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Neutrino–antineutrino asymmetry around rotating black holes

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Abstract. Propagation of fermion in curved space-time generates gravitational interaction due to the coupling between spin of the fermion and space-time curvature. This gravitational interaction appears as a CPT violating term in the Lagrangian. It is seen that this space-time interaction can generate neutrino asymmetry in the Universe. If the background metric is spherically asymmetric, say, of a rotating black hole, this interaction is non-zero, thus the net difference to the number density of the neutrino and anti-neutrino is non-zero.

Keywords. Neutrino asymmetry; rotating black hole; CPT violation.

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1. Introduction

Generation of neutrino asymmetry in the early Universe is a well-known fact. If the baryon and lepton numbers are different in our Universe, and since our Universe is electrically neutral, it is argued that lepton asymmetry manifests itself in the form of neutrino asymmetry. Large lepton asymmetries can arise in the early Universe through for e.g. Affleck–Dine mechanism [1]. Since relic neutrino asymmetry has important effects and hence can be constrained by Big Bang nucleosynthesis and power spectrum of cosmic microwave background, it becomes an important issue by itself to study new mechanisms which can generate neutrino asymmetry specially in the present epoch. When Dirac neutrinos propagate in gravitational backgrounds, then depending upon the form of the background metric there is always a possibility of the origin of neutrino degeneracy. This neutrino degeneracy or asymmetry is additional to the relic neutrino asymmetry from the Big–Bang era. Such an effect can have wide ranging implications for our understanding of the phenomenon in neutrino astrophysics.

When a spinning test particle propagates in the curved space-time, the coupling of its spin with the spin connection of the background field produces an interaction term. This interaction appears in a manner like a test spinor propagates in an electromagnetic field on a background of flat space [2]. The spin connection in curved space-time plays a similar role as of electromagnetic four-vector potential in the flat space. It is very interesting to note that the interaction term would not preserve CPT if the background space-time contribution does not flip sign under CPT transformation. When the spinor under consideration is chosen as a Dirac neutrino, this interaction under CPT will give rise to opposite sign for a left-handed (neutrino) and right-handed (anti-neutrino) fields.

2. Formalism

The most general Dirac Lagrangian density can be given as

$$\mathcal{L} = \sqrt{-g}\left(i\,\bar{\psi}\,\gamma^a D_a\psi - m\,\bar{\psi}\psi\right), \tag{1}$$

where the covariant derivative and spin connection are defined as

$$D_a = \left(\partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}\right), \qquad \omega_{bca} = e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu}e^\gamma_c e^\mu_a\right), \tag{2}$$

where $\sigma^{bc} = \frac{i}{2}\left[\gamma^b, \gamma^c\right]$ is the generator of tangent space Lorentz transformation, vierbiens are defined as $e^\mu_a\, e^\nu_b\, g_{\mu\nu} = \eta_{ab}$. The Latin and Greek alphabets indicate the flat and curved space coordinate respectively and we would work in units $c = \hbar = k_{\mathrm{B}} = 1$ and signature as $(+\,-\,-\,-)$.

Expanding (1), we get the Dirac Lagrangian separated into free and (gravitational) interaction parts as

$$\mathcal{L} = \sqrt{-g}\;\bar{\psi}\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi = \mathcal{L}_f + \mathcal{L}_I, \tag{3}$$

where $B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$. Clearly, $\mathcal{L}_I$ is an axial-vector term multiplied by a gravitational four vector potential coupling coefficient, which is odd under CPT transformation, if B_a does not flip its sign. According to the standard model, neutrino (particle) has left chirality and anti-neutrino (anti-particle) has right chirality and thus on further expansion $\mathcal{L}_I$ picks up different sign for neutrino (ψ) and anti-neutrino (ψ^c) as

$$\overline{\psi}\gamma^a\gamma^5\psi = \overline{\psi}_{\mathrm{L}}\gamma^a\psi_{\mathrm{L}}, \quad \overline{\psi^c}\gamma^a\gamma^5\psi^c = -\overline{\psi^c_{\mathrm{R}}}\gamma^a\psi^c_{\mathrm{R}}. \tag{4}$$

Therefore, the dispersion relation becomes

$$E_{\nu,\overline{\nu}} = \sqrt{|\vec{p}|^2 \pm 2\left(B_0 p^0 + B_i p^i\right) + B_a B^a - m^2}. \tag{5}$$

Finally, if the neutrinos are traveling between two extreme points, R_i and R_f, their asymmetry in number density can be generated as [3]

$$\Delta n = \frac{g}{(2\pi)^3}\int_{R_i}^{R_f}\mathrm{d}V\int \mathrm{d}^3|\vec{p}|\left[\frac{1}{1+\exp(E_\nu/T)} - \frac{1}{1+\exp(E_{\overline{\nu}}/T)}\right], \tag{6}$$

where $\mathrm{d}V$ is the small volume element in that space. If we consider cartesian coordinate system, for $B_0 = 0$, $\Delta n = 0$.

Kerr metric in cartesian coordinate system can be given as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 - \frac{2Mr^3}{r^4 + a^2 z^2} \times \left[\mathrm{d}t - \frac{1}{r^2 + a^2}\left(r(x\,\mathrm{d}x + y\,\mathrm{d}y) + a(x\,\mathrm{d}y - y\,\mathrm{d}x)\right) - \frac{z}{r}\,\mathrm{d}z\right]^2, \quad (7)$$

where r is defined through, $r^4 - r^2(x^2 + y^2 + z^2 - a^2) - a^2 z^2 = 0$. As this metric yields a non-vanishing space–space cross terms, $B_0 \neq 0$ and the asymmetry. For detailed discussions, see [4].

3. Result

Let us consider a special case of Kerr geometry where $\vec{B} \cdot \vec{p} \ll B_0 p^0$, $B_a B^a \ll 1$ and $B_0 \ll T$. Thus from (6), in ultra-relativistic regime $\Delta n \sim \overline{B_0} T^2$, where $\overline{B_0}$ indicates the integrated value of B_0 over the space.

In case of accretion disk around the black hole, $T \sim 10^{11}$ K ~ 10 MeV $\sim 10^{-5}$ erg. To satisfy the approximation of metric, black hole parameters have to be tuned in such a manner that $\overline{B_0} \lesssim 10^{-6}$ erg and thus $\Delta n \lesssim 10^{-16}$.

In case of Hawking radiation bath, $T \sim 10^{-7}(M_\odot/M)$ K. The primordial black hole of mass, $M > 10^{15}$ g, exist even today. The temperature of those black holes is $T > 10^{11}$ K $= 10^{-5}$ erg. Thus if $\overline{B_0} = 10^{-6}$, $\Delta n \gtrsim 10^{-16}$, depending on the mass of the black holes.

If there are N_i number of i-kind black holes with corresponding curvature scalar coupling as $\overline{B_0}_i$ and temperature T_i, the neutrino asymmetry for that particular kind of black hole can be written as

$$\Delta n_i = 10^{-10}\left(\frac{N_i}{10^6}\right)\left(\frac{\overline{B_0}_i}{10^{-6}\,\mathrm{erg}}\right)\left(\frac{T_i}{10^{-5}\,\mathrm{erg}}\right)^2. \quad (8)$$

The sign of the asymmetry strictly depends on the different black hole parameters. Thus the overall asymmetry for all kinds of black hole becomes

$$\Delta n = \sum_i \Delta n_i. \quad (9)$$

If we check this asymmetry in the earth, $\overline{B_0}_{\mathrm{earth}} = 10^{-40}$ erg, $T_{\mathrm{earth}} = 10^{-14}$ erg. Then $\Delta n_{\mathrm{earth}} = 10^{-68}$ which is very small over relic asymmetry and thus we do not see its effect. However, in a laboratory of the earth, if we are able to thermalise the neutrino, asymmetry could be raised.

4. Summary

Thus we can propose criteria to generate neutrino asymmetry in the presence of gravitational field as:

(1) The space-time should be axially symmetric.
(2) The interaction Dirac Lagrangian must be CPT violating.
(3) The temperature scale of the system should be large with respect to the energy scale of the space-time curvature.

References

[1] I Affleck and M Dine, *Nucl. Phys.* **B249**, 361 (1985)
[2] B Mukhopadhyay, *Class. Quantum Gravit.* **17**, 2017 (2000)
[3] P Singh and B Mukhopadhyay, *Mod. Phys. Lett.* **A**, (2003) (in Press)
[4] B Mukhopadhyay and P Singh, *Nucl. Phys. B*, (2003) submitted; gr-qc/0301002