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Scalar field dynamics on a brane

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Abstract. The dynamics of a flat isotropic brane Universe with two-component matter source – perfect fluid with the equation of state $p = (\gamma - 1)\rho$ and a scalar field with a power-law potential $V \sim \phi^{\alpha}$ is investigated. We describe solutions for which the scalar field energy density scales as a power-law of the scale factor. We also describe solutions existing in regions of the parameter space where these scaling solutions are unstable or do not exist.

Keywords. Brane; scalar field; scaling solution.

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In this report we describe solutions with scalar field energy density scales exactly as some power of the scale factor. The case of scalar field dominance on a brane have been described in [1]. Therefore we concentrate our effort to the fluid dominance case. Our work generalizes [1] to fluids with $\gamma \neq 4/3$.

We follow the method of [2], where a similar class of solutions has been found in the standard cosmology. Suppose that the scalar field energy density behaves as $\rho_\phi \sim a^{-n}$, where a is the scale factor. Since the Klein–Gordon equation for the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{1}$$

is not modified on a brane, this property, as in the standard case, requires that the ratio of a scalar field kinetic energy density and a total scalar field energy density remains constant [2]:

$$\frac{\dot{\phi}^2/2}{\rho_\phi} = \frac{n}{6}. \tag{2}$$

The perfect fluid with equation of state $p = (\gamma - 1)\rho$ has the energy density $\rho \sim a^{-m}$ where $m = 3\gamma$. In the non-standard brane regime the fluid dominance leads to

$$a \sim t^{1/m}. \tag{3}$$

All our considerations remain valid also for the case of the 'dark energy' $\mathcal{U}$ dominance (if $\mathcal{U} > 0$), because the 'dark energy' behaves as a 'normal' matter with $m = 2$. Using (1) and (2) we obtain

$$\phi = At^{1-n/2m}. \tag{4}$$

Then, solving for $V(\phi)$ we find the potentials which allow the scaling behavior in the non-standard brane regime

$$V(\phi) = A^{2-\alpha}\frac{2(3(\alpha-2)-\alpha m)}{\alpha m(\alpha-2)^2}\phi^{\alpha}, \tag{5}$$

where

$$\alpha = \frac{2n}{n-2m}. \tag{6}$$

It means that, as in the standard cosmology, scaling solutions exist for power-law potentials $V(\phi) \sim \phi^{\alpha}$. If α is given, the scalar field energy density is proportional to a^{-n}, where

$$n = \frac{2\alpha}{\alpha-2}m. \tag{7}$$

Though this expression differs only for the factor 2 in the numerator from its analog in the standard cosmology [2], the properties of brane scaling solutions are rather different. Indeed, the positivity of the potential in (5) requires that $\alpha(3-m) < 6$, which indicates for negative α that for $m > 3$ (a matter with a positive pressure) and

$$\alpha < \frac{6}{3-m} \tag{8}$$

the scaling solution does not exist. On the contrary, in the standard cosmology the scaling solution exists for an arbitrary negative α independently on m.

Consider now the case of positive α. The positivity of (5) leads to $\alpha(3-m) > 6$. It means that for $m > 3$ scaling solutions do not exist for any α. In particular, they do not exist for the case of $\gamma = 4/3$, investigated in [1]. On the other hand, for $m < 3$ the situation is similar to the standard cosmology – scaling solution exists for sufficiently steep potential with α satisfying in the brane case $\alpha > 6/(3-m)$.

Stability analysis carried out in [3] gives the following results. For $\alpha < 0$ the scaling solution is a stable attractor for all region of existence (8). And for $\alpha > 0$ there is an additional inequality

$$\alpha > \frac{2m+6}{3-m}\,. \tag{9}$$

Zones of stability of the scaling solutions on the plane (α, m) are plotted in figure 1.

It is also interesting, that in the case of negative α the scalar field energy density may fall more rapidly or more slowly than the fluid density, depending on the sign of $\alpha + 2$, whereas in the standard scaling solution with negative α the ratio of scalar field and fluid energy densities always increases. This was discovered in the particular case of radiation fluid in [4], where it was argued that this fact may be

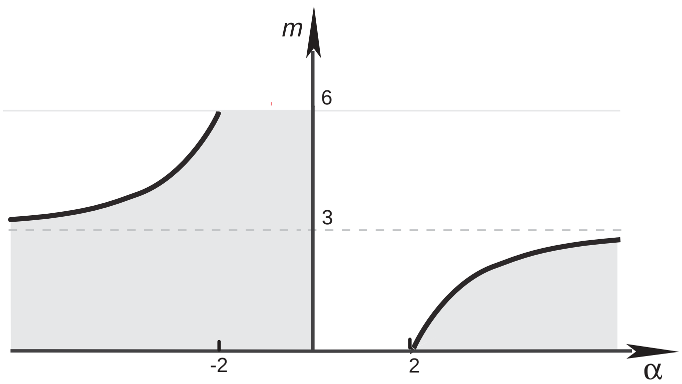

Figure 1. Stability zones of the scaling solutions (dark). In the white zone with negative α the kinetic-term dominated solution is stable whereas in white zone with positive α the scalar field infinitely oscillates.

important for quintessence scenario. In our report we confirm that this property do not depend on the state equation of the perfect fluid.

It is interesting also that in the particular case of $V(\phi) \sim \phi^{-2}$ there exists a tracker solution ($\rho_\phi \sim \rho$) which is a global attractor in the whole phase space (see [3] for details). This solution is an analog of a tracker solution in the standard cosmology existing for exponential scalar field potentials [5]. This fact and other similarities of dynamics with $V(\phi) \sim \phi^{-2}$ on a brane and $V(\phi) \sim \exp \lambda\phi$ in the standard cosmology (both potentials are bounding cases for inflation and chaotic behavior [6]) indicate that dynamics of a scalar field with inverse square potential on a brane shares many features with dynamics of a scalar field with exponential potential in the standard cosmology.

Now we briefly describe solutions, existing in the regions of the plane (α, m) where scaling solutions do not exist.

For positive α with even n we have infinite damping oscillations of the scalar field, similar to the standard case. Some asymptotic formulae for potentials $V = \frac{1}{2}m^2\phi^2$ and $\frac{1}{4}\lambda\phi^4$ have been given in [1].

For negative α the other type of solution exists. In this kinetic-term dominated solution the contribution of the potential for the energy density of the scalar field is negligible in comparison to the contribution of the kinetic term. Its explicit form is

$$\phi = \phi_0 \left(\frac{t}{t_0}\right)^{(m-3)/m}, \tag{10}$$

where ϕ_0 and t_0 are integration constants. This solution does not exist for $m \leq 3$. In this case the scaling solution is stable for all negative α. For $m = 4$ we recover the known behavior $\phi \sim t^{1/4}$ [1].

References

[1] S Mizuno, K-I Maeda and K Yamamoto, *Phys. Rev.* **D67**, 023516 (2003)
[2] A R Liddle and R J Scherrer, *Phys. Rev.* **D59**, 023509 (1999)
[3] N Yu Savchenko and A V Toporensky, *Scaling solutions on a brane*, gr-qc/0212104
[4] K-I Maeda, *Phys. Rev.* **D64**, 123525 (2001)
[5] E J Copeland, A R Liddle and D Wands, *Phys. Rev.* **D57**, 4686 (1998)
[6] A V Toporensky, P V Tretyakov and V O Ustiansky, *Astron. Lett.* **29**, 1 (2003)