Baryogenesis via density fluctuations with a second-order electroweak phase transition

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Abstract. We consider the presence of cosmic string-induced density fluctuations in the early universe at temperatures below the electroweak phase transition temperature. Resulting temperature fluctuations can restore the electroweak symmetry locally depending on the amplitude of fluctuations and the background temperature. The symmetry will be spontaneously broken again in a given region as the temperature drops there (for fluctuations with length scales smaller than the horizon), resulting in the production of baryon asymmetry. The time-scale of the transition will be governed by the wavelength of fluctuation and, hence, can be much smaller than the Hubble time. This leads to strong enhancement in the production of baryon asymmetry for a second-order electroweak phase transition as compared to the case when transition happens due to the cooling of the universe via expansion. For a two-Higgs doublet model (with appropriate CP violation), we show that one can get the required baryon asymmetry if fluctuations propagate without getting significantly damped. If fluctuations are damped rapidly, then a volume factor suppresses the baryon production, though it is still 3–4 orders of magnitude larger than the conventional case of second-order transition.

1. Introduction

Density fluctuations produced by moving cosmic strings can have significant effect on the phase transition in the early universe (see, e.g. ref. [1,2]). Here we discuss the effect of these fluctuations on electroweak baryogenesis for the case of a second-order electroweak phase transition [3]. We show that for a $10^{16}$ GeV GUT cosmic string, the resulting density fluctuations can give the required value of baryon-to-entropy ratio (in the context of a two-Higgs doublet model as in [4], with the CP violation parameter of order 10) if fluctuation propagates without getting significantly damped. If on the other hand, if fluctuations are dissipated rapidly, then a volume suppression factor reduces the produced baryon asymmetry, though it is still 3–4 orders of magnitude larger than the conventional case where baryons are produced during the cooling of the universe via expansion.

2. Baryon-to-entropy ratio with a second-order electroweak transition

For the conventional case of second-order electroweak phase transition when transition proceeds by the cooling of the universe due to its expansion, it was shown
in ref. [4] that maximum baryon asymmetry created in the context of two-Higgs doublet model is

\[ B_0 = \left| c_n T \epsilon \right|_{\text{max}}, \]

where \( c_n \simeq 0.4 \) [4] for two-Higgs doublet model and \( \epsilon \) characterizes CP violation parameter in the model value of which lies in the range 1–200 (ref. [4]).

If the departure from equilibrium is due to the expansion of the universe, then one can calculate \( T \) from the Friedman equation and finally get the baryon-to-entropy ratio, \( \frac{B}{s} \simeq \frac{45 c_n \epsilon}{2 \pi g_s^2 T_b^4}. \) With \( g_s \simeq 100 \) and \( T_c = 110 \text{ GeV} \), we get [3], \( \frac{B}{s} \simeq 10^{-18} \epsilon. \) Thus, as discussed in ref. [4], even with most optimistic estimates of \( \epsilon \sim 200 \), required baryon asymmetry cannot be generated for the case of standard second-order phase transition.

3. Baryogenesis via density fluctuations produced by moving cosmic string

To study the generation of density fluctuations due to cosmic string moving through the relativistic fluid, we have followed ref. [5]. It was shown in ref. [5], that, if the velocity of the cosmic string reaches the ultra-relativistic limit then the density fluctuation becomes of order 1 (\( \delta \rho / \rho \sim 1 \)) and the angle of the wake approaches deficit angle \( \simeq 8 \pi G \mu. \) Here, \( \mu \) is the string tension. If the wavelength of the fluctuation is much smaller than the horizon, then evolution of density fluctuation can be written as [3], \( \delta \rho / \rho \sim A e^{i(k \cdot x - \omega t)}. \) Here, \( A \) is the amplitude of the fluctuation, \( k \) is the wave vector and \( \omega = 2 \pi c_s / \lambda \) is the angular frequency of the fluctuation with wavelength \( \lambda. \) For density fluctuations generated due to GUT cosmic string moving with ultra-relativistic speed, the amplitude \( A \sim 1 \), and the shortest wavelength (\( \lambda_{\text{smc}} \)) is given by the average width of the generated wake \( \sim \frac{1}{\pi} 8 \pi G \mu \lambda H \simeq 10^{-6} \text{ cm}, \) where, \( \lambda_H (\simeq 0.1 \text{ cm}) \) is the horizon size at that time.

With the use of local thermal equilibrium assumption, one can relate the density fluctuation to the temperature fluctuation and finally can write the time-dependent temperature field, \( T(t) = T_0 \left[ 1 + A \sin \omega t \right]^{1/4} \), where \( T_0 \) is the background temperature of the plasma.

Using eq. (1) and from the above expression of temperature field, one can write [3],

\[ B_0 = (c_n T \epsilon T)_{\text{max}} = \frac{c_n \epsilon A \omega T_0^4}{4 T_b^3}. \]

For order of magnitude estimates, we take \( T \sim T_b \). The resulting baryon-to-entropy ratio can now be expressed in terms of the amplitude \( A \), and the wavelength \( \lambda \) of fluctuation [3],

\[ \frac{B}{s} = \frac{45 c_n \epsilon}{\pi g_s^2} \frac{A}{4 \lambda^2 T_b} \simeq \frac{0.01 \epsilon A}{\lambda T_b}. \]

Thus, for such fluctuations, the resulting value of baryon-to-entropy ratio is, \( \frac{B}{s} \simeq 10^{-11} \epsilon. \) For this case, with \( \epsilon \) of order 10, one is able to get the required baryon asymmetry.
Baryogenesis via density fluctuations

**Figure 1.** Left: Variation of temperature (in GeV) with time (in GeV\(^{-1}\)) for one oscillation period, resulting from density fluctuations with wavelength \(\lambda \sim 4\pi G\rho H\). Right: Evolution of the Higgs field \(\phi\) (solid line), and its vev (dots), both in GeV.

It is important to realize that, if the fluctuations propagate without getting significantly damped, then essentially all regions will participate in this generation of baryon asymmetry, so there is no volume suppression factor here. However, if the fluctuations do not propagate for large distances, and decay very rapidly, then there will be volume suppression factor \(f_0\). Taking account of about 10 long strings in a horizon volume, with typical wake thickness of order \(10^{-5}d_H\), we get \([3]\), \(f_0 \sim 10^{-4}\). With various factors taken as earlier, we get \(B/s \sim 10^{-15}\) for this case. With optimistic value of \(\epsilon \sim 100\), resulting value of \(B/s\) is \(10^{-15}\), about three orders of magnitude smaller than the required one.

An important assumption in deriving eq. (1) (using which eq. (2) has been derived) is that at any stage during the variation of the temperature, the value of the Higgs field is essentially given by the vacuum expectation value (vev) at that temperature. This assumption requires that the time-scale of temperature variation should be much larger than the typical time-scale of the Higgs field evolution. More importantly, as the temperature rises above \(T_c\), the Higgs field should settle down to value zero so that the electroweak symmetry is restored. For checking this requirement, we take a simple effective potential to model the second-order phase transition (ref. \([6]\]), \(V(\phi; T) = D(T^2 - T_c^2)\phi^2 + \frac{\lambda}{4}\phi^4\). We take the values of the parameters as suitable for the standard model \([6]\). \(T_c = 110\) GeV, \(D \approx 0.18\) and \(\lambda \approx 0.1\). The evolution equation of \(\phi\) (neglecting spatial dependence) is
\[
\ddot{\phi} + \frac{3}{2}\dot{\phi} + V'(\phi, T(t)) = 0.
\]
Using this equation, we have checked the evolution of \(\phi\) (figure 1) for one oscillation period and shown that \(\phi\) traces the evolution of the vev faithfully.

4. Conclusion

We have studied the implications of small wavelength density fluctuations produced by cosmic strings on electroweak baryogenesis in case of second-order phase transition. We have shown that for a \(10^{10}\) GeV GUT scale cosmic string, the resulting density fluctuations can give the required value of the baryon-to-entropy ratio (in the context of a two-Higgs doublet model as in \([4]\), with the CP violation parameter

of about 10). If density fluctuations decay away rapidly, then the volume factor suppresses the resulting baryon asymmetry by a factor of order $10^{-4}$ (though it is still larger by at least 3–4 orders of magnitude compared to the conventional case of second-order phase transition).

References