with a steady vertical field (orthogonal to the rotating field) and in a compact torus configuration with an additional steady toroidal field (rotamak-ST) [7]. The latter situation gives a magnetic structure like a low aspect ratio spherical tokamak [8]. This may give rise to the possibility of using rotating fields in conjunction with inductive current drive in spherical tokamaks to sustain the current, although an initial attempt to do this has been unsuccessful [9].

The purpose of this study is to investigate the current driven in two configurations by using a theoretical model based on a spherical geometry. Previous computational studies [5] have shown that in general in the rotamak-FRC configuration the current increases with increasing γ, but that there is a complex relationship between the driven current and the vertical magnetic field. Likewise, in both the experiments [3,7] and theory [10] the current drive efficiency is affected in a complex way by the presence of a toroidal field.

2. Theory

We use a model based on a common feature of non-inductive current drive mechanisms, viz. that the current involves the electrons moving though an ion backgound. We need to solve Maxwell's equations with the Ohm's law above, including the Hall term, in the presence of the applied fields. The externally applied fields consist of three terms: the rotating field, the vertical field and the toroidal field. Expressed in terms of the spherical coordinate system (r, θ, ϕ) this is

$$\begin{aligned}\mathbf{B}_{\rm app} = {} & [\sin\theta\cos(\phi-\omega t)\hat{\mathbf{r}} + \cos\theta\cos(\phi-\omega t)\hat{\boldsymbol{\theta}} - \sin(\phi-\omega t)\hat{\boldsymbol{\phi}}] \\ & + B_z(\cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}) + B_{\rm t}\frac{a}{r\sin\theta}\hat{\boldsymbol{\phi}},\end{aligned} \tag{4}$$

where ω is the angular frequency of the applied field. It is convenient to normalise all magnetic fields in terms of the rotating magnetic field strength, so that the rotating field has unit strength in the above equation and $B_{\rm t}$ is the normalised toroidal field at a radius a from a central current carrying rod. Expressed in Cartesian

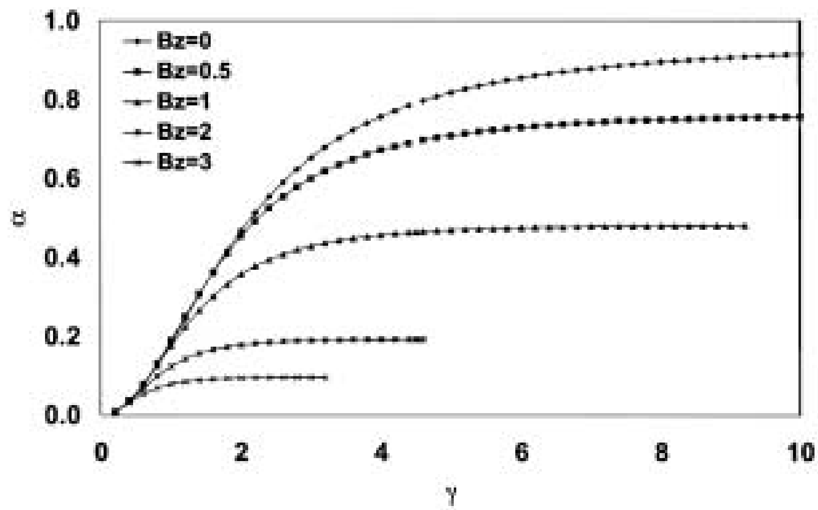

Figure 1. Driven current as a function of strength of rotating field for $\lambda = 1$ and various values of the steady vertical field (B_z). $B_{\mathrm{t}} = 0$ in this case.

coordinates the rotating and toroidal fields are parallel to the x–y plane and the vertical field, B_z, is parallel to the z-axis.

For the steady state situation the time derivative can be replaced by ω times a derivative with respect to ϕ; and if $\mathbf{B}$ is the time-dependent part of the magnetic field (with the steady fields subtracted) Maxwell's equations and Ohm's law give (in dimensionless variables)

$$-2\lambda^2 \frac{\partial \mathbf{B}}{\partial \phi} = \nabla \times \mathbf{J} + \gamma \nabla \times (\mathbf{J} \times \mathbf{B}) + \frac{\gamma B_{\mathrm{t}}}{r^2 \sin^2 \theta} \left(\frac{\partial \mathbf{J}}{\partial \phi} + 2 J_z \hat{\phi} \right), \tag{5}$$

where

$$\mathbf{J} = \nabla \times \mathbf{B} \quad \text{and} \quad J_z = (J_\theta \cos\theta + J_\phi \sin\theta). \tag{6}$$

These equations were solved numerically by expanding the vectors in terms of vector spherical harmonics [5,11,12] and matching the fields to the applied fields at infinity. This leads to a set of non-linear coupled equations which were solved by a finite difference technique with Piccard iteration [10,13]. A convergent solution was obtained for all except large values of γ.

3. Results

The results are most conveniently presented by calculating a normalised current, α, which is the ratio of the actual steady current driven, $I_{\phi 0}$, to the maximum possible current (if all the electrons move at the angular frequency ω). We define α by

$$\alpha = -2\pi \frac{I_{\phi 0}}{N_{\mathrm{e}} e \omega} \tag{7}$$

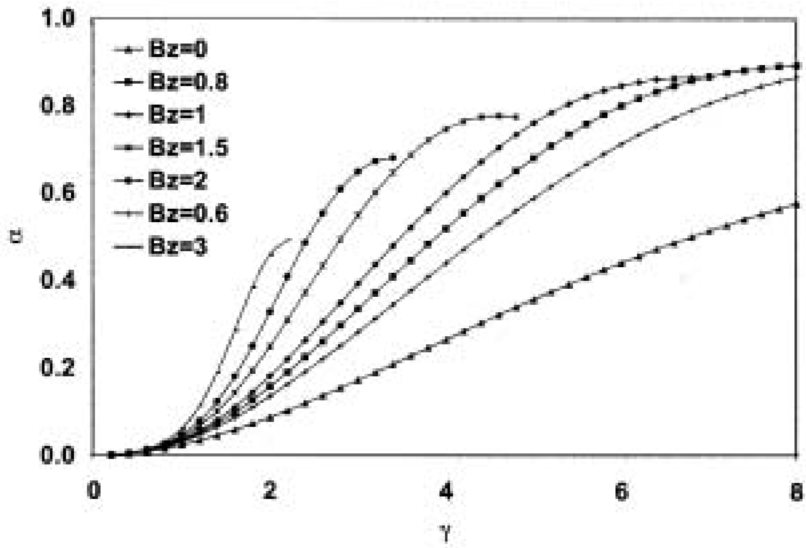

Figure 2. Driven current as a function of strength of rotating field for $\lambda = 4$ and various values of the steady vertical field (B_z). $B_{\mathrm{t}} = 0$ in this case.

where N_{e} is the total number of electrons in the plasma sphere. Figures 1 and 2 present the normalised current α as a function of the normalised rotating field strength γ for two different values of rotating field frequency (expressed in terms of the dimensionless parameter $\lambda \propto \sqrt{\omega}$). In both these cases the toroidal field is zero. It is of interest that in the first case ($\lambda = 1$) the driven current rises rapidly with increasing γ so that by $\gamma = 8$ it has reached 90% of the maximum (synchronous) current. Increasing the vertical magnetic field in this case leads to poorer driven current, in contrast to the situation for $\lambda = 4$ where for $B_z = 0$ the driven current rises only slowly with increasing γ, but where applying a vertical field increases the driven current over a wide range of γ. As shown in [5] (using a different numerical technique) some of the best results (e.g. at $B_z = 1.5$ and $\gamma \approx 4$) correspond to a compact-toroidal magnetic field structure characteristic of spherical tokamaks [2,8]. These results confirm the only previous computations [5] and so help to validate the new code which is designed to include toroidal fields.

As examples of the results of the new code which includes the effect of toroidal fields, figures 3 and 4 present calculations for cases where the vertical field is set to zero and the toroidal field is varied over a range of values. Again at $\lambda = 1$, increasing the toroidal field does not improve the current driven. However at $\lambda = 4$ the results are such that values of B_{t} up to about 3 lead to increased driven current over a range of values of γ. In figure 5 the driven current is shown as a function of the toroidal field at $\lambda = 4$ and various values of the rotating field.

The configuration of the fields and currents is fully three-dimensional and so the qualitative interpretation of these results is not simple. However, one can see from figure 5 that there is a value for the toroidal field which leads to the maximum current driven. Comparison of figures 3 (with $\lambda = 1$) and figure 4 (with $\lambda = 4$) indicates that the non-linear $\mathbf{J} \times \mathbf{B}$ term plays a significant role. That is, the larger

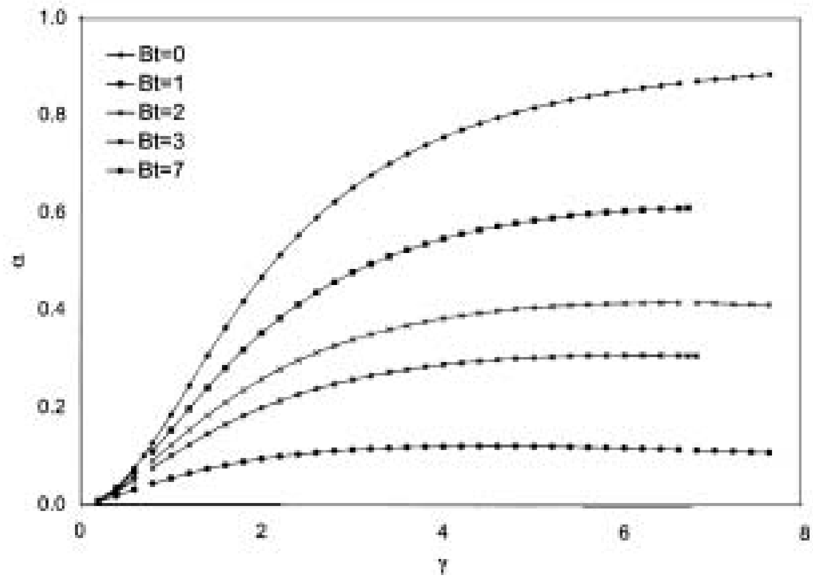

Figure 3. Driven current as a function of strength of rotating field for $\lambda = 1$ and various values of the steady toroidal field (B_t). $B_z = 0$ in this case.

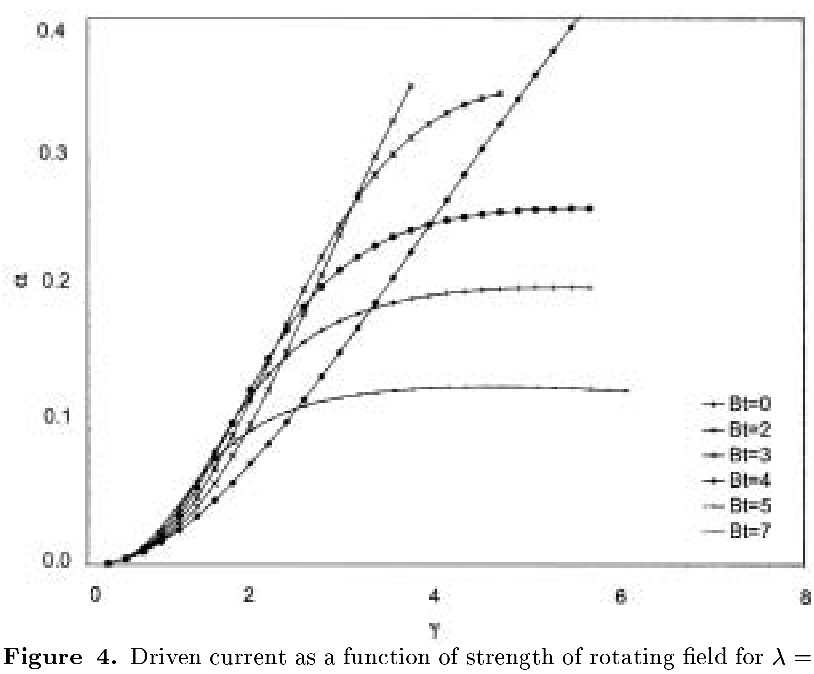

Figure 4. Driven current as a function of strength of rotating field for $\lambda = 4$ and various values of the steady toroidal field (B_t). $B_z = 0$ in this case.

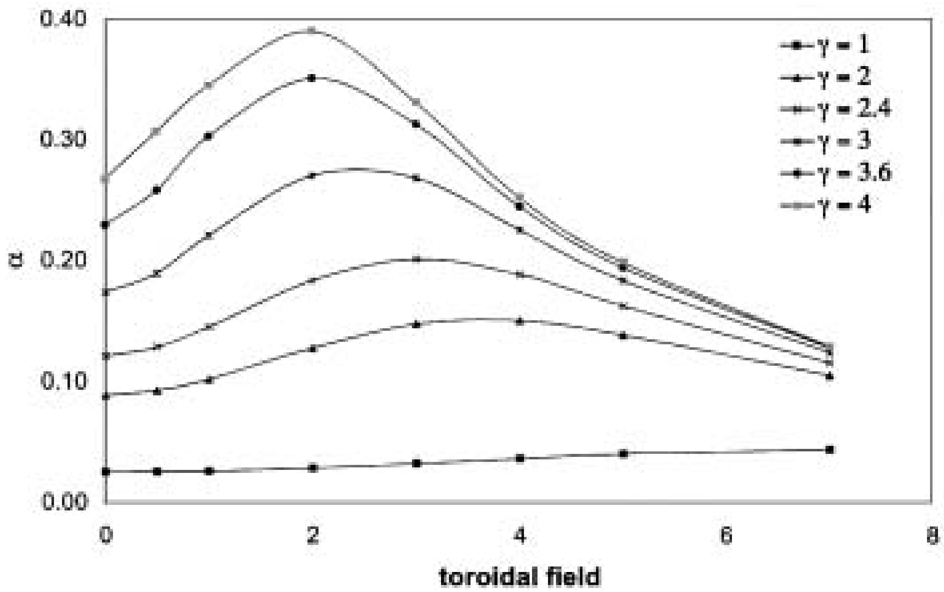

Figure 5. Driven current as a function of strength of toroidal field (B_t) for $\lambda = 4$ and various values of the rotating field. $B_z = 0$ in this case.

the non-linear term the more current is driven. The toroidal field strength B_t, on the other hand, can lead to an increase or a decrease in the driven current which probably depends on the change in angle between **J** and **B** and hence a change in the effect of the non-linear driving term $\mathbf{J} \times \mathbf{B}$.

4. Discussion and conclusion

The rotamak has proved to be an interesting device in which a unique current drive technique can be investigated. The current drive is a result of macroscopic electromagnetic phenomena, in contrast to the wave-particle interaction effects which form the basis of many other schemes. The calculations presented above indicate the sensitivity of the current drive to the choice of both the steady vertical and toroidal fields. It is clear that the current driven may depend on the actual experimental situation and that increasing either of these fields may not necessarily lead to better performance.

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