

Bounds on charged lepton mixing with exotic charged leptons

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Abstract. We examine the effects of mixing induced light heavy charged lepton neutral currents on the partial wave amplitude for the process $l^+l^- \rightarrow ZZ$ (with $l = e, \mu$ or τ). By imposing the constraints that the amplitude should not exceed the perturbative unitarity limit at high energy ($\sqrt{s} = \Lambda$), we obtain bounds on light heavy charged lepton mixing parameter $\sin^2(2\theta_L^a)$ where θ_L^a is the mixing angle of the ordinary charged lepton with its exotic partner. For $\Lambda = 1$ TeV, no bound is obtained on $\sin^2(2\theta_L^a)$ for $m_E < 0.69$ TeV. However, $\sin^2(2\theta_L^a) \leq 1.52 \times 10^{-5}$ for $m_E = 5$ TeV, $\sin^2(2\theta_L^a) \leq 2.41 \times 10^{-7}$ for $m_E = 10$ TeV. Similarity for $\Lambda = \infty$ no bound is obtained on $\sin^2(2\theta_L^a)$ for $m_E < 1.97$ TeV and $\sin^2(2\theta_L^a) \leq 0.15$ for $m_E = 5$ TeV and $\sin^2(2\theta_L^a) \leq 3.88 \times 10^{-2}$ for $m_E = 10$ TeV.

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1. Introduction

Many theories beyond the standard model (SM) predict the existence of new heavy leptons (NHL) with observable production cross-section and identifiable final states [1–4]. Searches of NHL are listed in the Review of Particle Physics [4], which reveal that the NHL masses are experimentally bounded to be greater than the standard Z -boson mass [4,5]. If NHL exist then even in the case in which these new particles are too heavy to be produced, their presence could still manifest through their mixing with the ordinary SM leptons.

A comprehensive analysis of the mixing between ordinary fermions with canonical $SU(2) \times U(1)$ assignments (i.e. left-handed (L) fermions as $SU(2)$ doublets, while right-handed (R) fermions as $SU(2)$ singlets) and possible heavy fermions with exotic (non-canonical) $SU(2) \times U(1)$ assignments (i.e. L-fermions as $SU(2)$ singlets, while R-fermions as $SU(2)$ doublets) has been performed by Langacker and London [1]. In the mixing formalism [1,6], the weak interaction eigenstates (ψ_L^0, ψ_R^0) and mass eigenstates (ψ_L, ψ_R) are related by the transformations,

$$\psi_L^0 = U_L \psi_L \quad \text{and} \quad \psi_R^0 = U_R \psi_R. \quad (1)$$

Here and below, the superscript 0 indicates the weak interaction basis and the mass basis is denoted by the absence of superscript 0. In the above $U_{L(R)}$ are unitary matrices, which diagonalize the lepton mass matrix.

In our previous publication we obtained the bounds on mixing parameter in neutral lepton sector [7]. In this paper we consider the process $l^+l^- \rightarrow ZZ$ (with $l = e, \mu$ or τ) with the inclusion of light heavy charged lepton mixing and obtain the bounds on mixings in charged lepton sector. The threshold of the process $\sqrt{s} = 2m_Z$ is high, and masses of light charged leptons are too small to be kinematically relevant. This allows us to neglect the masses of light (known) charged leptons. In the SM the process occurs at tree level via l -exchange in t - and u -channels. The inclusion of mixing between ordinary and heavy exotic leptons induces additional contribution to the process via heavy exotic lepton exchange in t - and u -channels. Any such additional contribution, in principle modifies the partial wave amplitude in comparison to the no mixing case. By using the constraint that the amplitude should not exceed the perturbative unitarity limit at high energies [7–10], we obtain theoretical bounds on the light heavy charged lepton mixing parameter.

The paper is organized as follows: In §2, following the mixing formalism of Langacker and London [1], we examine the light–heavy weak neutral currents and evaluate the amplitude for the process $l^+l^- \rightarrow ZZ$ with the inclusion of mixing. In §3, the unitarity bounds are obtained, and we summarize our results in §4.

2. Mixing formalism and amplitude for the process $l^+l^- \rightarrow ZZ$

2.1 Mixing formalism

We follow the mixing formalism and notations of Langacker and London [1]. All charged leptons of same charge and a given helicity (L, R) are grouped in a single column vector of ordinary and exotic weak interaction eigenstates

$$e_L^{0-} = \begin{pmatrix} e_{OL}^{0-} \\ e_{EL}^{0-} \end{pmatrix}, \quad e_R^{0-} = \begin{pmatrix} e_{OR}^{0-} \\ e_{ER}^{0-} \end{pmatrix}, \quad (2)$$

in which subscripts ‘O’ and ‘E’ stand for ordinary and exotic leptons respectively. Here we classify all charged leptons as either ordinary or exotic according to their $SU(2)$ transformation properties. In particular, we define all left-handed particles occurring in doublets to be *ordinary* (independent of whether the doublets are associated with known or sequential families or with vector doublets), and all left-handed $SU(2)$ singlets as *exotic* (associated with mirror families or vector singlets). Similarly, we define all right-handed singlets as *ordinary* (independent of whether the singlets are associated with known or sequential families or with vector singlets), and all right-handed particles occurring in doublets as *exotic* (associated with mirror families or vector doublets). In eq. (2) e_{OL}^{0-} is a column vector consisting of n_L ordinary fields, while e_{EL}^{0-} is a column vector of m_L exotic fields. In the same way, the e_{OR}^{0-} and e_{ER}^{0-} are column vectors of n_R ordinary fields and m_R exotic fields, i.e.,

$$e_{OL}^{0-} = (e_L^{0-} \mu_L^{0-} \tau_L^{0-} \cdots e_{n_L}^{0-})^T, \quad (3)$$

Charged lepton mixing with exotic charged leptons

$$e_{\text{OR}}^{0-} = (e_{\text{R}}^{0-} \mu_{\text{R}}^{0-} \tau_{\text{R}}^{0-} \cdots e_{n_{\text{R}}}^{0-})^T, \quad (4)$$

$$e_{\text{EL}}^{0-} = (E_{1\text{L}}^{0-} E_{2\text{L}}^{0-} E_{3\text{L}}^{0-} E_{m_{\text{L}}}^{0-})^T, \quad (5)$$

$$e_{\text{ER}}^{0-} = (E_{1\text{R}}^{0-} E_{2\text{R}}^{0-} E_{3\text{R}}^{0-} \cdots E_{m_{\text{R}}}^{0-})^T. \quad (6)$$

In general, $n_{\text{L}}(n_{\text{R}})$ and $m_{\text{L}}(m_{\text{R}})$ may be different, but for massive Dirac charged leptons, one has $n_{\text{L}} + n_{\text{R}} = m_{\text{L}} + m_{\text{R}}$. There will also be $n_{\text{L}} + n_{\text{R}} = m_{\text{L}} + m_{\text{R}}$ [11] mass eigenstates, which can be decomposed as

$$e_{\text{L}}^- = \begin{pmatrix} e_{\text{iL}}^- \\ e_{\text{hL}}^- \end{pmatrix}, \quad e_{\text{R}}^- = \begin{pmatrix} e_{\text{iR}}^- \\ e_{\text{hR}}^- \end{pmatrix}, \quad (7)$$

where $e_{\text{iL}}(e_{\text{iR}})$ is a column vector consisting of $n_{\text{L}}(n_{\text{R}})$ light mass eigenstates and $e_{\text{hL}}(e_{\text{hR}})$ is a column vector consisting of $m_{\text{L}}(m_{\text{R}})$ heavy mass eigenstates, i.e.,

$$e_{\text{iL}}^- = (e_{\text{iL}}^- e_{\text{2L}}^- e_{\text{3L}}^- \cdots e_{n_{\text{L}}}^-)^T, \quad (8)$$

$$e_{\text{hL}}^- = (E_{\text{1L}}^- E_{\text{2L}}^- E_{\text{3L}}^- \cdots E_{m_{\text{L}}}^-)^T. \quad (9)$$

The weak interaction eigenstates and the mass eigenstates are related by the unitary transformations

$$e_{\text{L(R)}}^{0-} = U_{\text{L(R)}} e_{\text{L(R)}}^-, \quad (10)$$

where $U_{\text{L(R)}}$ is a $(n_{\text{L(R)}} + m_{\text{L(R)}}) \times (n_{\text{L(R)}} + m_{\text{L(R)}})$ -dimensional unitary matrix, i.e.,

$$U_{\text{L(R)}} U_{\text{L(R)}}^\dagger = I, \quad (11)$$

$$U_{\text{L(R)}}^\dagger U_{\text{L(R)}} = I, \quad (12)$$

which diagonalize the charged lepton mass matrix. The matrix $U_{\text{L(R)}}$ can be written in the following block form [11]

$$U_{\text{L(R)}} = \begin{pmatrix} A_{\text{L(R)}} & E_{\text{L(R)}} \\ F_{\text{L(R)}} & G_{\text{L(R)}} \end{pmatrix}, \quad (13)$$

where $A_{\text{L(R)}}, E_{\text{L(R)}}, F_{\text{L(R)}}$ and $G_{\text{L(R)}}$ should satisfy certain relations obtained by requiring unitarity of the $U_{\text{L(R)}}$ (eqs (11) and (12)), namely

$$\begin{pmatrix} A_{\text{L(R)}}^\dagger A_{\text{L(R)}} + F_{\text{L(R)}}^\dagger F_{\text{L(R)}} & A_{\text{L(R)}}^\dagger E_{\text{L(R)}} + F_{\text{L(R)}}^\dagger G_{\text{L(R)}} \\ E_{\text{L(R)}}^\dagger A_{\text{L(R)}} + G_{\text{L(R)}}^\dagger F_{\text{L(R)}} & E_{\text{L(R)}}^\dagger E_{\text{L(R)}} + G_{\text{L(R)}}^\dagger G_{\text{L(R)}} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \quad (14)$$

and

$$\begin{pmatrix} A_{L(R)} A_{L(R)}^\dagger + E_{L(R)} E_{L(R)}^\dagger & A_{L(R)} F_{L(R)}^\dagger + E_{L(R)} G_{L(R)}^\dagger \\ F_{L(R)} A_{L(R)}^\dagger + G_{L(R)} E_{L(R)}^\dagger & F_{L(R)} F_{L(R)}^\dagger + G_{L(R)} G_{L(R)}^\dagger \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}. \quad (15)$$

Evidently the submatrices $A_{L(R)}$, $E_{L(R)}$, $F_{L(R)}$ and $G_{L(R)}$ are not unitary. In the above equation $A_{L(R)}$ is a $(n_{L(R)} \times n_{L(R)})$ -dimensional matrix relating ordinary weak states ($e_{OL(R)}^0$) and the light mass eigenstates ($e_{lL(R)}^-$), while $G_{L(R)}$ is a $(m_{L(R)} \times m_{L(R)})$ -dimensional matrix relating exotic weak states ($e_{eL(R)}^0$) and the heavy mass eigenstates ($e_{hL(R)}^-$). Similarly $E_{L(R)}$ and $F_{L(R)}$ are $(n_{L(R)} \times m_{L(R)})$ - and $(m_{L(R)} \times n_{L(R)})$ -dimensional matrices respectively, and describe the mixing between the two sectors (i.e., ordinary weak states ($e_{OL(R)}^0$) to the heavy mass eigenstates ($e_{hL(R)}^-$) and exotic weak states ($e_{eL(R)}^0$) to the light mass eigenstates ($e_{lL(R)}^-$) respectively). The matrices $E_{L(R)}$ and $F_{L(R)}$ are referred as light–heavy mixing matrices [1]. Here it may be noted that the matrix $A_{L(R)}$ describing the mixing between ordinary weak states and the light mass states is non-unitary by small terms quadratic in the light–heavy mixings present in the $E_{L(R)}$ and $F_{L(R)}$ matrices. Most of the phenomenological consequences of the mixing with exotic leptons are mainly associated with non-unitarity of $A_{L(R)}$.

The neutral current for charged leptons is

$$\begin{aligned} \frac{1}{2} J_Z^\mu &= (\bar{e}_{OL}^0 \quad \bar{e}_{eL}^0) \gamma^\mu (P_Z^L t_3 - IQ \sin^2 \theta_W) \begin{pmatrix} e_{OL}^0 \\ e_{eL}^0 \end{pmatrix} \\ &+ (\bar{e}_{OR}^0 \quad \bar{e}_{eR}^0) \gamma^\mu (P_Z^R t_3 - IQ \sin^2 \theta_W) \begin{pmatrix} e_{OR}^0 \\ e_{eR}^0 \end{pmatrix}, \end{aligned} \quad (16)$$

where

$$P_Z^L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \quad P_Z^R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

are projection operators onto the ordinary L-fields and exotic R-fields respectively. J_Z^μ involves $P_Z^{L(R)}$ rather than identity because we are considering mixing between doublets and singlets, therefore there is no Glashow–Iliopoulos–Maiani (GIM) mechanism, and will be FCNCs in general. t_3 and Q denote third isospin component and electrical charge of the corresponding fields. Using eqs (10) and (13) in eq. (16) we obtain neutral current in mass eigenstate basis as (here, repeated indices are summed over, with $\alpha = 1$ to n_L ; $\delta = 1$ to m_R ; $a = 1$ to n_L ; $b = 1$ to n_L ; $c = 1$ to m_L ; $d = 1$ to m_L ; $p = 1$ to n_R ; $q = 1$ to n_R ; $r = 1$ to m_R ; $s = 1$ to m_R),

$$\frac{1}{2} J_Z^\mu = \begin{pmatrix} \bar{e}_{aL} \gamma^\mu t_3 (A_L^\dagger)_{a\alpha} (A_L)_{\alpha b} e_{bL} + \bar{e}_{pR} \gamma^\mu t_3 (F_R^\dagger)_{p\delta} (F_R)_{\delta q} e_{qR} \\ + \bar{e}_{aL} \gamma^\mu t_3 (A_L^\dagger)_{a\alpha} (E_L)_{\alpha c} E_{cL} + \bar{e}_{pR} \gamma^\mu t_3 (F_R^\dagger)_{p\delta} (G_R)_{\delta r} E_{rR} \\ + \bar{E}_{cL} \gamma^\mu t_3 (E_L^\dagger)_{c\alpha} (A_L)_{\alpha a} e_{aL} + \bar{E}_{rR} \gamma^\mu t_3 (G_R^\dagger)_{r\delta} (F_R)_{\delta p} e_{pR} \\ + \bar{E}_{cL} \gamma^\mu t_3 (E_L^\dagger)_{c\alpha} (E_L)_{\alpha d} E_{dL} + \bar{E}_{rR} \gamma^\mu t_3 (G_R^\dagger)_{r\delta} (G_R)_{\delta s} e_{sR} \\ - \sin^2 \theta_W J_{em}^\mu \end{pmatrix}$$

Charged lepton mixing with exotic charged leptons

$$= \begin{pmatrix} \bar{e}_{\text{IL}}\gamma^\mu t_3(A_L^\dagger)(A_L)e_{\text{IL}} + \bar{e}_{\text{IR}}\gamma^\mu t_3(F_R^\dagger)(F_R)e_{\text{IR}} \\ + \bar{e}_{\text{IL}}\gamma^\mu t_3(A_L^\dagger)(E_L)e_{\text{hL}} + \bar{e}_{\text{IR}}\gamma^\mu t_3(F_R^\dagger)(G_R)e_{\text{hR}} \\ + \bar{e}_{\text{hL}}\gamma^\mu t_3(E_L^\dagger)(A_L)e_{\text{IL}} + \bar{e}_{\text{hR}}\gamma^\mu t_3(G_R^\dagger)(F_R)e_{\text{IR}} \\ + \bar{e}_{\text{hL}}\gamma^\mu t_3(E_L^\dagger)(E_L)e_{\text{hL}} + \bar{e}_{\text{hR}}\gamma^\mu t_3(G_R^\dagger)(G_R)e_{\text{hR}} \\ - \sin^2\theta_W J_{\text{em}}^\mu \end{pmatrix}. \quad (17)$$

In the above J_{em}^μ is the electromagnetic current, which is flavor diagonal. The first term in eq. (17) is the normal t_3 part of the neutral current involving left-handed light state, which is modified by the factor $A_L^\dagger A_L \neq 1$. Since A_L is non-unitary ($A_L^\dagger A_L$ is not necessarily diagonal) such a term allows the flavor-changing transitions between light charged leptons $\mu \rightarrow e$, $\tau \rightarrow \mu$, $\tau \rightarrow e$ etc. with strength

$$\lambda_{ab}^L = (A_L^\dagger A_L)_{ab} = -(F_L^\dagger F_L)_{ab}, \quad a \neq b \quad (18)$$

which is of second order in light-heavy mixing. Similarly, the second term in eq. (17) is an induced right-handed current, with off-diagonal (and diagonal) transitions of strength

$$\lambda_{ab}^R = (F_R^\dagger F_R)_{ab}. \quad (19)$$

Both λ_{ab}^L and λ_{ab}^R are second order in light-heavy mixing. The current experimental bounds on LFV transitions of charged leptons [4,12] are quite stringent and strongly constrain $\lambda_{ab}^{L,R}$ and force us to discard these terms, i.e.,

$$\lambda_{ab}^{L,R} = 0 \quad \text{for } a \neq b. \quad (20)$$

The above assumptions regarding the absence of FCNCs yield a simplification for the mixing matrix. As of now $A_L^\dagger A_L$ and $F_R^\dagger F_R$ are diagonal and $0 \leq (A_L^\dagger A_L)_{aa} \leq 1$, and $0 \leq (F_R^\dagger F_R)_{aa} \leq 1$ (see eqs (14) and (15)). Hence we can write [11]

$$A_L^\dagger A_L = \begin{pmatrix} (c_L^1)^2 & & 0 \\ & (c_L^2)^2 & \\ 0 & & (c_L^{n_L})^2 \end{pmatrix} \quad (21)$$

and

$$F_R^\dagger F_R = \begin{pmatrix} (s_R^1)^2 & & 0 \\ & (s_R^2)^2 & \\ 0 & & (s_R^{n_L})^2 \end{pmatrix}, \quad (22)$$

where $c_L^a = \cos\theta_L^a$ and $s_R^a = \sin\theta_R^a$, and $\theta_{L(R)}^a$ is the mixing angles of the a th L(R)-handed ordinary charged lepton with its exotic partner [1].

Further, it is convenient to define diagonal matrices [11]

$$c_{L(R)} = \text{diag} \left(c_{L(R)}^1 c_{L(R)}^2 \cdots c_{L(R)}^{n_{L(R)}} \right), \quad (23)$$

Jai Kumar Singhal

$$s_{L(R)} = \text{diag} \left(s_{L(R)}^1 s_{L(R)}^2 \cdots s_{L(R)}^{n_{L(R)}} \right), \quad (24)$$

so that (using eq. (14))

$$A_{L(R)}^\dagger A_{L(R)} = (c_{L(R)})^2 \quad (25)$$

and

$$F_{L(R)}^\dagger F_{L(R)} = (s_{L(R)})^2. \quad (26)$$

Equations (21) and (22) imply that there are no FCNCs if and only if the light-heavy mixing is restricted to distinct pairs of states, i.e., each light state mixed with its own exotic states with mixing angle $\theta_{L(R)}^\alpha$.

In eq. (17), the terms in the second and third line represent the transitions between light and heavy mass eigenstates (i.e. new mixing-induced couplings between the light and heavy mass eigenstates). These transitions (couplings) are phenomenologically important as they determine the production and decays of heavy particles.

The neutral current (eq. (17)) for charged lepton can also be written as

$$\frac{1}{2} J_Z^\mu = \frac{1}{2} \left(\begin{array}{l} \bar{e}_l \gamma^\mu \{ (t_3 A_L^\dagger A_L + t_3 F_R^\dagger F_R + 2 \sin^2 \theta_W) - (t_3 A_L^\dagger A_L - t_3 F_R^\dagger F_R) \gamma^5 \} e_l \\ + \bar{e}_l \gamma^\mu \{ (t_3 A_L^\dagger E_L + t_3 F_R^\dagger G_R) - (t_3 A_L^\dagger E_L - t_3 F_R^\dagger G_R) \gamma^5 \} e_h \\ + \bar{e}_h \gamma^\mu \{ (t_3 E_L^\dagger A_L + t_3 G_R^\dagger F_R) - (t_3 E_L^\dagger A_L - t_3 G_R^\dagger F_R) \gamma^5 \} e_l \\ + \bar{e}_h \gamma^\mu \{ (t_3 E_L^\dagger E_L + t_3 G_R^\dagger G_R + 2 \sin^2 \theta_W) - (t_3 E_L^\dagger E_L - t_3 G_R^\dagger G_R) \gamma^5 \} e_h \end{array} \right), \quad (27)$$

where $e_l = e_{lL} + e_{lR}$ and $e_h = e_{hL} + e_{hR}$. The first term in eq. (27) is the neutral current for the light charged lepton, in which the $A_L^\dagger A_L$ part represents a non-universal reduction of the strength of the normal neutral current, due to mixing with left-handed singlets, while the $F_R^\dagger F_R$ part is an induced right-handed current.

2.2 Amplitude for the process $l^+ l^- \rightarrow ZZ$

The process

$$l^-(k_1, \sigma) + l^+(\bar{k}_1, \bar{\sigma}) \rightarrow Z(k_2, \lambda) + Z(\bar{k}_2, \bar{\lambda}) \quad (28)$$

(where the arguments indicate the four momenta and helicities of the respective particles), in the SM occurs through the l -exchange in t - and u -channel Feynman diagrams (figure 1(a)). The inclusion of mixing between ordinary and heavy exotic leptons would result in two effects: (i) modification of the standard model Zll couplings and (ii) introduction of new vertices involving ordinary and heavy exotic leptons, t - and u -channel Feynman diagrams. The mixing-induced ZEl coupling would allow additional t - and u -channel Feynman diagrams (figure 1(b)), involving

Charged lepton mixing with exotic charged leptons

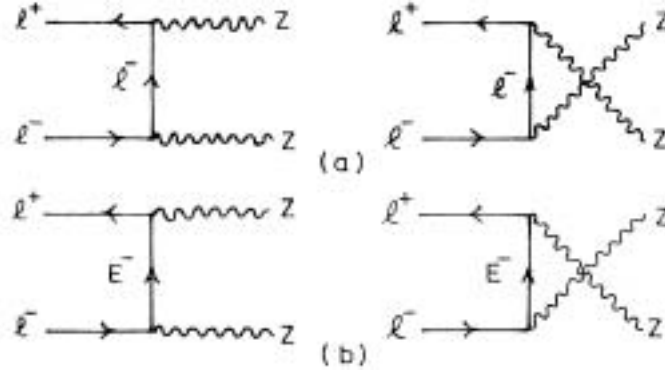


Figure 1. (a) Tree level ordinary charged lepton (l) exchange contribution to the process $l^+l^- \rightarrow ZZ$ and (b) mixing induced additional heavy charged lepton (E) contribution to the process $l^+l^- \rightarrow ZZ$.

exotic charged leptons (E). The relevant Feynman rules with the inclusion of mixing between ordinary and exotic charged leptons are obtained from the neutral current given in eq. (27). In our analysis we are confining to the possibility of extra charged leptons to e_{EL}^{0-} (i.e., left-handed $SU(2)$ singlets only).

As the threshold ($\sqrt{s} = 2m_Z$) of the process is sufficiently high, the masses of light leptons are neglected. Therefore $\sigma = -1$ and $\bar{\sigma} = +1$ giving total angular momentum $J = 1$ for the initial state in the center of mass frame. We evaluate the amplitude for the process in the helicity basis following the technique discussed in detail by Renard [13] and Hagiwara and Zeppenfeld [14]. In the c.m. frame, the momenta of the particles in eq. (28) are

$$k_1 = \frac{\sqrt{s}}{2} [1, 0, 0, 1], \quad \bar{k}_1 = \frac{\sqrt{s}}{2} [1, 0, 0, -1],$$

$$k_2 = \frac{\sqrt{s}}{2} [1, \beta \sin \theta, 0, \beta \cos \theta], \quad \bar{k}_2 = \frac{\sqrt{s}}{2} [1, -\beta \sin \theta, 0, -\beta \cos \theta],$$

where \sqrt{s} is the total c.m. energy, $\beta = \sqrt{1 - 4m_Z^2/s}$ and θ is the scattering angle of $Z(q, \lambda)$ with the incident lepton direction. The polarization vectors of the final states gauge bosons in Jacob-Wick phase convention are [13]

$$\varepsilon_1^* = \frac{1}{\sqrt{2}} [0, -\lambda \cos \theta, i, \lambda \sin \theta], \quad \text{for } \lambda = \pm 1$$

$$\varepsilon_1^* = \frac{\sqrt{s}}{2m_Z} [\beta, \sin \theta, 0, \cos \theta], \quad \text{for } \lambda = 0$$

$$\varepsilon_2^* = \frac{1}{\sqrt{2}} [0, \bar{\lambda} \cos \theta, i, -\bar{\lambda} \sin \theta], \quad \text{for } \bar{\lambda} = \pm 1$$

$$\varepsilon_2^* = -\frac{\sqrt{s}}{2m_Z} [\beta, -\sin \theta, 0, -\cos \theta], \quad \text{for } \bar{\lambda} = 0.$$

The $\lambda, \bar{\lambda} = \pm 1$ corresponds to transverse gauge bosons and, $\lambda, \bar{\lambda} = 0$ corresponds to longitudinal gauge bosons in the final state. The polarization vector of the

longitudinal gauge boson contains the factor $\gamma = \sqrt{s}/2m_Z$, which with its extra power of energy, is expected to give a possible breakdown of unitarity at high energies. Therefore we consider the production of longitudinal gauge bosons to examine the unitarity constraints. For the massless light leptons, $\Delta\sigma = (\sigma - \bar{\sigma})/2 = -1$, and for the final state longitudinal Z -bosons $\Delta\lambda = \lambda - \bar{\lambda} = 0$ giving the minimum angular momentum, $J = \max(|\Delta\lambda|, |\Delta\sigma|) = 1$. Following ref. [13] and confining ourselves to the specific case $\lambda = \bar{\lambda} = 0$, we separate the contributions to the amplitude from t - and u -channels ordinary and exotic lepton exchange as

$$M(l\ell \rightarrow ZZ) = M_Z^l + M_Z^E. \quad (29)$$

The ordinary lepton exchange contribution is found to be

$$M_Z^l = \frac{-e^2}{\sin^2 \theta_W \cos^2 \theta_W} (A_L^\dagger A_L - 2 \sin^2 \theta_W)^\dagger (A_L^\dagger A_L - 2 \sin^2 \theta_W) \times \frac{1}{\gamma^2} \frac{2 \sin \theta \cos \theta}{\left[4\beta^2 \sin^2 \theta + \frac{1}{\gamma^4}\right]} \delta_{J,1}. \quad (30)$$

Using eq. (25) in eq. (30), gives

$$M_Z^l = \frac{-e^2}{\sin^2 \theta_W \cos^2 \theta_W} (\cos^2 \theta_L^a - 2 \sin^2 \theta_W)^2 \frac{1}{\gamma^2} \frac{2 \sin \theta \cos \theta}{\left[4\beta^2 \sin^2 \theta + \frac{1}{\gamma^4}\right]} \delta_{J,1}. \quad (31)$$

The exotic lepton exchange contribution is found to be

$$M_Z^E = \frac{-e^2}{\sin^2 \theta_W \cos^2 \theta_W} (E_L^\dagger A_L)^\dagger (E_L^\dagger A_L) \left(\frac{1}{\gamma^2} + \frac{m_E^2}{m_Z^2} \right) \times \frac{2 \sin \theta \cos \theta}{\left[4\beta^2 \sin^2 \theta + \frac{1}{\gamma^4} + \frac{8m_E^2}{s} \left(1 + \beta^2 + \frac{4m_E^2}{s}\right)\right]} \delta_{J,1}. \quad (32)$$

Using the unitarity relations (15) and eq. (25) in the above relation we get

$$M_Z^E = \frac{-e^2}{\sin^2 \theta_W \cos^2 \theta_W} (\sin^2 \theta_L^a \cos^2 \theta_L^a) \left(\frac{1}{\gamma^2} + \frac{m_E^2}{m_Z^2} \right) \times \frac{2 \sin \theta \cos \theta}{\left[4\beta^2 \sin^2 \theta + \frac{1}{\gamma^4} + \frac{8m_E^2}{s} \left(1 + \beta^2 + \frac{4m_E^2}{s}\right)\right]} \delta_{J,1}, \quad (33)$$

where m_E is the mass of the heavy charged lepton. In the high-energy domain, $\sqrt{s} \gg m_Z$, we use the approximation $\beta \rightarrow 1$ and $\gamma \rightarrow \infty$, further we make no assumption regarding the relative size of m_E and \sqrt{s} . Then eqs (31) and (33) reduce to

$$M_Z^l \rightarrow 0, \quad (34)$$

Charged lepton mixing with exotic charged leptons

$$M_Z^E \approx \frac{-G_F}{\sqrt{2}} \sin^2 (2\theta_L^a) m_E^2 \frac{\sin \theta \cos \theta}{\left[\sin^2 \theta + \frac{4m_E^2}{s} \left(1 + \frac{2m_E^2}{s} \right) \right]} \delta_{J,1}. \quad (35)$$

As $M_Z^l \rightarrow 0$, we do not expect the unitarity violation if heavy exotic charged leptons are absent. However, if they exist then through mixing they induce non-zero amplitude M_Z^E which depend on mixing angle (θ_L^a) and heavy charged lepton mass (m_E). The total amplitude for the process, $l^+l^- \rightarrow ZZ$, in high energy limit with the inclusion of mixing is given by (using eqs (34) and (35) in (29))

$$M(l l \rightarrow ZZ) = \frac{-G_F}{\sqrt{2}} \sin^2 (2\theta_L^a) m_E^2 \frac{\sin \theta \cos \theta}{\left[\sin^2 \theta + \frac{4m_E^2}{s} \left(1 + \frac{2m_E^2}{s} \right) \right]} \delta_{J,1}. \quad (36)$$

3. Bounds on exotic charged lepton mixing

The unitarity of the scattering matrix together with optical theorem imposes certain unitarity restrictions for the partial waves [7–10]. For the process $a + b \rightarrow c + d$, in terms of partial wave decomposition the amplitude is written as [10]

$$M(s, t, u) = 16\pi \sum_{J=0}^{\infty} (2J + 1) P_J(\cos \theta) a_J(s), \quad (37)$$

where s, t, u are the Mandelstam variables, $a_J(s)$ is the J th partial wave amplitude and $P_J(\cos \theta)$ are the Legendre polynomials. The corresponding differential cross-section is given by [10]

$$\frac{d\sigma}{d \cos \theta} = \frac{1}{32\pi} |M(s, t, u)|^2. \quad (38)$$

Using the fact that the Legendre polynomials are orthogonal, we obtain the total cross-section as

$$\sigma = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J + 1) |a_J(s)|^2. \quad (39)$$

The optical theorem together with the expression gives the following unitarity constraints [10]:

$$|\operatorname{Re} a_J| \leq \frac{1}{2}. \quad (40)$$

The J th partial wave ($a_J(s)$) can be inversed from eq. (37), and is given by

$$a_J(s) = \frac{1}{32\pi} \int_{-1}^1 M(s, t, u) P_J(\cos \theta) d(\cos \theta). \quad (41)$$

Substituting the value of transition amplitude $M(l l \rightarrow ZZ)$ from eq. (37) in eq. (41), we obtain $J = 1$ partial wave amplitude for the process $l^+l^- \rightarrow ZZ$ as

Jai Kumar Singhal

$$a_1(ll \rightarrow ZZ) = \frac{-G_F}{32\sqrt{2}\pi} \sin^2(2\theta_L^a) m_E^2 \times \int_{-1}^1 \frac{\sin\theta \cos\theta}{\left[\sin^2\theta + \frac{4m_E^2}{s} \left(1 + \frac{2m_E^2}{s}\right)\right]} \cos\theta d(\cos\theta). \quad (42)$$

After integration and constraining by unitarity condition, eq. (42), we obtain

$$\sin^2(2\theta_L^a) m_E^2 \left[1 + \frac{8m_E^2}{s} \left(1 + \frac{2m_E^2}{s}\right)\right] \leq \frac{32\sqrt{2}}{G_F}. \quad (43)$$

In terms of energy scale Λ at which the perturbative unitarity is violated, eq. (43) reads as

$$\sin^2(2\theta_L^a) \leq \frac{32\sqrt{2}}{G_F m_E^2 \left[1 + \frac{8m_E^2}{\Lambda^2} \left(1 + \frac{2m_E^2}{\Lambda^2}\right)\right]}. \quad (44)$$

The upper limits on $\sin^2(2\theta_L^a)$ as a function of heavy charged lepton mass (m_E) obtained from eq. (44) with $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ [4] are shown in figure 2, for $\Lambda = 1 \text{ TeV}$, 2 TeV , and ∞ . No constraint on $\sin^2(2\theta_L^a)$ is obtained for $m_E < 0.69 \text{ TeV}$ (when $\Lambda = 1 \text{ TeV}$), $m_E < 0.99 \text{ TeV}$ (when $\Lambda = 2 \text{ TeV}$), and $m_E < 1.97 \text{ TeV}$ (when $\Lambda = \infty$). However, for heavy charged lepton mass, $m_E = 5 \text{ TeV}$, we obtain $\sin^2(2\theta_L^a) \leq 1.52 \times 10^{-5}$ ($\Lambda = 1 \text{ TeV}$), $\leq 2.29 \times 10^{-4}$ ($\Lambda = 2 \text{ TeV}$) and ≤ 0.15 ($\Lambda = \infty$). Similarly for $m_E = 10 \text{ TeV}$, we find $\sin^2(2\theta_L^a) \leq 2.41 \times 10^{-7}$ ($\Lambda = 1 \text{ TeV}$), $\leq 3.80 \times 10^{-6}$ ($\Lambda = 2 \text{ TeV}$) and $\leq 3.88 \times 10^{-2}$ ($\Lambda = \infty$).

4. Conclusions and discussion

In order to obtain the theoretical constraints on mixing parameter for charged lepton sector we used the notion of perturbative unitarity, requiring that the partial wave amplitude should not exceed the perturbative unitarity limit. We examined the $J = 1$ partial wave amplitude of the process $l^+l^- \rightarrow ZZ$ (where $l = e, \mu, \tau$), with the inclusion of mixings. The mixing modifies the weak neutral current for the process. If $\sqrt{s} = \Lambda$ is the energy scale at which perturbative unitarity is assumed to be violated, then the constraints on mixing parameters are obtained as a function of heavy exotic lepton mass for different values of Λ . We find the following:

- (i) For $\Lambda = 1 \text{ TeV}$, no unitarity constraint on $\sin^2(2\theta_L^a)$ is obtained for $m_E < 0.69 \text{ TeV}$. However for $m_E = 5 \text{ TeV}$, we obtain $\sin^2(2\theta_L^a) \leq 1.52 \times 10^{-5}$ and for $m_E = 10 \text{ TeV}$, $\sin^2(2\theta_L^a) \leq 2.41 \times 10^{-7}$.
- (ii) For $\Lambda = 2 \text{ TeV}$, no unitarity constraint on $\sin^2(2\theta_L^a)$ is obtained for $m_E < 0.99 \text{ TeV}$. For $m_E = 5 \text{ TeV}$, $\sin^2(2\theta_L^a) \leq 2.29 \times 10^{-4}$ and for $m_E = 10 \text{ TeV}$, $\sin^2(2\theta_L^a) \leq 3.80 \times 10^{-6}$.
- (iii) For $\Lambda = \infty$, no unitarity constraint is obtained for $m_E < 1.97 \text{ TeV}$. For $m_E = 5 \text{ TeV}$, $\sin^2(2\theta_L^a) \leq 0.15$ and for $m_E = 10 \text{ TeV}$ $\sin^2(2\theta_L^a) \leq 3.88 \times 10^{-2}$.

Charged lepton mixing with exotic charged leptons

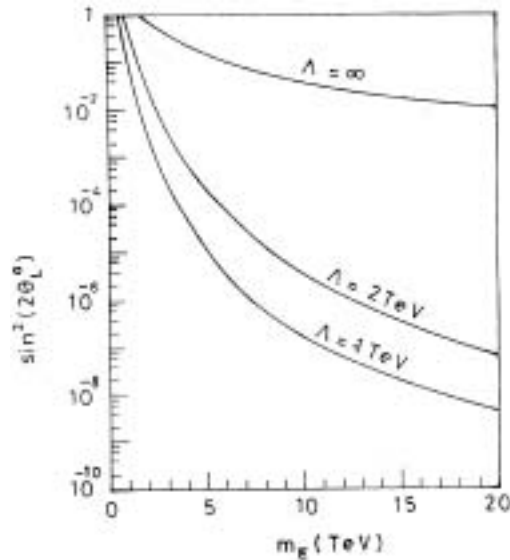


Figure 2. Upper bounds on light-heavy charged lepton mixing parameter $\sin^2(2\theta_L^a)$ as a function of heavy charged lepton mass m_E , at different values of Λ .

It is observed that (see figure 2) there is a stronger upper limit on mixing angle for higher values of exotic lepton mass whereas for lower exotic lepton mass the limits are poor. We understand this behavior by referring eq. (35). The exotic leptons through mixing induces non-zero amplitude (M_Z^E) for the process $l^+l^- \rightarrow ZZ$, which depends on mixing angle (θ_L^a) and exotic lepton mass (m_E). At very high energies ($\sqrt{s} \rightarrow \infty$) the amplitude M_Z^E grows like $G_F \sin^2(2\theta_L^a) m_E^2$. With increasing exotic lepton mass the amplitude M_Z^E becomes stronger and may lead to a possible breakdown of unitarity at higher exotic lepton mass. Thus at higher exotic lepton mass the unitarity requirements yield stronger bounds on mixing angle. On the other hand for lower values of exotic lepton mass we note that the amplitude M_Z^E vanishes as $m_E \rightarrow 0$ even if the mixing is large, as such for lower values of exotic lepton mass the unitarity requirements do not yield stringent bounds on mixing angle.

We have also looked for the implications of perturbative unitarity requirements on partial wave amplitudes for the processes $l^+l^- \rightarrow W^+W^-$ and $\nu\bar{\nu} \rightarrow W^+W^-$. It is found that the unitarity requirements on partial wave amplitudes for these processes yield the bounds on the combination of mixing angles in charged lepton sector and neutral lepton sector, not on the single mixing angle. As such these bounds are not of much interest.

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