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Late time phase transition as dark energy

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Abstract. We show that the dark energy field can naturally be described by the scalar condensates of a non-abelian gauge group. This gauge group is unified with the standard model gauge groups and it has a late time phase transition. The small phase transition explains why the positive acceleration of the universe is occurring only recently. The model has *no free* parameters but for the matter content of the group. The initial energy density at the unification scale and at the condensation scale are fixed by the number of degrees of freedom of the gauge group.

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In the past few years the SN1a [1] and CMB spectrum [2] have led to conclude that the universe is flat and filled with an energy density with negative pressure. The dark energy is perhaps best understood, from an elementary particle point of view, as the contribution from a scalar field that interacts with all other fields only gravitationally. The universe is now dominated by the dark energy density $\Omega_{\phi o} = 0.7 \pm 0.1$ (the subscript 'o' refers to present day quantities) with negative pressure. The SN1a data requires an equation of state $w_{\phi o} < -2/3$ while recent analysis on the CMBR peaks constrains the models to have $w_{\rm eff} = -0.82^{+.14}_{-.11}$ [3], where $w_{\rm eff}$ is an average equation of state. Structure formation also favors a non-vanishing cosmological constant consistent with SN1a and CMBR observations [4]. The evolution of scalar field has been widely studied and some general approach can be found in [5]. The evolution of the scalar field ϕ depends on the functional form of its potential $V(\phi)$ and a late time accelerating universe constrains the form of the potential [5,6].

One of the simplest and most interesting quintessence potentials are the inverse power law (IPL). In some special cases they can be derived from non-abelian gauge theories [6] and we can also have consistent models with a gauge coupling unified with the standard model (SM) couplings [6]. In this work we show that a non-abelian gauge group with N_c the number of colors and N_f that of chiral fields leads to an acceptable quintessence potential. We show that only the degrees of freedom are precisely the simple choice of N_c, N_f and the number of dynamically generated bilinear fields which set the scale of condensation and the power in the potential of the scalar field responsible for present day acceleration of the universe.

The model is quite simple, we start with a non-abelian gauge group (which we call the 'Q' group) at a high-energy scale, the unification scale of the standard model gauge groups, with massless matter fields and we let it evolve to lower scales. By lowering the energy scale, the Q gauge coupling constant becomes large and all fields become strongly interacting at the condensation scale Λ_c. Below this scale there are no more free elementary fields, chiral nor gauge fields, similar to what happens with QCD and we are left with gauge singlets bilinear fields $\phi^2 \equiv \langle Q\tilde{Q}\rangle$ (the square in ϕ is to give the field a mass dimension one). We use Afflek–Dine–Seiberg superpotential 'ADS' [7] to determine the form of the scalar potential V in terms of ϕ. Afterwards, we solve Einstein's general relativity equations in a Friedmann–Robertson–Walker flat metric and determine the cosmological evolution of ϕ. We show that a positive accelerating universe at present time with $\Omega_{\phi o} \simeq 0.7$ and $w_{\phi o} < -2/3$ is possible and we determine the CMB spectrum [2]. Furthermore, we constrain the model to have the same unification scale and gauge coupling as the standard model gauge groups. This is by all means not a necessary condition but it gives a very interesting model. We could think of this model as coming from string theory after compactifying the extra dimensions.

We start by assuming that the universe has a matter content of the supersymmetric gauge groups $SU(1) \times SU(2) \times SU(3) \times SU(Q)$ where the first three are the SM gauge groups while the last one corresponds to the 'quintessence group' Q and that the couplings are unified at $\Lambda_{\rm gut}$ with $g_1 = g_2 = g_3 = g_Q = g_{\rm gut}$. The condensation scale Λ_c of a gauge group $SU(N_c)$ with N_f (chiral + antichiral) matter fields has in $N = 1$ SUSY a one-loop renormalization group equation given by

$$\Lambda_c = \Lambda_{\rm gut} {\rm e}^{-(8\pi^2/b_o g_{\rm gut}^2)}, \tag{1}$$

where $b_o = (3N_c - N_f)$ is the one-loop beta function and $\Lambda_{\rm gut}, g_{\rm gut}$ are the unification energy scale and coupling constant, respectively. From gauge coupling unification we know that $\Lambda_{\rm gut} \simeq 10^{16}$ GeV and $g_{\rm gut} \simeq \sqrt{4\pi/25.7}$.

A phase transition takes place at the condensation scale Λ_c, since the elementary fields are free fields above Λ_c and condense at Λ_c. In order to study the cosmological evolution of these condensates, which we will call ϕ, we use Affleck's potential [7]. This potential is non-perturbative and exact [8]. The superpotential for a non-abelian $SU(N_c)$ gauge group with N_f (chiral + antichiral) massless matter fields is [7]

$$W = (N_c - N_f)\left(\frac{\Lambda_c^{b_o}}{\det\langle Q\tilde{Q}\rangle}\right)^{1/(N_c - N_f)}, \tag{2}$$

where b_o is the one-loop beta function coefficient. The scalar potential in global supersymmetry is $V = |W_\phi|^2$, with $W_\phi = \partial W/\partial\phi$, giving [6]

$$V = c^2 \Lambda_c^{4+n} \phi^{-n}, \tag{3}$$

where we have taken $\det\langle Q\tilde{Q}\rangle = \Pi_{j=1}^{N_f} \phi_j^2, \;\; c = 2N_f, \;\; n = 2 + 4\frac{N_f}{N_c - N_f}$ and Λ_c is the condensation scale of the gauge group $SU(N_c)$. We have taken ϕ canonically normalized, but the full Kähler potential K is not exactly known and for $\phi \simeq 1$ other terms may become relevant and could spoil the runaway and quintessence behavior

Table 1. We show the matter content for the four different models and we give the number of degrees of freedom for the SUSY and non-SUSY Q group in the last two columns, respectively. Notice that the condensation scale and b_o is the same for all models.

Num	N_c	N_f	ν	n	b_o	Λ_c (eV)	g_{Qs}	Ω_ϕ(NS)
I	3	6	1	2/3	3	42	97.5	0.13
II	6	15	3	2/3	3	42	468.5	0.42
III	7	18	4	6/11	3	42	652.5	0.5

of ϕ. However, for $n < 2$ the normalization does not present any problems [6]. If we wish to study models with $0 < n < 2$, which are cosmologically favored [6] we need to consider the possibility that not all N_f condensates ϕ_i become dynamical but only a fraction ν are (with $N_f \geq \nu \geq 1$) and we also require $N_f > N_c$ [6]. One can have $\nu \neq N_f$ with gauge group with unmatching number of chiral and anti-chiral fields or if some of the chiral fields are also charged under another gauge group. Another possibility is by giving a mass term to $N_f - \nu$ condensates $\varphi = \langle \bar{Q}_k Q_k \rangle$, $(k = 1, ..., N_f - \nu)$ while leaving ν condensates $\phi^2 = \langle \bar{Q}_j Q_j \rangle$, $(j = 1, ..., \nu)$ massless. In this case we have $c = 2\nu, n = 2 + 4\frac{\nu}{N_c - N_f}$ in eq. (3) [6].

Not all values of Λ_c, n will give an acceptable cosmology. The correct values of Λ_c depend on the cosmological evolution of the scalar condensate ϕ which is determined by the power n in eq. (3). The number of models that satisfy the unification and cosmological constrains of having $\Omega_{\phi o} \simeq 0.7, h_o \simeq 0.7$ and $w_{\phi o} < -2/3$ [3] is quite limited [6]. In fact there are only three models given in table 1. All of these three models have $n \leq 2/3$ and the quantum corrections to the Kähler potential are, therefore, not dangerous. All other combinations of N_c, N_f, ν do not lead to an acceptable cosmological model.

The cosmological evolution of ϕ with an arbitrary potential $V(\phi)$ can be determined from a system of differential equations describing a spatially flat Friedmann–Robertson–Walker universe in the presence of a barotropic fluid energy density ρ_γ that can be either radiation or matter, are $\dot{H} = -\frac{1}{2}(\rho_\gamma + p_\gamma + \dot{\phi}^2), \ddot{\phi} = -3H\dot{\phi} - \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}$, where H is the Hubble parameter ($H = 100h$ km/(Mpc s)), $\dot{f} = \mathrm{d}f/\mathrm{d}t$, $\rho(p)$ is the total energy density(pressure) and we are setting the reduced Planck mass $m_\mathrm{p}^2 = 1/8\pi G \equiv 1$. Solving the above equations, we have that the energy density of the Q group Ω_ϕ drops quickly, independently of its initial conditions, and it is close to zero for a long period of time, which includes nucleosynthesis (NS) if Λ_c is larger than the NS energy Λ_{NS} (or temperature $T_{\mathrm{NS}} = 0.1$–10 MeV), and becomes relevant only until very recently [6]. On the other hand, if $\Lambda_c < \Lambda_{\mathrm{NS}}$ then the NS bounds on relativistic degrees of freedom must be imposed on the models $\Omega_\phi(\mathrm{NS}) \leq 0.1$–$0.2$ [9]. Finally, the energy density of Q grows and it dominates at present time the total energy density with the $\Omega_{\phi o} \simeq 0.7$ and a negative pressure $w_{\phi o} < -2/3$ leading to an accelerating universe [3].

What is the energy density of Q at the condensation scale Λ_c? Using [10]

$$\Omega_{Qf} = \frac{g_{Qf}(T'/T)^4}{g_{\mathrm{sm}f} + g_{Qf}(T'/T)^4}, \tag{4}$$

where $g_{Qf}, g_{\mathrm{sm}f}$ are the final degrees of freedom of the dark group and the SM, respectively. For model I, one has $g_{\mathrm{sm}f} = 3.36, g_{Qf} = 97.5, T'/T = (43/11/228.75)^{1/3}$ given at $\Lambda_c = 42$ eV and $\Omega_\phi(\Lambda_c) = \Omega_Q(\Lambda_c) = 0.11$. Evolving FRW equations with initial condition $\Omega_\phi(\Lambda_c) = 0.11$ gives at present time with $h_o = 0.7$, $\Omega_{\phi o} = 0.7$ a value of $w_{\mathrm{eff}} \equiv \int \mathrm{d}a\ \Omega(a) w_\phi(a) / \int \mathrm{d}a\ \Omega(a) = -0.93$ with $w_{\phi o} = -0.90$ in agreement with SN1a and CMBR data. Furthermore, a large range of initial condition of $\Omega_{\phi i}$ [6], with upper limit $\Omega_{\phi i} = 0.20$ due to NS, and no 'reasonable' lower limit still gives an acceptable model and there is clearly no fine-tuning in these models.

In table 1, we give the values of n, b_o, the degrees of freedom of g_Q, the condensation scale Λ_c. Notice that all models have the same b_o, Λ_c but n differs slightly. Model I is the minimal model, in the sense that it has the smallest number of degrees of freedom.

We have shown that an unification scheme, where all coupling constants are unified, as predicted by string theory, leads to an acceptable cosmological constant parameterized in terms of the condensates of a non-abelian gauge group. These fields play the role of quintessence. We would like to stress out that there are no free parameters, not even the Q initial energy density at unification nor at the condensation scale. Only a requirement on the minimum number of degrees of freedom on the SUSY breaking gauge group is needed which poses no problems to the models since the number is in agreement with existing examples. The condition of gauge coupling unification fixes the values of N_c, N_f, ν of the Q group and all parameters are then fixed.

The value of Λ_c is small (i.e. we have a late time phase transition) and this explains why we have a late time (present day) accelerating universe since the appearance and evolution of the scalar field is only recently.

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