Tachyon field in cosmology

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Abstract. We study cosmological effects of homogeneous tachyon field as dark energy. We concentrate on two different scalar field potentials, the inverse square potential and the exponential potential. These models have a unique feature that the matter density parameter and the density parameter for tachyon remain comparable for a large range in red-shift. It is shown that there exists a range of parameters for which the universe undergoes an accelerated expansion and the evolution is consistent with structure formation requirements. For a viable model we require fine tuning of parameters comparable to that in $\Lambda$CDM or in quintessence models. For the exponential potential, the accelerated phase is followed by a phase with $a(t) \propto t^{2/3}$ thus eliminating a future horizon.

Keywords. Cosmology; tachyon field; dark energy; structure formation.

PACS Nos 98.80.Cq; 95.35.+d

This report is based on a recent work in collaboration with Bagla and Padmanabhan [1]. In this paper, we construct cosmological models with homogeneous tachyon matter [2] to provide the dark energy component which drives acceleration of the universe (for a recent review of dark energy models, see [3]). We assume that the tachyon matter coexists with normal non-relativistic matter and radiation (for other work on cosmological aspects of tachyon field, see [4]). For a spatially flat universe, the Friedman equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$

where $\rho = \rho_{\text{nr}} + \rho_{\text{r}} + \rho_{\phi}$, with respective terms denoting non-relativistic, relativistic and tachyon matter densities.

For the tachyon field $\phi$ the energy density and pressure are given by

$$\rho_{\phi} = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad p_{\phi} = -V(\phi)\sqrt{1 - \dot{\phi}^2}.$$

The equation of state for tachyon is $p = w\rho$ with $w = \dot{\phi}^2 - 1$. The evolution of the scalar field is described by
\[ \ddot{\phi} = -(1 - \dot{\phi}^2) \left[ 3H \dot{\phi} + \frac{1}{V(\phi)} \frac{dV}{d\phi} \right]. \] (3)

We discuss cosmological models with two different potentials. The first one is an inverse square potential given by \( V(\phi) = \frac{n}{1 + \phi^2}, (1 - \frac{2}{\alpha^2})^{1/2} \phi^{-2} \). The above potential leads to the power-law expansion \( a(t) = t^n \) if \( \phi \) is the only source \([5,6]\). The term in the square bracket in the scalar field equation vanishes in the asymptotic limit when tachyons dominate and \( \phi \propto t \). All initial conditions eventually converge to this asymptotic solution. If normal matter or radiation dominates, \( \phi \) stays close to the transition point \( 2/(3H \phi) \), unless we start the field very close to \( \dot{\phi}^2 = 1 \). For a viable model with matter domination at high red-shifts and an accelerating phase at low red-shifts, we need to start the tachyon field such that \( \dot{\phi} \approx 1 \), and the field \( \phi \) is very large.

The second form of potential is the exponential one with \( V(\phi) = V_0 e^{-\phi/\phi_0} \). In a radiation-dominated or a matter-dominated universe, \( H(t) \) is a monotonically decreasing function of time. As \( H(t) \) keeps decreasing, \( \phi \) increases slowly and asymptotically approaches unity. However, in a tachyon-dominated scenario, the universe expands rapidly and \( H(t) \) varies much more slowly than in the matter-dominated or in the radiation-dominated era. Thus \( \phi \) changes at a slower rate but it still approaches unity and hence we eventually get a dust-like equation of state for the tachyon field. Whether the evolution undergoes an accelerating phase or not depends on the initial values of \( \dot{\phi}, \phi_0 \) and \( \Omega_\phi \). The present values of density parameter for non-relativistic matter and for tachyons fix the epoch at which tachyons start to dominate the energy density. The parameter \( \phi_0 \) sets the time when \( \phi \) approaches unity and the asymptotic dust-like phase for tachyons is reached and the duration of the accelerating phase is fixed by the initial value of \( \phi \). If this initial value is very close to unity, it departs very little from it and if it starts far away from unity, the equation of state for the tachyon field leads to a significantly long accelerating phase. It is possible to fine tune the evolution by choosing a sufficiently large value for \( \phi_0 \), so that \( \phi \) is much smaller than unity even at the present epoch, and by requiring that the tachyon field starts to dominate the energy density of the universe at the present epoch. For the exponential potential, the accelerating phase is a transient between the matter-dominated era and the tachyon-dominated era both with \( a(t) \propto t^{2/3} \). These models have the attractive feature that they do not asymptotically approach de Sitter-like universe and hence do not possess a future horizon (for other attempts, see \([7]\)).

A comparison of these models with supernova type-Ia data shows that the models are consistent with observations for a wide range of parameters. Therefore it is difficult to make a definite statement about constraints on the parameters. To further check the viability and to constrain the range of parameters, we study structure formation in tachyons. The unique feature in tachyon models is that at high red-shift, matter density parameter does not saturate at unity for all the models. This holds true for both the potentials. For models where the matter density parameter does not reach unity, the growth of perturbations is slow. The models in which density parameter is unity at high red-shifts, the growth of perturbations is closer to that in the \( \Lambda \)CDM model. The slower growth of perturbations implies that rms fluctuations in mass distribution were larger at the time of recombination as compared to conventional models. This will have
an impact on the temperature anisotropies in the microwave background in these models.

The density parameter for matter is almost a constant at high red-shifts ($3 < z < 10^3$), and therefore in the linear limit we can solve for the rate of growth of density contrast. The equation for the density contrast is given by

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho\delta,$$

(4)

where $\delta = (\rho - \bar{\rho})/\bar{\rho}$, the factor $\bar{\rho}$ being the average density. The density contrast increases as $\delta \propto t^m$ where $m = (1/6)(\sqrt{1 + 2H_0^2M} - 1)$. The rate of growth slows down once the universe begins to approach an accelerated phase from the matter-dominated phase and comes to a halt when the universe starts accelerating. This behavior is similar to what happens in most models with late time acceleration of the universe. In the framework of tachyon cosmology, one can indeed construct viable models in which the growth of perturbation is very similar to that in $\Lambda$CDM models. These models are confined to a narrow range of parameters. For parameters lying outside this range, the perturbation growth is slower and should have higher amplitude in the past in order to maintain a given amplitude today and consequently the models are ruled out by cosmic microwave background observations.

We have shown that it is possible to construct viable models where tachyons contribute significantly to the energy density of the universe. Here we have considered models in which matter, radiation and tachyons coexist. It is shown that a subset of these satisfy the constraints on the accelerated expansion of the universe. However, for the accelerating phase to occur at the present epoch, it is necessary to fine tune the initial conditions. The density parameter for tachyons does not become negligible at high red-shifts, hence the growth of perturbations in non-relativistic matter is slower for most models than, e.g., the $\Lambda$CDM model. This problem does not exist in a small subset of models. Given that the density parameter of tachyons cannot be ignored in the matter-dominated era, it is essential to study the fate of fluctuations in the tachyon field.

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